Advances in Industrial Engineering, December 2025, 59(2): 215-240
DOI: 10.22059/aie.2025.391393.1940

RESEARCH PAPER

A Novel Model for the Synchromodal Hub Location Problem

Ali Ghorbani Farami Nezhad[1], **Mohammad Mahdi Nasiri**[2] **and Fatemeh Sabouhi**[3 *]

[1] *Ph.D. Candidate, Department of Industrial Engineering, Kish International Campus, University of Tehran, Tehran, Iran*
[2] *Professor, School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.*
[3] *Assistant Professor, School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.*

Received: 8 March 2025, Revised: 27 April 2025, Accepted: 29 April 2025

Abstract

With advancements in technology and growing customer demand, the optimal design of hub networks in distribution systems, along with flow synchronization across the entire network, play a critical role in reducing costs and enhancing overall efficiency. These networks significantly contribute to optimizing delivery times and improving responsiveness to customer needs, particularly in the transportation of time-sensitive goods. This study develops a mixed-integer linear programming model for the synchromodal hub location problem. We synchronize the flow throughout the entire network, which consists of origin points, a sender hub, a receiver hub, and demand points. To replicate real-world conditions, we also consider the synchronization of product flow within the distribution hub distribution networks. The proposed model aims to minimize the total cost, which includes transportation costs, operational costs across the distribution network, fixed costs of establishing Hubs and deploying vehicles, and potential penalties incurred as a result of failures to meet customer demand in terms of quantity and timeliness. To aggregate long-, mid-, and short-term decisions, we examine several decisions across different time periods. These decisions include the incomplete hub location problem, the service network design problem involving the scheduling of all network nodes, the synchronization of shipment flows in an intermodal transportation system, as well as integration and sorting operations on all components of the hub distribution network. The model's performance is assessed using data from an actual case study in the Iranian food industry. We conduct various sensitivity analyses on key parameters of the problem and present the numerical findings.

Keywords:
Synchromodal Hub,
Synchronization,
Scheduling,
Optimization Model,
Time Window.

Introduction

In today's world, with the rapid advancement of technology, the demand for goods and services worldwide is on the rise. Therefore, it is arguably essential to establish hub distribution networks that ensure maximum responsiveness to customer needs. Among the functions of hub distribution networks are collecting goods directly from origin points, consolidating and sorting

---

[*] Corresponding author: (Fatemeh Sabouhi)
Email: fatemehsabouhi@ut.ac.ir

them, transferring them among different hub distribution networks, and finally delivering them directly to demand points. Utilizing hub distribution networks in distribution systems can reduce the number of direct connections between nodes, thereby decreasing the complexity of the network. The hub location (HL) problem was introduced by O'Kelly [1], along with its initial mathematical model for the median p-hub problem. Since then, various subproblems have branched out from the main HL problem, including the single/multiple allocation median p-hub problems [2], hub covering problems, p-hub center problems, and uncapacitated HL problems [3]. Additionally, other types of problems have been developed by introducing various constraints, such as multi-objective p-hub problems, capacitated p- HL problems [4], hub-arc location problems [5], continuous p- HL problems [6], and maximum-profit HL problems [7]. In recent years, a novel type of hub distribution network, known as the intermodal hub distribution network, has been proposed [8]. In these networks, at least two distinct types of transportation modes are considered for the transfer of shipments along the network's edges. Examples of intermodal hub distribution networks include rail-road, sea-rail, and sea-road transportation systems.

In intermodal hub distribution networks, a.k.a. many-to-many distribution networks [9], origin-destination pairs require that facilities in the hub distribution network be allocated for transporting shipments from the origin point to the destination. For this purpose, a small transportation fleet is assigned the task of carrying shipments from the origin points to the origin hub, while after relevant consolidation operations at the origin hub, the shipments are transferred to the receiver hub by higher-capacity vehicles. At the destination hub, after further consolidation, shipments are dispatched to their final destinations by smaller vehicles. The difference between the vehicles used between the origin-hub and hub-destination edges compared to the hub-hub edges is not necessarily restricted to capacity, as the type of vehicles might also differ regardless of their capacity or size across different edges. Nevertheless, intermodal hub distribution networks have proven to be highly efficient, and numerous practical applications have been designed and proposed for this type of network, with Crainic et al [10], Caris et al [11], and Crainic et al [12] being prominent examples.

The issue of time and synchronization is also a critical and influential factor in the economies of scale within transportation networks. Moreover, time can serve as a suitable measure to evaluate service levels in HL problems. Although its role and influence are significant in hub system operations, time has not been extensively considered in the majority of studies on HL problems. Synchronization in a hub distribution network can be done in two main areas. The first is the synchronization of shipment flows within the transportation fleet, where the carriers (vehicles, data transfer tokens, etc.) are synchronized to achieve objectives such as maximizing service levels and minimizing cost and delivery times of shipments. The other type of synchronization occurs in consolidation operations within hubs. Synchronization thus plays a significant role in making the problem space closely resemble real-world scenarios. This effect can be extended to passenger transportation distribution networks and goods distribution networks (road-air, rail-sea, road-rail), in which establishing direct connections between each pair of demand and destination points is not feasible, and time and cost play a crucial role across the network. To sum up, the synchromodal hub distribution network extends the intermodal hub distribution network by synchronizing flows. Based on our review of the literature, no research in this field has yet examined or modeled hub distribution networks that integrate both types of synchronization.

In this research, a novel mathematical formulation is presented for the synchromodal HL problem with the primary objective of minimizing the total cost. In the modeled problem, the overall flow within the network is synchronized. To be precise, the overall flow consists of the transportation fleet's flows between origin points and sender hubs, sender hubs and receiver hubs, and receiver hubs and destination nodes, in addition to the flow of internal operations within the hubs. This study thus attempted to simultaneously synchronize the flow of shipments both within and outside the hubs, synchronize the transportation fleet, including at least two transportation modes, and incorporate these synchronizations into the design of the hub distribution network. Moreover, to aggregate long-, mid-, and short-term decisions, the proposed model considers decisions from the HL problem and service network design (ND) problems across different time periods. These decisions include scheduling the start and end times of activities at all network nodes, synchronizing shipment flows, and consolidating operations within the hub distribution network. Data from an actual case problem of the Iranian food industry supplying perishable products were applied to assess the performance of our model.

The remainder of this research is structured as follows: A comprehensive review of the literature on the HL problem is presented in the State of the Art section. The Problem Statement section introduces the mathematical model for the synchromodal HL problem. The Case Study section presents the numerical results of solving the mathematical model based on data obtained from the case study, followed by a sensitivity analysis on key parameters. Finally, the Conclusion section is dedicated to the conclusion and summary of this research, with a number of suggestions for future studies.

State of the Art

In location problems, hubs refer to specific points in a network that are used for connecting and establishing communication among other points. These central point’s create pathways for the quick and efficient transfer of shipments and information among network components. Numerous researchers have examined the HL problem from various standpoints. Here, some prominent examples of studies published on this subject are reviewed.

HL Problems

Recently, there has been a surge in the number of articles addressing the subfield of HL. Ullmert et al. [13] designed a traffic and communication network based on HL concepts. Mokhtarzadeh et al. [14] investigated the location-allocation problem for p-mobile hubs, considering the depreciation of facilities located in the hub. Wandelt et al. [15] presented a heuristic approach for HL in a contracted network due to the problem's complexity and the limited ability of exact methods in solving it on a large scale. Bütün et al. [16] studied the routing of goods and capacitated HL in a directed cycle network under congestion.

Kayışoğlu and Akgün [17] examined the multiple allocation tree problem to reduce the origin-destination transportation cost. Shang et al. [18] proposed a robust, hierarchical, cluster-oriented model for HL in urban and rural transportation systems. Monemi et al. [19] studied the distribution of humanitarian relief items among refugee camps from a ND perspective. Korani and Eydi [20] introduced a bi-level model and a Karush-Kuhn-Tucker penalty function approach for reliable HL problems. Stokkink and Geroliminis [21] investigated a hub-based collective transportation system aimed at attracting enough carriers to meet the high demand in small packages and mail sectors. Considering demand uncertainty, Li et al. [22] presented a HL model focusing on urban and rural logistics networks.

Rahmati et al. [23] investigated a two-stage HL model with uncertain demand aimed to maximize profit. The authors used advanced sample average approximation to determine the number of scenarios, along with the k-means and the self-organizing map to cluster the samples. Wu [24] analyzed the HL problem by examining multi-plant concepts to reduce costs. The concepts included decision-making on manufacturing plant locations, lot sizing, inventory management, HL, and multi-product distribution to customers. Li et al. [25] proposed a nonlinear model for HL accounting for carbon emission coefficients. To solve the developed model, the authors used an adaptive genetic algorithm. Wang et al. [26] developed several models for HL. The models addressed service levels, route capacities, direct access points, multi-modal transportation, and single-allocation incomplete hub graphs. Li et al. [27] presented a multi-modal hub distribution network to improve intercontinental transportation under the Belt and Road Initiative during the COVID-19 pandemic. Pourghader et al. [28] proposed a multi-objective model for HL in military equipment logistics under uncertainty. The objectives of their model included minimizing costs and route congestions, and maximizing demand fulfillment. The authors employed metaheuristic methods, such as the Grey Wolf Optimizer and Non-dominated Sorting Genetic Algorithm II, to solve the model for mid- and large-scale instances. However, the authors did not address synchronization and the element of time in their model, focusing primarily on the minimization of costs and congestions.

Time in HL Problems

Despite the critical role of time in hub distribution ND, many prior studies have not adequately addressed various aspects of this crucial factor. In this subsection, we examine some of the research on the role of time in HL problems.

Mohammadi et al. [29] proposed the reliable single-allocation capacitated p- HL problem and presented a bi-objective mathematical model subject to uncertainty in hubs and linking edges. The model's objectives included minimizing total cost and the maximum transportation time. Motamedi et al. [30] presented a scheduled HL problem with six different strategies for customer prioritization. Domínguez-Bravo et al. [31] dealt with the location-allocation problem in a hub distribution network, taking into account time-sensitive demand and network congestion. Musavi and Bozorgi-Amiri [32] examined the scheduling and sequencing of service to customer nodes by vehicles dispatched from intermediate hubs within a perishable food supply chain. Khaleghi and Eydi [33] provided a model to design a multi-period hub distribution network in a continuous time horizon. The model's objectives included maximizing both variable and fixed job opportunities and minimizing costs and pollutant emissions.

Zhang et al. [34] introduced a multi-modal HL problem with an incomplete connection graph and random values for customizing HL models in many-to-many transportation and distribution systems. The researchers examined different allocation scenarios and considered delivery time constraints. Considering linear time-dependent demand, Khaleghi et al. [35] presented a mathematical model to design a multi-period hub distribution network in a continuous time horizon. Roozkhosh et al. [36] explored the HL problem under both scheduled and real-time modes. However, the authors did not incorporate operation flow synchronization into their model, treating time only as the factor determining the departure of goods from hubs. Khalilzadeh et al. [37] developed a bi-objective mathematical model for HL with maximum coverage, aiming to minimize transportation times and environmental risks. However, the model did not account for the synchronization of network components.

As was observed, the foundation of the studies reviewed above lay in the HL problem. Although the authors have addressed the time factor, the primary focus appears to have been on determining the schedule and sequence of service provision rather than flow synchronization within the hub distribution network.

Synchronization in Distribution Networks

Flow synchronization in networks can be done over two dimensions: synchronization of shipment flows within the transportation fleet and synchronization of consolidation operations within the hub distribution network. Rupp et al. [38] worked on a general optimization problem to simplify shipment consolidation processes in the hub-input-output problem. The authors focused on scheduling the arrival of shipments to a distribution networks such as transshipment yards, airports, or train stations and coordinating incoming and outgoing vehicles. Anderluh et al. [39] synchronized distribution in a supply chain and demonstrated that spatial synchronization may be as important as temporal synchronization in distribution networks.

Karimi and Setak [40] integrated the scheduling of shipment flows into the incomplete HL problem. Masaeli et al. [41] studied the scheduling of shipment flows in the complete HL problem. However, these studies both overlooked two key aspects: the synchronization of operations within the hubs as part of the shipment transfer flow and the diversity of vehicles and their differences across various network levels and edges. Synchronization of distribution flows within the network, particularly in the context of planning and designing hub distribution networks, is a topic that requires further investigation.

Nolz et al. [42] dealt with synchronizing goods distribution at two levels of a supply chain, demonstrating that synchronization at both the first and second levels can affect the overall performance of the supply chain. Other areas where synchronization has been utilized include route design problems in transportation networks. For instance, Estrada et al. [43] focused on operational planning for transportation corridors, showing that proper planning and synchronization can lead to performance improvements and cost reductions. However, the latter two evidently cannot be classified as HL problems.

Elbert et al. [44] integrated the distribution network location problem with the service ND problem. Typically, the location problem is considered a strategic decision, while ND is tactical. The authors attempted to overcome this challenge through various analyses. Another category where synchronization has been considered includes transshipment problems, with cross-docking a particular case in point. Synchronizing inbound-outbound doors as well as consolidation operations at cross-docking yards is a significant concern for researchers in this field (e.g., Giusti et al. [45]; Crainic et al. [46]). Considering the synchronization of operations in cross-docking yards can contribute to targets like just-in-time logistics.

Guo et al. [47] relied on job synchronization in parallel machine scheduling problems to synchronize inbound-outbound doors at intermediate facilities in a transportation network. Crainic et al. [46] addressed the synchronized location-transshipment problem, which falls within the category of synchromodal logistics problems. Luo et al. [48] studied the synchronization of production and storage in a transshipment problem. Wu et al. [49] examined the service ND problem based on the temporal synchronization of inbound-outbound doors of intermediate facilities. The transportation network explored by the researchers could also fall within the category of transshipment problems. Although these studies have dealt with the synchronization of various operations, none of them could be categorized as a HL problem.

The model developed by Kara et al. [50] and later expanded by Tan and Kara [51], titled as the latest arrival HL problem, fits into the category of HL problems with deterministic delivery times. In this model, the delivery time of shipments to demand points determines how the transportation system is managed. Yaman et al. [52] presented another model based on the latest arrival HL problem by considering intermediate stops. However, like the two previous studies, Yaman et al. did not fully explore flow synchronization. Moreover, the starting point of the flow from the origin, which is the first determinant of flow in any part of the origin-destination network, was overlooked in all three of these studies.

Among recent works, Giusti et al. [53] introduced the synchronized multi-commodity and multi-service level HL problem. The authors presented scheduling in a periodic manner and used a space-time diagram to simplify the complex integration of synchronization and HL problems. Although Giusti et al. considered the start times of vehicles from network components, they disregarded the synchronization of flow inside distribution hubs and only employed the models proposed by Meraklı and Yaman [54] and Ebery et al. [55]. Ogazón et al. [56] presented a model for the time-definite hub-line location problem, which represented an innovative expansion of the hub line location problem. The main objective of the problem is to incorporate common delivery time constraints under real-world logistical conditions. The authors developed this model for a study in the field of healthcare. Consequently, the model focused on optimizing the processes of collection, transshipment, and delivery with the emphasis on precise scheduling. However, the proposed model failed to consider capacity constraints across the network, such as the number of vehicles, hub capacities, or the capacity of the entry and exit gates.

A classification of articles related to the present study is provided in Table 1.

Table 1. Classification of the literature relevant to this study

						Capacity			Minimum		Synchronize			
Reference	Cross docking	HL	Distribution network	Inter/multi-modal	Time factor	Facility	vehicle	Inbound door/ Outbound door	Number of distribution service	Number of vehicle	transportation	Facility	All of network	Multi-period
Anderluh et al [39]			✓		✓	✓	✓			✓	✓			
Guo P et al [47]			✓		✓	✓		✓				✓		
Luo H et al [48]	✓				✓							✓		
Giusti R et al [45]			✓			✓			✓			✓		✓
Crainic TG et al [46]	✓				✓	✓			✓		✓	✓		✓
Wu XB et al [49]			✓		✓			✓	✓		✓	✓	✓	
Rupp J et al [38]			✓		✓						✓	✓	✓	
Giusti R et al [53]		✓		✓	✓	✓	✓				✓		✓	✓
Khaleghi A et al [33]		✓			✓	✓								✓
Wang S et al [26]		✓		✓		✓			✓					
Ogazón et al [56]		✓			✓							✓		
This thesis		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Research Gaps

Based on the literature review, summarized in Table 1, the gaps in the literature on HL problems are outlined as follows:

- Although numerous research works have been conducted on HL, and the time factor has indeed been considered in some of these, problems like the latest arrival HL, which emphasizes the arrival time of the last vehicle to Hubs, or allowable time windows have not been incorporated into the proposed models in a precise and practical manner. In real-world scenarios, it is essential to consider the start and arrival times of vehicles for loading and unloading at each network to achieve coordination across the entire network.
- Thus far, researchers have only employed homogeneous vehicle fleets without capacity constraints throughout the network, and the use of intermodal fleets has rarely been considered. However, in practice, reputable transportation companies like DHL, UPS, and FedEx utilize various transportation modes for delivering different types of shipments.
- Lastly, the time factor in synchronizing consolidation operations within hubs, which is a key part of the flow synchronization process, has not been considered in existing models. In other words, researchers have thus far not considered incorporating an allowable limit for the number goods at inbound-outbound doors within an allowable time window for

consolidation operations into HL problems. Moreover, failure to synchronize the arrivals and departures of vehicles at hub points not only can cause delays across the entire system but also may lead to penalties for potential shortfalls in meeting customer demand. Similarly, lack of synchronization in consolidation operations within hubs can lead to or exacerbate internal disorganization in hubs and further prolong the delivery times of shipments from origin to destination.

Therefore, based on our review of the literature, no study has comprehensively investigated the issue of flow synchronization both across the network as a whole and within its components in HL problems. This research may thus be the first of its kind to comprehensively examine the synchronization of shipment flows in the transportation fleet and the synchronization of consolidation operations within hubs in intermodal transportation networks while introducing a novel variant of the problem termed the Synchromodal HL Problem. To this end, we present a mathematical model that encompasses long-term HL decisions, as well as allocation, flow synchronization, and scheduling as mid- and short-term decisions. The applicability of the proposed model will also be evaluated through a case study in Iran.

Problem Statement

Synchronized distribution networks have found widespread applications, especially in industries where time plays a key role, such as companies specializing in the transportation of perishable goods. In these sectors, it is crucial to deliver products at the specified time to preserve their quality. In the problem explored in this study, assume a transportation company handling perishable food products is in the process of implementing a comprehensive strategic, tactical, and operational planning project for all potential network nodes. A distribution network typically consists of origin points that act as suppliers, distribution points that handle the collection and distribution of goods, and demand points.

The problem addresses the following tactical and operational decisions:

- The number and type of transportation vehicles at each network node;
- Allocating nodes to their respective Hubs in each time period;
- Scheduling network component activities; and
- The capacity of the transportation fleet and Hubs in each time period.

It is thus essential to consider the time factor in all network components and synchronize the entire network. On the other hand, considering the capacity of network components alongside the temporal synchronization of operations is crucial in real-world contexts. In view of the various strategic and operational dimensions of the problem, we divide the multimodal transportation distribution network into three segments:

- From origin points to the Sender hub with direct shipment and a single-allocation policy.
- Between the sender and receiver hubs with different transportation vehicles from both the previous and next segments in each time period.
- From the receiver hub to demand points, where the transportation mode changes compared to the second segment. For example, it may switch from rail to road, which involve entirely different vehicles with varying capacities and costs.

All Sender and receiver hubs have separate inbound and outbound doors, and each door can have different capacities such that the combined capacity of the doors does not exceed the hub's total capacity. Creating a synchronized schedule for receiving, loading, and unloading goods at each segment of the network is part of the strategic and operational design of the network. As can be observed, time is a critically important factor, as the synchronization of shipment flows, network components, and the transportation fleet would make no sense without considering the role of time.

The problem is formulated under the following assumptions:

- Origin points and destination points (a.k.a. demand points) are fixed and specified.
- Goods demanded in each time period can be sourced from different origins.
- The network follows a single-allocation distribution policy.
- No storage is allowed in Hubs.
- A deadline is set at each destination for meeting customer demand.
- Connections between hubs in the network follow an incomplete graph to prevent network overhead costs from increasing.
- Each network component has its own specific time windows in each period.
- Each time period is distinct from the others.
- The shipment delivery policy is direct and no vehicle routing is considered in the problem.

The structure of the synchromodal HL and allocation problem is illustrated in Figure 1.

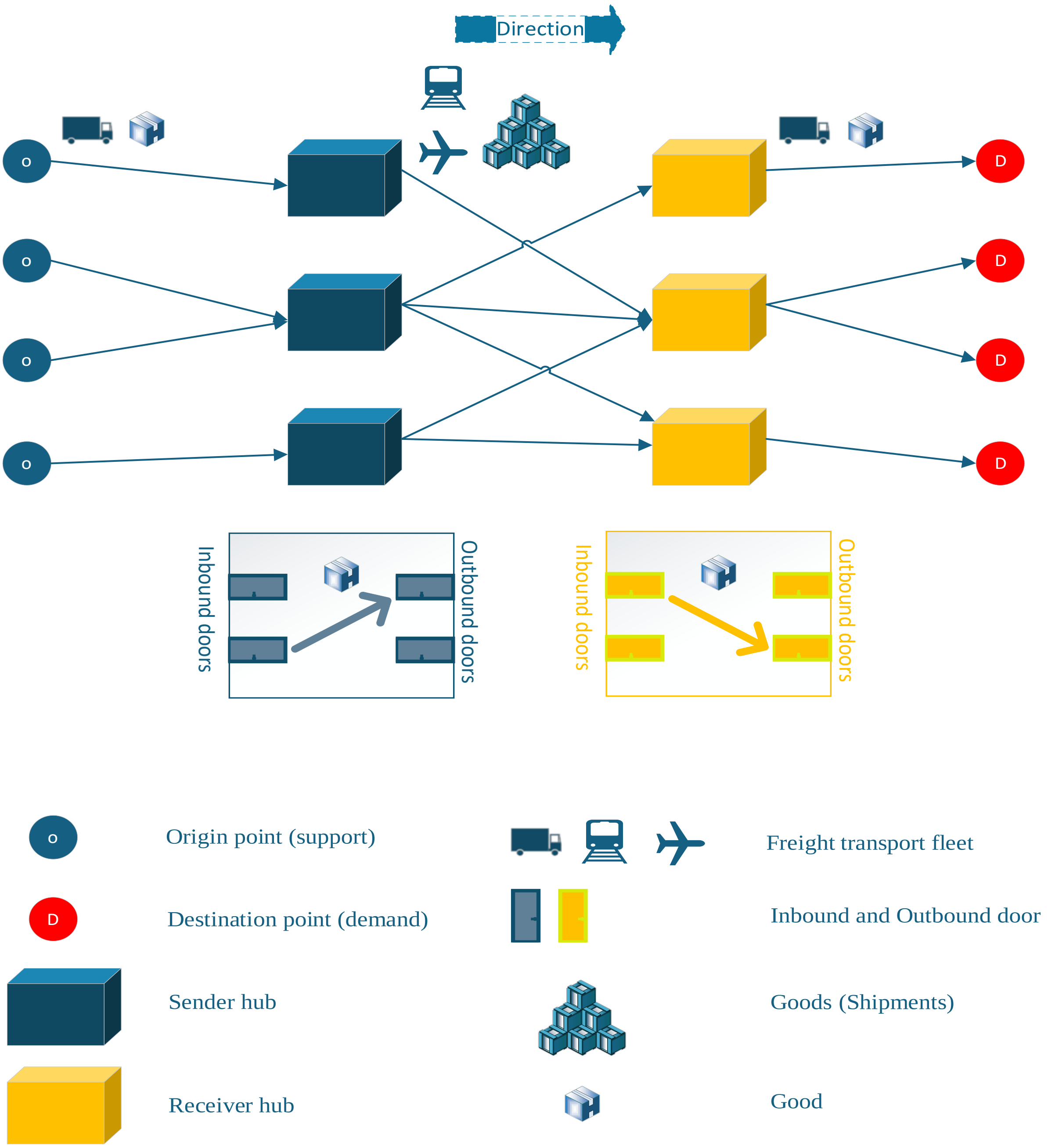

Figure 1. Structure of the synchromodal HL and allocation problem.

Symbols and Indices

The following notations are employed in the problem formulation.

Indices and Sets

O	Set of origin points $o \in O$
D	Set of destination points $d \in D$
K	Set of candidate points for establishing sender hubs $k \in K$
L	Set of candidate points for establishing receiver hubs $l \in L$
U	Set of vehicles operating on the routes between origin points and sender hubs $u \in U$
V	Set of vehicles operating on the routes between sender hubs and receiver hubs $v \in V$
W	Set of vehicles operating on the routes between receiver hubs and destination points $w \in W$
I	Set of inbound doors (IDs) of the sender hub $I_k \in k$
J	Set of outbound doors of the sender hub $J_k \in k$
E	Set of IDs of the receiver hub $E_l \in l$
N	Set of outbound doors of the receiver hub $N_l \in l$
S	Set of time periods $S \in s$

Parameters

TC_{okus}	The transportation cost of shipping one unit of cargo from origin o to sender hub k by vehicle u during period s
TC_{klvs}	The transportation cost of shipping one unit of cargo from sender hub k to receiver hub l by vehicle v during period s
TC_{ldws}	The transportation cost of shipping one unit of cargo from receiver hub l to destination d by vehicle w during period s
CC_{ijks}	The transportation cost of shipping cargo from ID i to outbound door j of sender hub k during period s
CC_{enls}	The transportation cost of shipping cargo from ID e to outbound door n of receiver hub l during period s
HC_{ks}	capacity of sender hub k during period s
HC_{ls}	capacity of receiver hub l during period s
IC_{iks}	capacity of ID i at the sending hub k during period s
OC_{jks}	capacity of ID j at the sending hub k during period s
IC_{els}	capacity of ID e at the receiver hub l during period s
OC_{nls}	capacity of outbound door n at the receiver hub l during period s
SW_{iks}	The service start time at ID i of the sending hub k during period s
EW_{iks}	The service end time at ID i of sending hub k during period s
SW_{jks}	The service start time at outbound door j of receiver hub k during period s
EW_{jks}	The service end time at outbound door j of sending hub k during period s
SW_{els}	The service start time at ID e of receiver hub l during period s
EW_{els}	The service end time at ID e of receiver hub l during period s
SW_{nls}	The service start time at outbound door n of receiver hub l during period s
EW_{nls}	The service end time at outbound door n of receiver hub l during period s
VC_v	Capacity of vehicle v
VC_u	Capacity of vehicle u
VC_w	Capacity of vehicle w
ES_{okus}	The earliest departure time of vehicle u from origin o to sending hub k during period s
LS_{okus}	The latest departure time of vehicle u from origin o to sending hub k during period s
ES_{klvs}	The earliest departure time of vehicle v from sending hub k to receiver hub l during period s
LS_{klvs}	The latest departure time of vehicle v from sending hub k to the receiver hub l during period s
ES_{ldws}	The earliest departure time of vehicle w from the receiver l to destination d during period s
LS_{ldws}	The latest departure time of vehicle w from the receiver hub l to destination d during period s
ER_{os}	The earliest time the shipment is ready for dispatch from origin o during period s
EA_{ds}	The earliest time the shipment is received at destination d during period s
LA_{ds}	The latest time the shipment is received at destination d during period s
TT_{okus}	The travel time from origin o and sending hub k by vehicle u during period s
TT_{klvs}	The travel time from sending hub k and receiver hub l by vehicle v during period s
TT_{ldws}	The travel time from receiver hub l and destination d by vehicle w during period s
Dm_{ds}	Expected demand at destination d during period s

Sp_{os}	The number of shipments available at origin o during period s
FH_k	The fixed cost of establishing the sender hub k
FH_l	The fixed cost of establishing the receiver hub l
Pq_{ds}	The penalty cost due to not meeting the demand at destination d during period s
Pt_{ds}	The penalty cost due to delay in delivering the demand at destination d during period s
DT_{ds}	The maximum desired time for receiving shipments at destination d during period s
CT_{ijk}	The transport time of one unit of shipment from ID i to outbound door j at sending hub k
CT_{enl}	The transport time of one unit of shipment from ID e to outbound door n at receiver hub l
UT_u	The time needed to unload one unit of shipment from vehicle u
LT_u	The time needed to load one unit of shipment to vehicle u
UT_v	The time needed to unload one unit of shipment from vehicle v
LT_v	The time needed to load one unit of shipment to vehicle v
UT_w	The time needed to unload one unit of shipment from vehicle w
LT_w	The time needed to load one unit of shipment to vehicle w
FV_u	The fixed cost of using vehicle u
FV_v	The fixed cost of using vehicle v
FV_w	The fixed cost of using vehicle w
ϕ	A large number

Decision variables

x_k	Equals 1 if a sender hub is establishing at point k; 0, otherwise
x_l	Equals 1 if a receiver hub is establishing at point l; 0, otherwise
a_{oks}	Equals 1 if the origin point o is assigned to the sender hub k during period s; 0, otherwise
a_{dls}	Equals 1 if the destination point d is assigned to the receiver hub l during period s; 0, otherwise
η_{kls}	Equals 1 if sender hub k is allocated to receiver hub l during period s; 0, otherwise
b_{okus}	Equals 1 if vehicle u is allocated to the origin point o during period s for dispatch to the sender hub k; 0, otherwise
b_{klvs}	Equals 1 if vehicle v is allocated to the sender hub point k during period s for dispatch to the receiver hub l; 0, otherwise
b_{ldws}	Equals 1 if vehicle w is allocated to the receiver hub point l during period s for dispatch to destination d; 0, otherwise
λ_{ikus}	Equals 1 if the ID i of the sender hub k is assigned to the incoming vehicle u during period s; 0, otherwise
λ_{jkvs}	Equals 1 if the outbound door j of the sender hub k is assigned to the incoming vehicle v during period s; 0, otherwise
λ_{elvs}	Equals 1 if the ID e of the receiver hub l is assigned to the incoming vehicle v during period s; 0, otherwise
λ_{nlws}	Equals 1 if the outbound door n of the receiver hub l is assigned to the incoming vehicle w during period s; 0, otherwise
σ_{ijks}	Equals 1 if a shipment is sent from ID i of the sender hub k to outbound door j of the same hub during period s; 0, otherwise
σ_{enls}	Equals 1 if a shipment is sent from ID e of the sender hub l to outbound door n of the same hub during period s; 0, otherwise
Q_{okus}	The number of shipments sent from origin o to the sender hub k by vehicle u during period s
Q_{klvs}	The number of shipments sent from sender hub k to the receiver hub l by vehicle v during period s
Q_{ldws}	The number of shipments sent from receiver hub l to the destination d by vehicle w during period s
q_{ijks}	The number of shipments sent from ID i to outbound door j of sender hub k during period s
q_{enls}	The number of shipments sent from ID e to outbound door n of sender hub l during period s
β_{okus}	The start time of vehicle u from origin o to sender hub k during period s
β_{klvs}	The start time of vehicle v from sender hub k to receiver hub l during period s
β_{ldws}	The start time of vehicle w from receiver hub l to destination d during period s
A_{ikus}	The reaching time of vehicle u at ID i of sender hub k during period s
A_{elvs}	The reaching time of vehicle v at ID e of receiver hub l during period s
A_{ldws}	The reaching time of vehicle w at the destination point d from receiver hub l during period s
θ_{ikus}	The time availability of shipments at ID i of sender hub k after being unloaded from vehicle u during period s
θ_{elvs}	The time availability of shipments at ID e of receiver hub l after being unloaded from vehicle v during period s

Mathematical Modeling

$$\begin{aligned}
Min\ Z = {} & \sum_{k=1}^{K} FH_k x_k + \sum_{l=1}^{L} FH_l x_l + \sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{o=1}^{O}\sum_{u=1}^{U} FV_u b_{uoks} + \sum_{v=1}^{V}\sum_{s=1}^{S}\sum_{l=1}^{L}\sum_{k=1}^{K} FV_v b_{vkls} \\
& + \sum_{w=1}^{W}\sum_{s=1}^{S}\sum_{d=1}^{D}\sum_{l=1}^{L} FV_w b_{wlds} + \sum_{u=1}^{U}\sum_{k=1}^{K}\sum_{o=1}^{O}\sum_{s=1}^{S} TC_{okus} Q_{okus} + \sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{j=1}^{J} CC_{ijks} q_{ijks} \\
& + \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{v=1}^{V}\sum_{s=1}^{S} TC_{klvs} Q_{klvs} + \sum_{e=1}^{E}\sum_{n=1}^{N}\sum_{l=1}^{L}\sum_{s=1}^{S} CC_{enls} q_{enls} + \sum_{s=1}^{S}\sum_{d=1}^{D}\sum_{w=1}^{W}\sum_{l=1}^{L} TC_{ldws} Q_{ldws} \\
& + \sum_{d=1}^{D}\sum_{s=1}^{S} Pq_{ds} \,.\, max\left(\left(Dm_{ds} - \sum_{w=1}^{W}\sum_{l=1}^{L} Q_{ldws}\right), 0\right) + \sum_{d=1}^{D}\sum_{s=1}^{S} Pt_{ds} \,.\, max\left(\left(\sum_{w} A_{ldws} - DT_{ds}\right), 0\right)
\end{aligned} \tag{1}$$

S.To:

$$\sum_{k=1}^{K} a_{oks} = 1 \qquad \forall s, o \tag{2}$$

$$\sum_{l=1}^{L} a_{dls} = 1 \qquad \forall d, s \tag{3}$$

$$a_{oks} \le x_k \qquad \forall o, k, s \tag{4}$$

$$\eta_{kls} \le x_k \qquad \forall k, l \tag{5}$$

$$\eta_{kls} \le x_l \qquad \forall k, l \tag{6}$$

$$a_{dls} \le x_l \qquad \forall l, d, s \tag{7}$$

$$\sum_{u=1}^{U}\sum_{k=1}^{K} Q_{okus} = Sp_{os} \qquad \forall\ s, o \tag{8}$$

$$\sum_{l=1}^{L}\sum_{d=1}^{D}\sum_{w=1}^{W} Q_{ldws} \le \sum_{o=1}^{O} Sp_{os} \qquad \forall s \tag{9}$$

$$Q_{okus} \le VC_u b_{okus} \qquad \forall s, k, u, o \tag{10}$$

$$Q_{klvs} \le VC_v b_{klvs} \qquad \forall s, v, l, k \tag{11}$$

$$Q_{ldws} \le VC_w b_{ldws} \qquad \forall s, d, w, l \tag{12}$$

$$Q_{ldws} \le VC_w a_{dl} \qquad \forall s, w, d, l \tag{13}$$

$$\sum_{o=1}^{O}\sum_{k=1}^{K} b_{okus} \le 1 \qquad \forall s, u \tag{14}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} b_{klvs} \le 1 \qquad \forall v, s \tag{15}$$

$$\sum_{l=1}^{L}\sum_{d=1}^{D} b_{ldws} \le 1 \qquad \forall w, s \tag{16}$$

$$\sum_{o=1}^{O}\sum_{u=1}^{U} Q_{okus} \le HC_{ks} x_k \qquad \forall s, k \tag{17}$$

$$\sum_{k=1}^{K}\sum_{v=1}^{V} Q_{klvs} \le HC_{ls} x_l \qquad \forall s, l \tag{18}$$

$$b_{okus} \le a_{oks} \qquad \forall u, s, o, k \tag{19}$$

$$b_{ldws} \le a_{dls} \qquad \forall l, s, d, w \tag{20}$$

$$b_{ldws} \le a_{dls} \qquad \forall w, s, d, l \tag{21}$$

$$\beta_{okus} \ge ER_{os} + LT_u - \phi(1 - b_{okus}) \qquad \forall s, o, u, k \tag{22}$$

$$\beta_{okus} \ge ES_{okus} - \phi(1 - b_{okus}) \qquad \forall o, k, u, s \tag{23}$$

$$\beta_{okus} \le LS_{okus} + \phi(1 - b_{okus}) \qquad \forall o, k, u, s \tag{24}$$

$$\beta_{okus} + TT_{okus} - \phi(2 - b_{okus} - \lambda_{ikus}) \le A_{ikus} \qquad \forall i, o, k, u, s \tag{25}$$

$$\beta_{klvs} \ge SW_{jks} - \phi\big(2 - \lambda_{jkvs} - b_{klvs}\big) \qquad \forall v, j, l, k, s \tag{26}$$

$$\beta_{klvs} \le EW_{jks} + \phi\big(2 - \lambda_{jkvs} - b_{klvs}\big) \qquad \forall s, j, v, l, k \tag{27}$$

$$\beta_{klvs} \ge ES_{klvs} - \phi(1 - b_{klvs}) \qquad \forall j, l, k, v, s \tag{28}$$

$$\beta_{klvs} \leq LS_{klvs} + \phi(1 - b_{klvs}) \qquad \forall v,j,l,k,s \tag{29}$$
$$\beta_{klvs} + TT_{klvs} - \phi(2 - b_{klvs} - \lambda_{elvs}) \leq A_{elvs} \qquad \forall e,s,v,k,l \tag{30}$$
$$\beta_{ldws} \geq ES_{ldws} - \phi(1 - b_{ldws}) \qquad \forall w,d,s,l \tag{31}$$
$$\beta_{ldws} \leq LS_{ldws} + \phi(1 - b_{ldws}) \qquad \forall d,s,l,w \tag{32}$$
$$\beta_{ldws} \geq SW_{nls} - \phi(2 - \lambda_{nlws} - b_{ldws}) \qquad \forall s,w,d,n,l \tag{33}$$
$$\beta_{ldws} \leq EW_{nls} + \phi(2 - \lambda_{nlws} - b_{ldws}) \qquad \forall l,w,d,s,n \tag{34}$$
$$\beta_{ldws} + TT_{ldws} - \phi(1 - b_{ldws}) \leq A_{ldws} \qquad \forall l,d,w,s \tag{35}$$
$$\sum_{i=1}^{I} \lambda_{ikus} \leq x_k \qquad \forall s,u,k \tag{36}$$
$$\sum_{e=1}^{E} \lambda_{elvs} \leq x_l \qquad \forall v,l,s \tag{37}$$
$$\sum_{j=1}^{J} \lambda_{jkvs} \leq x_k \qquad \forall k,v,s \tag{38}$$
$$\sum_{n=1}^{N} \lambda_{nlws} \leq x_l \qquad \forall l,w,s \tag{39}$$
$$\sum_{k=1}^{K} \sum_{i=1}^{I} \lambda_{ikus} \leq 1 \qquad \forall s,u \tag{40}$$
$$\sum_{l=1}^{L} \sum_{e=1}^{E} \lambda_{elvs} \leq 1 \qquad \forall s,v \tag{41}$$
$$\sum_{j=1}^{J} \sum_{k=1}^{K} \lambda_{jkvs} \leq 1 \qquad \forall s,v \tag{42}$$
$$\sum_{n=1}^{N} \sum_{l=1}^{L} \lambda_{nlws} \leq 1 \qquad \forall w,s \tag{43}$$
$$\lambda_{ikus} \leq \sum_{o=1}^{O} b_{okus} \qquad \forall s,u,k,i \tag{44}$$
$$\sum_{i=1}^{I} \lambda_{ikus} \leq \sum_{o=1}^{O} b_{okus} \qquad \forall k,u,s \tag{45}$$
$$\sum_{j=1}^{J} \lambda_{jkvs} \leq b_{klvs} \qquad \forall v,l,k,s \tag{46}$$
$$\sum_{j=1}^{J} \lambda_{jkvs} \leq \sum_{l=1}^{L} b_{klvs} \qquad \forall v,s,k \tag{47}$$
$$\lambda_{elvs} \leq \sum_{k=1}^{k} b_{klvs} \qquad \forall s,e,v,l \tag{48}$$
$$\sum_{e=1}^{E} \lambda_{elvs} \leq \sum_{k=1}^{k} b_{klvs} \qquad \forall v,s,l \tag{49}$$
$$\sum_{n=1}^{N} \lambda_{nlws} \leq \sum_{d=1}^{D} b_{ldws} \qquad \forall w,l,s \tag{50}$$
$$\lambda_{nlws} \leq \sum_{d=1}^{D} b_{ldws} \qquad \forall w,n,l,s \tag{51}$$
$$A_{ikus} \leq \phi\lambda_{ikus} \qquad \forall i,k,s,u \tag{52}$$
$$A_{elvs} \leq \phi\lambda_{elvs} \qquad \forall s,e,v,l \tag{53}$$
$$A_{ikus} \geq SW_{iks} - \phi(1 - \lambda_{ikus}) \qquad \forall s,k,u,i \tag{54}$$
$$A_{ikus} + UT_u \leq EW_{iks} + \phi(1 - \lambda_{ikus}) \qquad \forall i,k,u,s \tag{55}$$
$$A_{ikus} + UT_u - \phi(1 - \lambda_{ikus}) \leq \theta_{ikus} \qquad \forall u,i,k,s \tag{56}$$

$$A_{elvs} \geq SW_{els} - \phi(1 - \lambda_{elvs}) \qquad \forall s,l,v,e \tag{57}$$

$$A_{elvs} + UT_v \leq EW_{els} + \phi(1 - \lambda_{elvs}) \qquad \forall v,l,e,s \tag{58}$$

$$A_{elvs} + UT_v - \phi(1 - \lambda_{elvs}) \leq \theta_{elvs} \qquad \forall e,l,v,s \tag{59}$$

$$\theta_{ikus} \leq \phi\lambda_{ikus} \qquad \forall k,u,i,s \tag{60}$$

$$\theta_{ikus} \leq \phi \sum_{j=1}^{J} \sigma_{ijks} \qquad \forall u,i,k,s \tag{61}$$

$$\theta_{elvs} \leq \phi\lambda_{elvs} \qquad \forall s,v,e,l \tag{62}$$

$$\theta_{elvs} \leq \phi \sum_{n=1}^{N} \sigma_{enls} \qquad \forall v,e,l,s \tag{63}$$

$$\theta_{ikus} + CT_{ijk} \geq SW_{jks} - \phi\left(2 - \lambda_{ikus} - \sigma_{ijks}\right) \qquad \forall s,u,i,j,k \tag{64}$$

$$\theta_{ikus} + CT_{ijk} \leq EW_{jks} + \phi\left(2 - \lambda_{ikus} - \sigma_{ijks}\right) \qquad \forall s,u,i,j,k \tag{65}$$

$$\theta_{elvs} + CT_{enl} \geq SW_{nls} - \phi(2 - \lambda_{elvs} - \sigma_{enls}) \qquad \forall s,v,n,e,l \tag{66}$$

$$\theta_{elvs} + CT_{enl} \leq EW_{nls} + \phi(2 - \lambda_{elvs} - \sigma_{enls}) \qquad \forall s,v,n,e,l \tag{67}$$

$$\sum_{j=1}^{J} q_{ijks} = \sum_{o=1}^{O}\sum_{u=1}^{U} Q_{okus}\lambda_{ikus} \qquad \forall s,i,k \tag{68}$$

$$\sum_{n=1}^{N} q_{enls} = \sum_{k=1}^{K}\sum_{v=1}^{V} Q_{klvs}\lambda_{elvs} \qquad \forall s,e,l \tag{69}$$

$$\beta_{klvs} \leq \phi b_{klvs} \qquad \forall l,k,s,v \tag{70}$$

$$\beta_{klvs} \leq \phi \sum_{j=1}^{J} \lambda_{jkvs} \qquad \forall v,s,k,l \tag{71}$$

$$\beta_{ldws} \leq \phi b_{ldws} \qquad \forall s,d,w,l \tag{72}$$

$$\beta_{ldws} \leq \phi \sum_{n=1}^{N} \lambda_{nlws} \qquad \forall s,d,w,l \tag{73}$$

$$\sum_{u=1}^{U}\sum_{o=1}^{O} Q_{okus}\lambda_{ikus} \leq IC_{iks} \qquad \forall k,i,s \tag{74}$$

$$\sum_{v=1}^{V}\sum_{k=1}^{K} Q_{klvs}\lambda_{elvs} \leq IC_{els} \qquad \forall s,l,e \tag{75}$$

$$\sum_{i=1}^{I} q_{ijks} \leq OC_{jks} \qquad \forall j,k,s \tag{76}$$

$$\sum_{e=1}^{E} q_{enls} \leq OC_{nls} \qquad \forall n,l,s \tag{77}$$

$$\sum_{l=1}^{L}\sum_{v=1}^{V} Q_{klvs}\lambda_{jkvs} = \sum_{i=1}^{I} q_{ijks} \qquad \forall k,j,s \tag{78}$$

$$\sum_{d=1}^{D}\sum_{w=1}^{W} Q_{ldws}\lambda_{nlws} = \sum_{e=1}^{E} q_{enls} \qquad \forall l,n,s \tag{79}$$

$$\sigma_{ijks} \leq q_{ijks} \qquad \forall j,i,s,k \tag{80}$$

$$\sigma_{enls} \leq q_{enls} \qquad \forall s,e,n,l \tag{81}$$

$$q_{ijks} \leq HC_{ks}\sigma_{ijks} \qquad \forall k,j,i,s \tag{82}$$

$$q_{enls} \leq HC_{ls}\sigma_{enls} \qquad \forall s,e,n,l \tag{83}$$

$$\theta_{ikus} + CT_{ijk} + LT_v - \phi\left(4 - \lambda_{ikus} - \sigma_{ijks} - \lambda_{jkvs} - b_{klvs}\right) \leq \beta_{klvs} \qquad \forall i,s,v,j,l,k,u \tag{84}$$

$$\theta_{elvs} + CT_{enl} + LT_w - \phi(4 - \lambda_{elvs} - \sigma_{enls} - \lambda_{nlws} - b_{ldws}) \leq \beta_{ldws} \qquad \forall w,v,s,d,e,n,l \tag{85}$$

$$\sum_{v=1}^{V} \lambda_{jkvs} \leq \sum_{i=1}^{I} q_{ijks} \qquad \forall k,j,s \tag{86}$$

$$\sum_{w=1}^{W} \lambda_{nlws} \leq \sum_{e=1}^{E} q_{enls} \qquad \forall s,n,l \qquad (87)$$

$$a_{dls} \leq \sum_{w=1}^{W} Q_{ldws} \qquad \forall l,s,d \qquad (88)$$

$$A_{ldws} \leq \phi b_{ldws} \qquad \forall d,w,l,s \qquad (89)$$

$$A_{ldws} \geq EA_{ds} - \phi(1 - b_{ldws}) \qquad \forall s,w,d,l \qquad (90)$$

$$A_{ldws} \leq LA_{ds} + \phi(1 - b_{ldws}) \qquad \forall l,d,w,s \qquad (91)$$

$$x_k, x_l, b_{uos}, b_{vks}, b_{wlds}, a_{oks}, a_{dls}, \eta_{kl}, \lambda_{ikus}, \lambda_{jkvs}, \lambda_{elvs}, \lambda_{nlws} \in \{0,1\} \qquad \forall v,s,o,u,l,k,w,d,i,j,e,n \qquad (92)$$

$$Q_{okus}, Q_{klvs}, Q_{ldws}, q_{ijks}, q_{enls}, \beta_{okus}, \beta_{klvs}, \beta_{ldws}, A_{ikus}, A_{elvs}, A_{ldws}, \theta_{ikus}, \theta_{elvs} \geq 0 \qquad \forall v,s,o,u,l,k,w,d,i,j,e,n \qquad (93)$$

Equation (1) aims to minimize total cost, which includes location and setup costs for sender and receiver hubs, costs of securing and operating different types of vehicles, transportation costs from origin points to origin hubs, from origin hubs to destination hubs, and finally from destination hubs to destination points, as well as consolidation costs between the inbound-outbound doors of origin and destination hubs. Constraints (2) and (3) guarantee that each origin or destination point is assigned to exactly one origin and one destination hub, respectively. Constraints (4) to (7) specify that an origin or destination point can only be connected to an origin or destination hub if that hub has already been established. Constraint (8) guarantees that all shipments at origin points are transferred within the hub distribution network. Constraint (9) ensures that the shipments transferred to any destination do not exceed the demand at those destinations in each period. Constraints (10) to (12) define the maximum capacity of different types of vehicles. Constraint (13) indicates that the load carried by each vehicle must not exceed its capacity.

Constraints (14) to (16) guarantee that each vehicle can be allocated to at most one origin point for loading, one route between the origin and the secondary hub, or one route between the destination hub and the demand point within each time period in each part of the network. Constraints (17) and (18) ensure that the total quantity of shipments transported by vehicles to an origin or destination hub does not exceed the operational capacity of the that hub in that time period. Constraints (19) to (21) guarantee that a vehicle can be allocated to transport shipments from one point in the network to another only if a connection exists between the two points. The time window constraints are expressed by Constraints (22) to (25).

Constraints (26) to (29) guarantee the time a vehicle is dispatched to the outbound door of an origin hub based on the respective time window. Constraint (30) determines the reaching time of a vehicle on the route connecting the origin and destination hubs. Constraints (31) to (34) ensure the time window for dispatching vehicles to the outbound door of the destination hub and the demand point. Constraint (35) calculates the reaching time of a vehicle from the destination hub to the demand point.

Constraints (36) to (39) ensure that a hub's inbound or outbound doors can only be allocated to a vehicle if that hub has been opened as an origin or destination hub. Constraints (40) to (43) guarantee that in one time period, each vehicle can be assigned to at most one inbound or outbound door. Constraints (44) to (51) ensure that the inbound-outbound doors of established hubs can only be assigned to a vehicle if that vehicle has been assigned to that route for that time period.

Constraints (52) and (53) ensure the relationship between the reaching time of a vehicle at the intended destination hub in each part of the network and the assignment of that vehicle to an ID of the hub. Constraints (54) to (56) ensure that the reaching time of a vehicle at the origin hub is within the allowed time window for that hub. Constraints (57) to (59) ensure that the reaching time of a vehicle at the destination hub in each time period must be within the allowed time window of its ID.

Constraints (60) and (61) express the relationship between the availability time of goods for operations within the hubs and the assignment of vehicles to them and its ID. Constraints (62) and (63) refer to the connection between the inbound and outbound doors of the destination hub and the vehicle assigned to its ID. Constraints (64) and (65) show the availability time of shipments at an outbound door of the origin hub based on the time window. Constraints (66) and (67) ensure that the delivery time of shipments to the destination hub's outbound is within the allowed operational time window.

Constraints (68) and (69) express the total quantity of consolidated shipments moving between the inbound and outbound doors of the distribution hubs, which must not exceed the total quantity of shipments delivered to the respective IDs. Constraints (70) to (73) define the relationship between the dispatch time of vehicles and their assignment to designated routes within the network.

Constraints (74) and (75) determine the operational capacity of the established hubs in such a way that the total load entering the hub's IDs should not exceed the maximum capacity of each ID. Constraints (76) and (77) state that the number of shipments transported from the IDs of one hub to the outbound doors of another should not exceed the operational capacity of the outbound doors. Constraints (78) and (79) guarantee that the quantity of shipments dispatched from the outbound doors of distribution hubs to other network components does not exceed the quantity of shipments delivered to those outbound doors. Constraints (80) and (81) ensure that the assignment of inbound-outbound doors to each other only occurs when there are shipments to be moved between them.

Constraints (82) and (83) define the relationship between the allowable quantity of shipments to move through each hub door and the total capacity of that hub. Constraints (84) and (85) ensure that the sequence of shipment transportation operations occurs after the loading time of those shipments. Constraints (86) and (87) define the assignment of vehicles to the outbound doors of hubs.

Constraint (88) determines the relationship between the assignment of an origin hub to a destination hub and the quantity of shipments moving between them. Constraints (89) to (91) indicate the allowed time window for a vehicle to arrive at a demand point. Finally, Constraints (92) and (93) indicate the type of decision variables.

Case Study

For the purposes of this research, a distribution network for Iranian food products is used as a case study.

In this network, seven origin points for dispatching and eight destination points are defined for one type of product. There are three candidate points for the location of origin hubs and three for destination hubs. In this network, shipment refers to goods that should be delivered to target markets in the southern regions of Iran within 48 hours. Given the number of origin points (suppliers of goods) and differences in the production levels and the level of supply support, the origin and destination hubs are utilized on two sides of the network. This approach also ensures the implementation of economies of scale throughout the distribution process. The actual case of the Iranian HL network is illustrated in Figure 2. Next, Figure 3 illustrates the synchromodal flow within the network, including on the network edges, start times, and delivery times at destination points in origin and destination hubs. Finally, Figure 4 displays the synchronization of transportation flows by the transportation fleet and the consolidation operations carried out within the network.

In Figure 2, goods are collected from the origin points (red circles) via road transportation and delivered to the sending hubs (blue squares). Next, the sending hubs ship the goods via air transportation to the receiving hubs (yellow squares). After sorting, the goods are delivered to the destination points (purple points) through road transportation. In this model, each time period spans 48 hours. Therefore, in the following description, all time references are given in hourly sequence within a 48-hour operation cycle.

In Figures 3 and 4, Vehicle 6 departs from the red-colored Origin Points 1 in West Azerbaijan province in the 3rd hour, carrying 300 units of goods. Following a 7-hour journey, the shipment is delivered to ID 2 of Sending Hub 1 in Hamedan in the 10th hour. Following a one-hour unloading process, the goods are prepared for sorting operations, which is a two-hour process. The shipment is then delivered to Outbound Door 1 of the same sending hub. Subsequently, Aircraft 8 transports the aforementioned 300 units of goods from the sending hub to the receiving hub. The aircraft is loaded and commences its journey in the 13th hour, and arrives at Receiving Hub 2 in the 16th hour. Unloading finishes in the 17th hour through ID 2 and the goods are prepared for intra-hub processes. Given that the demand at each destination point is 250 units of goods, a portion of this shipment is directed to Outbound Door 2, and another portion to Outbound Door 3. The sorting process, which lasts two hours, is followed by a two-hour loading process into Vehicle 5. In the 21st hour, 250 units of goods are transported from Outbound Door 3 by Vehicle 5 to Destination Point 5 in Bushehr. At the end of a three-hour journey, the goods are delivered to the destination in the 24th hour. Concurrently, at Outbound Door 2, 150 units of goods from ID 1 and 50 units from ID 2 and 3 undergo a two-hour sorting process. Following a two-hour loading process into Vehicle 6, which finishes in the 23rd hour, the goods are transported to Destination Point 8 in Kohgiluyeh and Boyer-Ahmad, reaching their destination in the 27th hour.

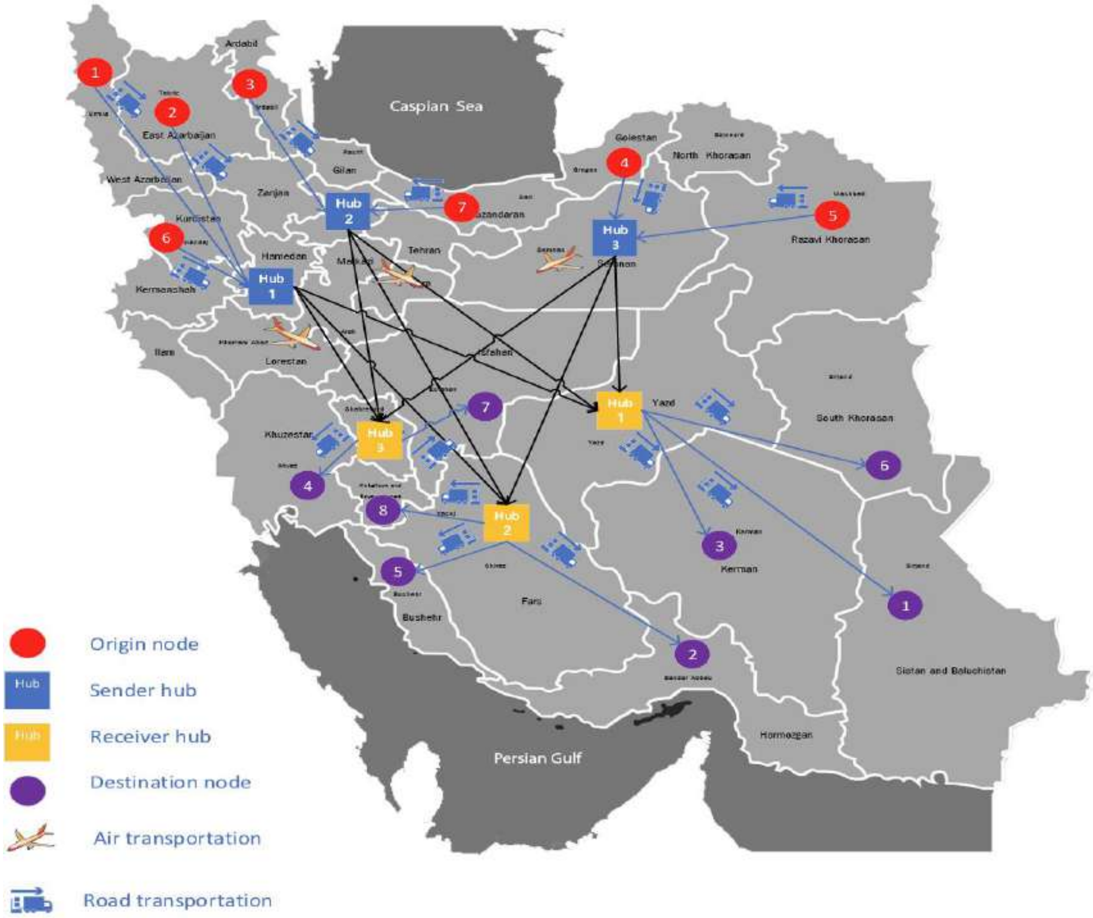

Figure 2. HL and allocation network for the case study

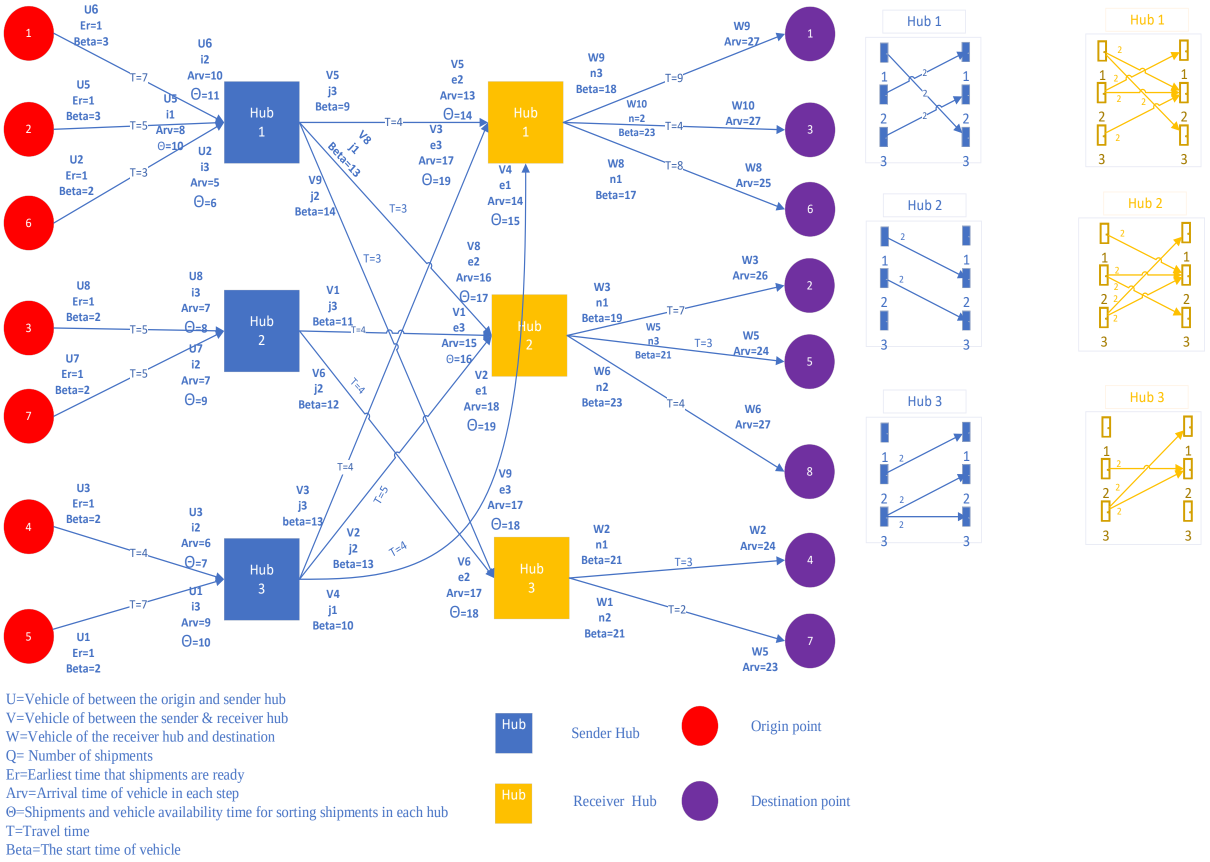

Figure 3. Synchromodality of time flows in hubs and the entire network

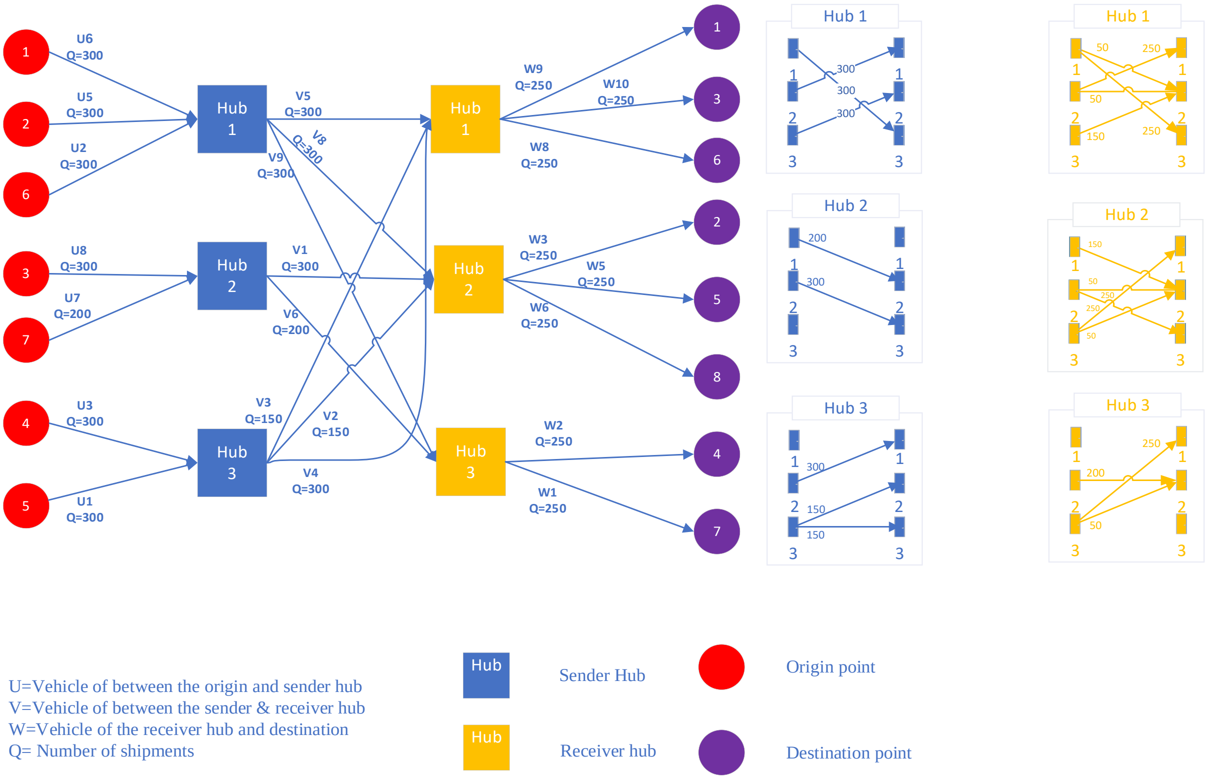

Figure 4. Synchromodal quantity of shipments transported and goods sorting and sequencing in hubs and the entire network

Sensitivity Analyses

In this subsection, sensitivity analyses on key parameters of the problem are performed to assess the efficiency of the proposed model. One of these parameters is the optimal delivery time of shipments from the origin to destination point, denoted as parameter DT_{ds}. This is the most significant parameter of the model due to its impact on various factors, including the number of vehicles employed, the number of located hubs, and the objective function (OF) value. Figure 5 indicates the number of vehicles employed in the network in response to changes in the value of parameter DT_{ds}.

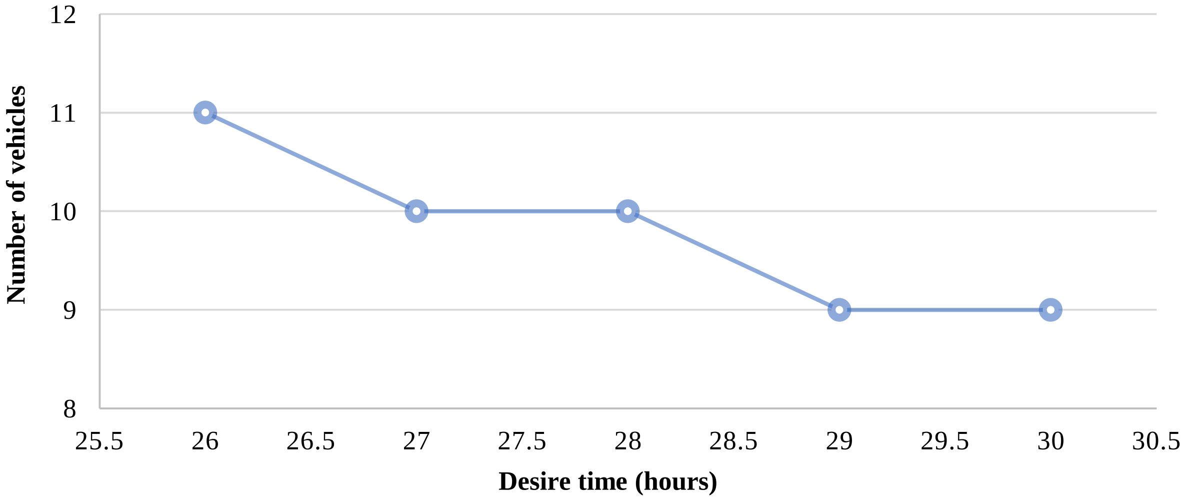

Figure 5. Sensitivity analysis of the number of vehicles relative to variations in parameter DT_{ds}.

As observed in Figure 5, an increase in the optimal delivery time results in a decrease in the number of vehicles used in the network. Evidently, expanding each delivery time window allows for completing the delivery of customer shipments with fewer vehicles. Therefore, the model is able to meet customer demand and deliver shipments to destinations with fewer vehicles. However, the slope of this graph is gentle, and from DT_{ds}=29 onward, it converges to a specific number in this case, nine. The reason for this is the constraint that dictates customer demand should be met with the minimum number of vehicles. In this context, minimum denotes that using fewer vehicles than this would render the model infeasible. Thus, the minimum number of vehicles needed in the network, as determined by this sensitivity analysis, is nine. Consequently, through the model proposed in this study and using sensitivity analysis tools, we can determine the minimum total number of vehicles required in the network. Moving forward, the number of located hubs in the network in response to changes in the value of parameter DT_{ds} is shown in Figure 6.

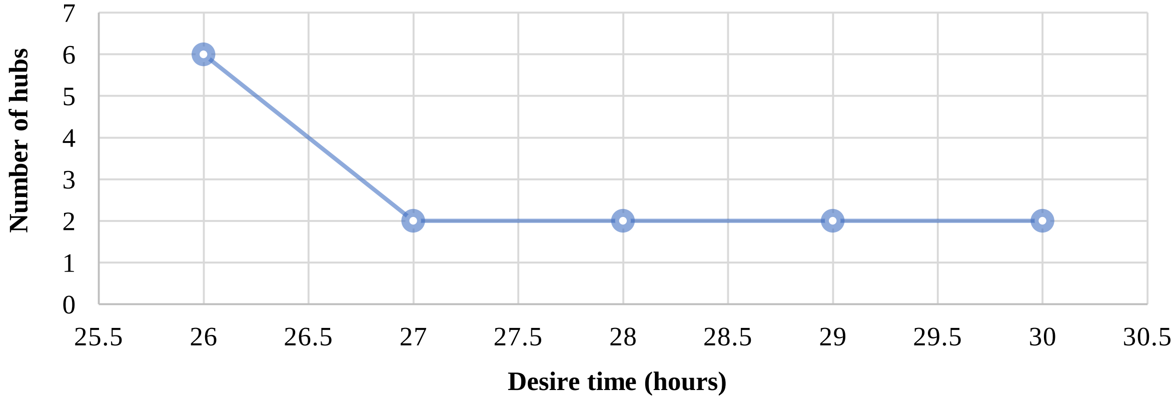

Figure 6. Sensitivity analysis of the number of located hubs relative to variations in parameter DT_{ds}

As Figure 6 indicates, an increase in the optimal delivery time for shipments over a short period results in a decrease in the number of located hubs in the network, causing the graph to quickly converge to a specific number in this case, two. The reason for this is the necessity of using the minimum amount of resources and facilities for managing the flow within the hub distribution network. According to Figure 6, the minimum number of distribution hubs established for both dispatching and receiving shipments is two. Figure 7 demonstrates the outcome of the sensitivity analysis conducted on the model's OF relative to changes in the value of parameter DT_{ds}.

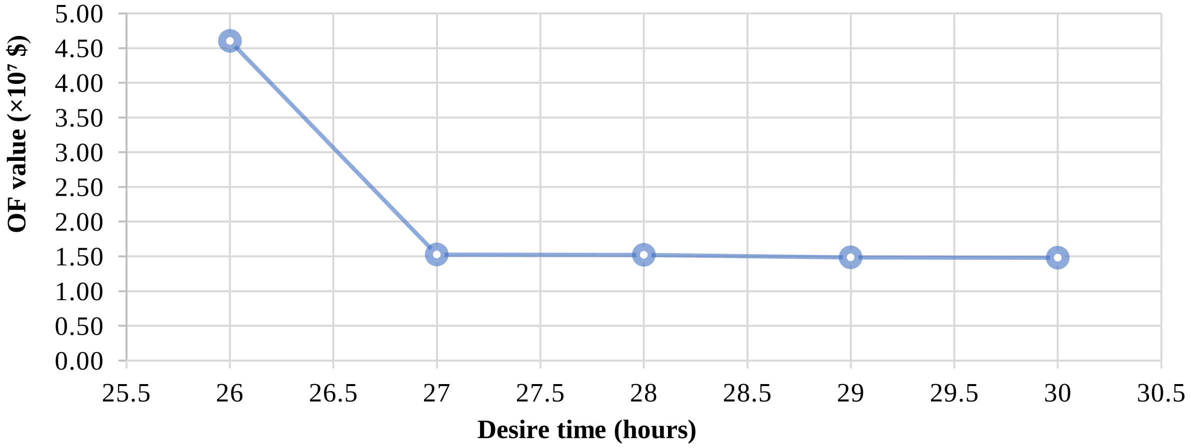

Figure 7. Sensitivity analysis of the OF of the model relative to variations in parameter DT_{ds}.

As Figure 7 suggests, an increase in the optimal delivery time for shipments over a short period leads to a decrease in the OF value and convergence to a specific value. This happens because, firstly, with the increase in delivery time, the number of employed vehicles and established hubs decreases, thus reducing the fixed costs of the network. Secondly, an increase in the value of parameter DT_{ds} reduces the penalties incurred for failing to deliver the agreed quantity of shipments in a timely fashion. This also reduces the OF value.

Figure 8 examines the impact of penalty costs on shortages.

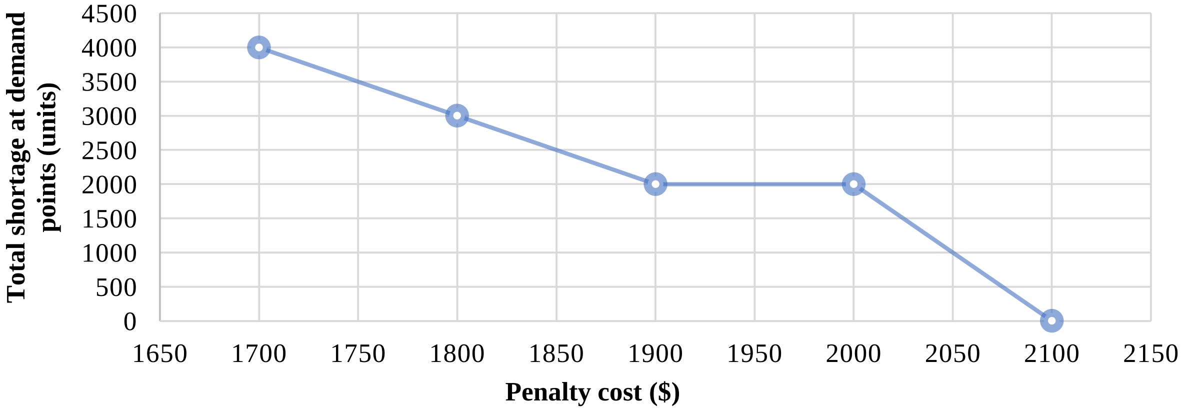

Figure 8. Sensitivity analysis of total shortage at demand points relative to variations in parameter Pq_{ds}

As can be observed in Figure 8, an increase in the penalty cost for failing to meet customer demand results in reduced shortages at destination points. In other words, in response to the rise of penalty costs, the model attempts to fulfil more demand – and thereby reduce shortages – by employing more vehicles or hub centers. While this does lead to enhanced customer satisfaction, it also increases the total network cost. Figure 9 examines the variations in delivery delays relative to the increase in time-related penalties.

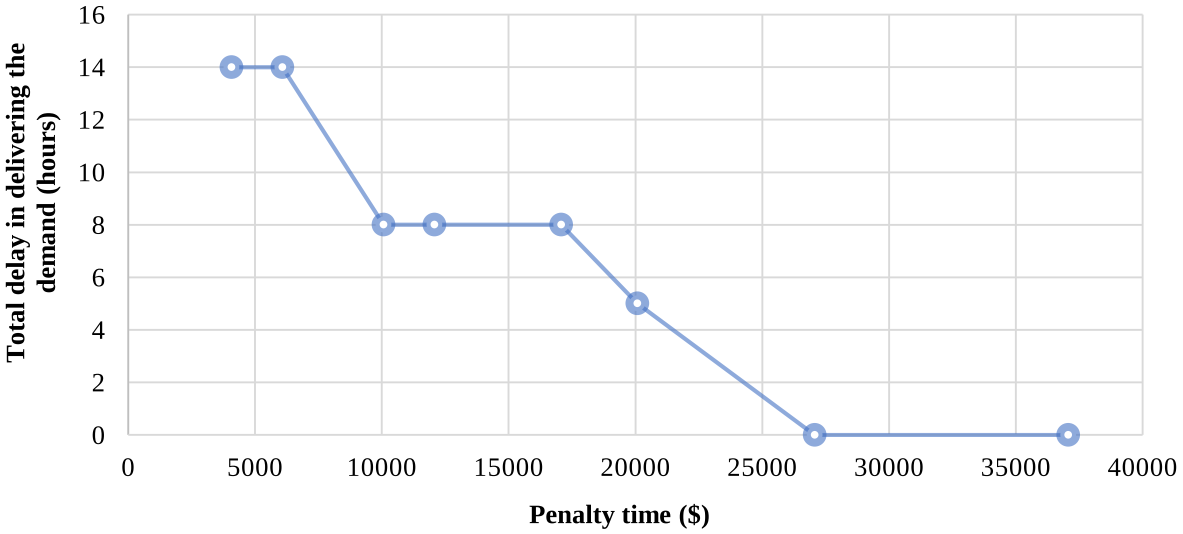

Figure 9. Sensitivity analysis of total delay in delivering the demand of the model relative to variations in parameter Pq_{ds}

According to Figure 9, as the time-related penalty cost for failing to meet customer demand increases, delivery times to destination points are shortened; however, this also increases the total network cost. To achieve this reduction in delivery times, the model employs strategies such as increasing the number of hubs and vehicles, or deploying vehicles with higher capacity that typically incur higher operational costs. Additionally, in some cases, a reduction in shortages is observed at destination points. These results demonstrate the practical applicability of the proposed model in enhancing system performance and regulating the tradeoff between costs and delivery times. Figure 10 examines the effect of different fleet capacity levels on the number of vehicles utilized by the model.

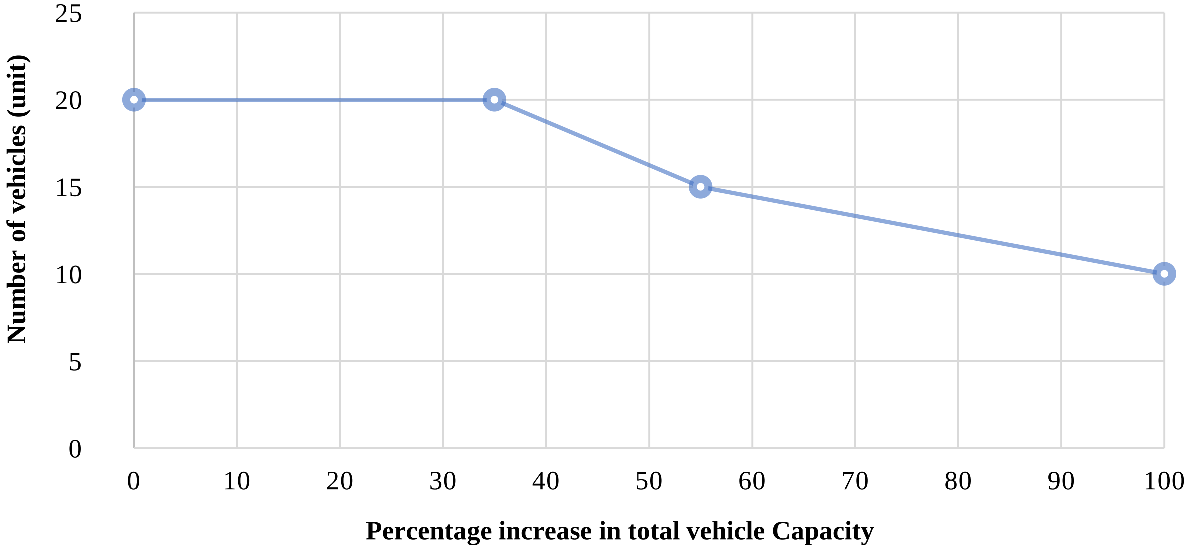

Figure 10. Sensitivity analysis of the number of vehicles relative to the increase in total vehicle capacity

As discussed in previous sections, the proposed model incorporates three transportation modes: road transportation between origin points and sending hubs, air transportation between sending and receiving hubs, and road transportation between receiving hubs and demand points. According to Figure 10, increasing the total capacity of the network's transportation vehicles leads to a reduction in the number of vehicles deployed in the network. Moreover, reducing the number of vehicles can occasionally decrease the network's overhead costs.

The results of the sensitivity analyses performed on the problem's key parameters indicate that the model's behavior aligns with the expectations stated by experts.

Conclusion

In this research, a novel problem termed the Synchromodal HL was explored. In this problem, the flow within the network, including the transportation fleet flow between origin points and Sender hubs, Sender and receiver hubs, as well as receiver hubs and demand nodes, along with the internal operations flow within the hubs, were synchronized. To achieve this, an optimization model was proposed with the objective of minimizing the total network cost, aiming to organize the flow of shipments both within and outside the hub distribution network using a multimodal transportation fleet. This study combined decisions from both HL and service ND problems to consider long-, mid-, and short-term decisions simultaneously. An actual case study from the Iranian food industry was provided to evaluate the applicability of the model under real-world conditions, and the results of implementing the model on it were presented. Lastly, different sensitivity analyses were conducted on the main parameters of the problem. In classical HL models, it is typically assumed that, once allocated to their respective hubs, goods are distributed across the network based on fixed costs or a function of distance. These models typically do not account for the following:

- Coordination of transportation flows: Classical HL models primarily focus on minimizing construction and operational costs, while often neglecting the scheduling of transportation vehicles, coordination of dispatching and receiving shipments, and synchronization of intra-hub operations. This can lead to incoordination in the flow of goods and increased operational costs.
- Impact of delivery times on decision-making: Classical models generally overlook the element of time in transportation processes. However, for time-sensitive goods, such as perishable food items, the lack of optimal coordination between dispatch and receipt times can lead to quality degradation and increased waste, thus imposing financial losses on the distribution system.
- Flexibility in selecting transportation modes: Classical models typically involve a single transportation mode (e.g., road or rail), neglecting multimodal fleets and the possibility of utilizing the right transportation mode for different shipments that can improve overall network efficiency.

The findings and the results of sensitivity analyses conducted on the proposed model's key parameters confirm that integrating long-term (HL) with mid- and short-term (transportation scheduling and resource allocation) decisions enables process synchronization in transportation networks, which can significantly reduce costs and improve delivery times. Furthermore, optimal timing of deliveries can play a critical role in minimizing the costs incurred by coordination failures and delivery delays. However, it was also observed that stricter delivery time requirements might lead to increased transportation costs. Penalties for delays and shortages, as well as the number of vehicles deployed, directly impact network performance. This finding highlights the need for an optimized fleet allocation strategy. Adopting multimodal transportation and dynamic allocation strategies can contribute to cost reduction and delivery time optimization. Operational costs of hubs and transportation fleets are also considered key factors in decision-making. In this regard, the proposed model minimizes the operational costs of the hub distribution network and vehicle deployment costs at each stage, thus optimizing the total network cost.

While this study is arguably a comprehensive investigation of the novel Synchromodal HL Problem, it comes with several limitations that future research could address to better equip the model for dealing with real-world conditions. This study focuses on a basic unilateral distribution network, with origin points on one side and destination points on the other. Extending the model to become a bilateral network, where each point can serve as both an origin point and a destination, is thus a potential avenue for further work. This problem can be applicable to various industries, such as the pharmaceutical industry, which is considered time-sensitive. This is because, following their arrival to demand points, medications may need to be returned to distribution centers due to non-consumption or expiration. Taking this into account helps may not only reduce costs but also can make the problem more practical. Additionally, developing the model into a multi-product network and exploring the application of heuristic or metaheuristic algorithms to solve large-scale instances can represent a promising direction for future research. In real-world logistics networks, which typically handle multiple types of goods, extending the proposed model to a multi-product one can facilitate its adaptation to real-world conditions and thus improve the accuracy of analysis and optimization for multi-product networks. The model presented in this study was solved for a case study using the CPLEX solver in the commercial software GAMS. Due to the inherent complexity of the HL problem and its NP-hard nature, employing exact methods to solve large-scale instances within reasonable timeframes may not be feasible. For large-scale problems, it is possible to employ metaheuristic algorithms such as the Genetic Algorithm, Particle Swarm Optimization, the Grey Wolf Optimizer, and Simulated Annealing. Nowadays, minimizing greenhouse gas emissions and optimizing sustainable transportation have become critically important concerns in the logistics industry. Therefore, future research could incorporate sustainability dimensions, such as minimizing CO_2 emissions, energy consumption management, and transitioning to green transportation modes (e.g., electric vehicles) to reduce the current reliance on fossil fuels. Finally, considering the proposed model under uncertainty in parameters such as demand fluctuations, unexpected delays, and fuel price volatility can bring the problem closer to real-world conditions.

References

1. O'kelly, M.E., *The location of interacting hub facilities.* Transportation science, 1986. **20**(2): p. 92-106.
2. Campbell, J.F., *Hub location and the p-hub median problem.* Operations research, 1996. **44**(6): p. 923-935.
3. Campbell, J.F. and M.E. O'Kelly, *Twenty-five years of hub location research.* Transportation Science, 2012. **46**(2): p. 153-169.
4. Hoff, A., et al., *Heuristics for the capacitated modular hub location problem.* Computers & Operations Research, 2017. **86**: p. 94-109.
5. Campbell, J.F., et al., *Solving hub arc location problems on a cluster of workstations.* Parallel Computing, 2003. **29**(5): p. 555-574.
6. Campbell, J.F., *Continuous and discrete demand hub location problems.* Transportation Research Part B: Methodological, 1993. **27**(6): p. 473-482.
7. Taherkhani, G. and S.A. Alumur, *Profit maximizing hub location problems.* Omega, 2019. **86**: p. 1-15.
8. Basallo-Triana, M.J., C.J. Vidal-Holguín, and J.J. Bravo-Bastidas, *Planning and design of intermodal hub networks: A literature review.* Computers & Operations Research, 2021. **136**: p. 105469.
9. Campbell, J.F. *Modeling economies of scale in transportation hub networks*. in *2013 46th Hawaii International Conference on System Sciences*. 2013. IEEE.
10. Crainic, T.G. and K.H. Kim, *Intermodal transportation.* Handbooks in operations research and management science, 2007. **14**: p. 467-537.
11. Caris, A., C. Macharis, and G.K. Janssens, *Decision support in intermodal transport: A new research agenda.* Computers in industry, 2013. **64**(2): p. 105-112.
12. Crainic, T.G., G. Perboli, and M. Rosano, *Simulation of intermodal freight transportation systems: a taxonomy.* European Journal of Operational Research, 2018. **270**(2): p. 401-418.
13. Ullmert, T., S. Ruzika, and A. Schöbel, *On the p-hub interdiction problem.* Computers & Operations Research, 2020. **124**: p. 105056.
14. Mokhtarzadeh, M., et al., *A hybrid of clustering and meta-heuristic algorithms to solve a p-mobile hub location–allocation problem with the depreciation cost of hub facilities.* Engineering Applications of Artificial Intelligence, 2021. **98**: p. 104121.
15. Wandelt, S., et al., *An efficient and scalable approach to hub location problems based on contraction.*

Computers & Industrial Engineering, 2021. **151**: p. 106955.
16. Bütün, C., S. Petrovic, and L. Muyldermans, *The capacitated directed cycle hub location and routing problem under congestion.* European journal of operational research, 2021. **292**(2): p. 714-734.
17. Kayışoğlu, B. and İ. Akgün, *Multiple allocation tree of hubs location problem for non-complete networks.* Computers & Operations Research, 2021. **136**: p. 105478.
18. Shang, X., et al., *Distributionally robust cluster-based hierarchical hub location problem for the integration of urban and rural public transport system.* Computers & Industrial Engineering, 2021. **155**: p. 107181.
19. Monemi, R.N., et al., *Multi-period hub location problem with serial demands: A case study of humanitarian aids distribution in Lebanon.* Transportation Research Part E: Logistics and Transportation Review, 2021. **149**: p. 102201.
20. Korani, E. and A. Eydi, *Bi-level programming model and KKT penalty function solution approach for reliable hub location problem.* Expert systems with applications, 2021. **184**: p. 115505.
21. Stokkink, P. and N. Geroliminis, *A continuum approximation approach to the depot location problem in a crowd-shipping system.* Transportation Research Part E: Logistics and Transportation Review, 2023. **176**: p. 103207.
22. Li, Z.-C., X. Bing, and X. Fu, *A hierarchical hub location model for the integrated design of urban and rural logistics networks under demand uncertainty.* Annals of Operations Research, 2023: p. 1-22.
23. Rahmati, R., et al., *Stochastic green profit-maximizing hub location problem.* Journal of the Operational Research Society, 2024. **75**(1): p. 99-121.
24. Wu, T., *Exact Method for Production Hub Location.* INFORMS Journal on Computing, 2024.
25. Li, H. and Y. Wang, *Hierarchical multimodal hub location problem with carbon emissions.* Sustainability, 2023. **15**(3): p. 1945.
26. Wang, S., S. Wandelt, and X. Sun, *Stratified $ p $-Hub Median and Hub Location Problems: Models and Solution Algorithms.* IEEE Transactions on Intelligent Transportation Systems, 2024.
27. Li, L., et al., *Intermodal transportation hub location optimization with governments subsidies under the Belt and Road Initiative.* Ocean & Coastal Management, 2023. **231**: p. 106414.
28. Pourghader Chobar, A., H. Bigdeli, and N. Shamami, *A Mathematical Model of Hub Location for War Equipment under Uncertainty Using Meta-Heuristic Algorithms.* Journal of Industrial Engineering and Management Studies, 2024. **11**(1): p. 62-83.
29. Mohammadi, M., P. Jula, and R. Tavakkoli-Moghaddam, *Reliable single-allocation hub location problem with disruptions.* Transportation Research Part E: Logistics and Transportation Review, 2019. **123**: p. 90-120.
30. Motamedi, Z., et al., *Scheduling of transportation fleet based on the customer's priority in a hub location problem.* Scientia Iranica, 2023.
31. Domínguez-Bravo, C.-A., E. Fernández, and A. Lüer-Villagra, *Hub location with congestion and time-sensitive demand.* European Journal of Operational Research, 2024. **316**(3): p. 828-844.
32. Musavi, M. and A. Bozorgi-Amiri, *A multi-objective sustainable hub location-scheduling problem for perishable food supply chain.* Computers & Industrial Engineering, 2017. **113**: p. 766-778.
33. Khaleghi, A. and A. Eydi, *Hybrid solution methods for a continuous-time multi-period hub location problem with time-dependent demand and sustainability considerations.* Journal of Ambient Intelligence and Humanized Computing, 2024. **15**(1): p. 115-155.
34. Zhang, H., et al., *Accelerating Benders decomposition for stochastic incomplete multimodal hub location problem in many-to-many transportation and distribution systems.* International Journal of Production Economics, 2022. **248**: p. 108493.
35. Khaleghi, A. and A. Eydi, *SUSTAINABLE MULTI-PERIOD HUB LOCATION: A DYNAMIC PROGRAMMI NG APPROACH.* Sharif Journal of Industrial Engineering & Management, 2022. **38**(1): p. 95-110.
36. Roozkhosh, P. and N. Motahari Farimani, *Designing a new model for the hub location-allocation problem with considering tardiness time and cost uncertainty.* International Journal of Management Science and Engineering Management, 2023. **18**(1): p. 36-50.
37. Khalilzadeh, M., M. Ahmadi, and O. Kebriyaii, *A Bi-Objective Mathematical Programming Model for a Maximal Covering Hub Location Problem Under Uncertainty.* SAGE Open, 2025. **15**(1): p. 21582440251324335.
38. Rupp, J., N. Boysen, and D. Briskorn, *Optimizing consolidation processes in hubs: The hub-arrival-departure problem.* European Journal of Operational Research, 2022. **298**(3): p. 1051-1066.
39. Anderluh, A., V.C. Hemmelmayr, and P.C. Nolz, *Synchronizing vans and cargo bikes in a city distribution network.* Central European Journal of Operations Research, 2017. **25**: p. 345-376.
40. Karimi, H. and M. Setak, *A bi-objective incomplete hub location-routing problem with flow shipment scheduling.* Applied Mathematical Modelling, 2018. **57**: p. 406-431.
41. Masaeli, M., S.A. Alumur, and J.H. Bookbinder, *Shipment scheduling in hub location problems.* Transportation Research Part B: Methodological, 2018. **115**: p. 126-142.
42. Nolz, P.C., et al., *Two-echelon distribution with a single capacitated city hub.* EURO Journal on Transportation

and Logistics, 2020. **9**(3): p. 100015.
43. Estrada, M., J. Mension, and M. Salicrú, *Operation of transit corridors served by two routes: Physical design, synchronization, and control strategies.* Transportation Research Part C: Emerging Technologies, 2021. **130**: p. 103283.
44. Elbert, R., J. Rentschler, and J. Schwarz, *Combined hub location and service network design problem: A case study for an intermodal rail operator and structural analysis.* Transportation Research Record, 2023. **2677**(1): p. 730-740.
45. Giusti, R., D. Manerba, and R. Tadei, *Multi-period transshipment location-allocation problem with stochastic synchronized operations.* Networks (to appear), 2020.
46. Crainic, T.G., et al., *The synchronized location-transshipment problem.* Transportation Research Procedia, 2021. **52**: p. 43-50.
47. Guo, P., F. Weidinger, and N. Boysen, *Parallel machine scheduling with job synchronization to enable efficient material flows in hub terminals.* Omega, 2019. **89**: p. 110-121.
48. Luo, H., X. Yang, and K. Wang, *Synchronized scheduling of make to order plant and cross-docking warehouse.* Computers & Industrial Engineering, 2019. **138**: p. 106108.
49. Wu, X.B., et al., *Synchronizing time-dependent transportation services: Reformulation and solution algorithm using quadratic assignment problem.* Transportation Research Part B: Methodological, 2021. **152**: p. 140-179.
50. Kara, B.Y. and B.Ç. Tansel, *The latest arrival hub location problem.* Management Science, 2001. **47**(10): p. 1408-1420.
51. Tan, P.Z. and B.Y. Kara, *A hub covering model for cargo delivery systems.* Networks: An International Journal, 2007. **49**(1): p. 28-39.
52. Yaman, H., B.Y. Kara, and B.Ç. Tansel, *The latest arrival hub location problem for cargo delivery systems with stopovers.* Transportation Research Part B: Methodological, 2007. **41**(8): p. 906-919.
53. Giusti, R., et al., *The synchronized multi-commodity multi-service Transshipment-Hub Location Problem with cyclic schedules.* Computers & Operations Research, 2023. **158**: p. 106282.
54. Meraklı, M. and H. Yaman, *A capacitated hub location problem under hose demand uncertainty.* Computers & Operations Research, 2017. **88**: p. 58-70.
55. Ebery, J., et al., *The capacitated multiple allocation hub location problem: Formulations and algorithms.* European journal of operational research, 2000. **120**(3): p. 614-631.
56. Ogazón, E., A.M. Anaya-Arenas, and A. Ruiz, *The time-definite hub line location problem.* Socio-Economic Planning Sciences, 2025. **98**: p. 102185.

Appendix

Linearization

The OF of the model, as presented in the Problem Statement section, is nonlinear. To linearize the function, we use variables DD_{ds} and Sh_{ds}. Consequently, the following constraints are added to the model:

$$Sh_{ds} \geq Dm_{ds} - \sum_{l=1}^{L}\sum_{w=1}^{W} Q_{ldws} \qquad \forall s\,, d \qquad (94)$$

$$Sh_{ds} \geq \ 0 \qquad \forall s, d \qquad (95)$$

$$DD_{ds} \geq \sum_{w} A_{dws} - DT_{ds} \qquad \forall s, d \qquad (96)$$

$$DD_{ds} \geq \ 0 \qquad \forall s, d \qquad (97)$$

Ultimately, we add Constraints (94) to (97) to the model and modify the OF to the form of Equation (98) as follows:

$$\begin{aligned}
Min\ Z = &\sum_{k=1}^{K} FH_k x_k + \sum_{l=1}^{L} FH_l x_l + \sum_{u=1}^{U}\sum_{s=1}^{T}\sum_{o=1}^{O} FV_u b_{uos} + \sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{s=1}^{T} FV_v b_{vks} \\
&+ \sum_{w=1}^{W}\sum_{l=1}^{L}\sum_{s=1}^{T} FV_w b_{wls} + \sum_{o=1}^{O}\sum_{k=1}^{K}\sum_{u=1}^{U}\sum_{s=1}^{T} TC_{okus} Q_{okus} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{s=1}^{T} CC_{ijks} q_{ijks} \\
&+ \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{s=1}^{T}\sum_{v=1}^{V} TC_{klvs} Q_{klvs} + \sum_{l=1}^{L}\sum_{e=1}^{E}\sum_{n=1}^{N}\sum_{s=1}^{T} CC_{enls} q_{enls} \\
&+ \sum_{w=1}^{W}\sum_{d=1}^{D}\sum_{l=1}^{L}\sum_{s=1}^{T} TC_{ldws} Q_{ldws} + \sum_{d=1}^{D}\sum_{s=1}^{T} Pq_{ds}\, SH_{ds} + \sum_{d=1}^{D}\sum_{s=1}^{T} Ps_{ds} DD_{ds}
\end{aligned} \tag{98}$$

Among the constraints, Constraint (74) causes the nonlinearity as it multiplies Q_{okus} and λ_{ikus}, which are a positive variable and a binary variable, respectively. To linearize this constraint, we first define the auxiliary variable QD_{oikus}, then replace Constraint (74) with Constraints (99) to (102) as follows:

$$\sum_{o=1}^{O}\sum_{u=1}^{U} QD_{oikus} \leq IC_{iks} \qquad \forall s, i, k \tag{99}$$

$$QD_{oikus} \leq \phi\ \lambda_{ikus} \qquad \forall u, s, o, i, k \tag{100}$$

$$QD_{oikus} \geq Q_{okus} - \phi(1 - \lambda_{ikus}) \qquad \forall o, i, k, u, s \tag{101}$$

$$QD_{oikus} \leq Q_{okus} + \phi(1 - \lambda_{ikus}) \qquad \forall o, i, k, u, s \tag{102}$$

We also linearize Constraint (68) by replacing it with Constraint (103) as follows:

$$\sum_{j=1}^{J} q_{ijks} = \sum_{u=1}^{U}\sum_{o=1}^{O} QD_{oikus} \qquad \forall s, k, i \tag{103}$$

Constraint (78) is another constraint that causes nonlinearity due to the multiplication of two variables, and it requires a similar operation to the one that linearized Constraint (74). Therefore, to linearize Constraint (78), we define the auxiliary variable QT_{jklvs}, then replace constraint (78) with Constraints (104) to (107) as follows:

$$\sum_{l=1}^{L}\sum_{v=1}^{V} QT_{jklvs} = \sum_{i=1}^{I} q_{ijks} \qquad \forall s, k, j \tag{104}$$

$$QT_{jklvs} \leq \phi\ \lambda_{jkvs} \qquad \forall s, v, j, k, l \tag{105}$$

$$QT_{jklvs} \geq Q_{klvs} - \phi\left(1 - \lambda_{jkvs}\right) \qquad \forall s, v, j, k, l \tag{106}$$

$$QT_{jklvs} \leq Q_{klvs} + \phi\left(1 - \lambda_{jkvs}\right) \qquad \forall s, v, j, k, l \tag{107}$$

To linearize Constraint (75), we follow the same approach as for Constraint (74). We first define the auxiliary variable QD_{kelvs}, then replace Constraint (75) with Constraints (108) to (110) as follows:

$$\sum_{k=1}^{K}\sum_{v=1}^{V} QD_{kelvs} \leq IC_{els} \qquad \forall s, l, e \tag{108}$$

$$QD_{kelvs} \leq \phi\ \lambda_{elvs} \qquad \forall s, v, e, k, l \tag{109}$$

$$QD_{kelvs} \geq Q_{klvs} - \phi(1 - \lambda_{elvs}) \qquad \forall v, e, k, l, s \tag{110}$$

$$QD_{kelvs} \leq Q_{klvs} + \phi(1 - \lambda_{elvs}) \qquad \forall s, e, k, l, v \tag{111}$$

Constraint (69) also causes nonlinearity because of the multiplication of an integer decision variable by a binary variable. To linearize this constraint, we replace it with Constraint (112) as follows:

$$\sum_{n=1}^{N} q_{enls} = \sum_{v=1}^{V}\sum_{k=1}^{K} QD_{kelvs} \qquad \forall s, l, e \qquad (112)$$

Constraint (79) is linearized in a similar manner to Constraint (78). We first define the auxiliary variable QT_{nldws}, then replace Constraint (79) with Constraints (113) to (116) as follows:

$$\sum_{w=1}^{W}\sum_{d=1}^{D} QT_{nldws} = \sum_{e=1}^{E} q_{enls} \qquad \forall n, s, l \qquad (113)$$

$$QT_{nldws} \leq \phi\, \lambda_{nlws} \qquad \forall l, s, d, n, w \qquad (114)$$

$$QT_{nldws} \geq Q_{ldws} - \phi(1 - \lambda_{nlws}) \qquad \forall w, n, s, d, l \qquad (115)$$

$$QT_{nldws} \leq Q_{ldws} + \phi(1 - \lambda_{nlws}) \qquad \forall d, n, l, s, w \qquad (116)$$