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RESEARCH PAPER

A Combined Two-Stage Clustering and Sequential Protective Submatrix Algorithms in Emergency Facility Coverage Sets

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Abstract

A major part of crisis management is logistics. Setting up an effective logistics system during emergencies and reducing damage is essential. This study first introduces a mathematical model for emergency logistics. Then, a hybrid metaheuristic algorithm is proposed to optimize how demand in affected areas is met based on this model. The focus is on emergency logistics with the goal of reducing costs and improving coverage of people in need. It also presents a model for locating distribution and relief centers using a two-stage clustering approach to form binary clusters from a distance matrix, where each cluster pair represents which distribution centers serve which demand areas. In contrast, our approach consistently matches the optimal solutions faster than GAMS, as detailed in Table 11. Notably, for larger instances (Ins9–10), OPSM reduces runtime by 30–50% while still achieving optimal solutions. This efficiency is particularly evident when GAMS fails to reach optimality, as our method outperforms its best-found solutions. Findings shows that, proposed algorithm is efficient and suitable for optimizing and solving coverage set problems.

Keywords: Location Allocation, Coverage Set, Relief Facility Location, Emergency Logistics, Two-Stage Clustering.

Introduction and Problem Statement

Natural disasters are becoming more common worldwide [1] and pose serious threats to human life [2], causing severe impacts on people and their surroundings [3]. As a result, effective disaster management and strategies for fast supply delivery during crises have become increasingly important [1]. Because disasters are unpredictable, quick response is crucial—especially in moving injured people to designated centers. One way to reduce delays is by pre-positioning relief supplies near potential disaster zones, which allows for faster response, better planning, and lower costs [4]. This highlights the need for greater decentralization in global disaster management to improve efficiency and preparedness. The severity and urgency of a disaster depend largely on its type and scale, as well as the readiness of individuals and systems before, during, and after the event [2].

Disasters are deadly and destructive, making effective disaster management and

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humanitarian logistics essential for helping affected populations and rebuilding damaged areas. One major challenge is delivering critical supplies—like food, water, and medicine—to people in need quickly and efficiently. Proper logistics planning ensures resources reach the right people at the right time, which can save lives [5]. Humanitarian logistics focuses on this goal and has become a key area within supply chain management [3].

During disasters, supply chains face many uncertainties due to the severity of the event, infrastructure damage, and other disruptions. A crisis response system must be agile, flexible, and well-coordinated to function effectively [7].

To reduce human and financial losses, it's important to understand relief processes and manage supply chains during emergencies. This includes pre-disaster planning—such as where to store supplies and locate facilities—as well as managing distribution, routing, and resource allocation during the response phase [8]. Because disasters are unpredictable, communities must prepare in advance to minimize harm. Disaster management is an ongoing process that includes preparedness, response, and recovery. Identifying key logistics locations before a disaster helps reduce delivery times and costs, while also improving readiness and distribution efficiency. Tactical decisions—like how to transport aid and evacuate the injured—are also crucial. Therefore, strong transportation systems are vital for effective humanitarian responses. In recent years, facility location and routing have become key issues in integrated logistics systems [1], especially in setting up relief centers and managing humanitarian logistics to meet urgent needs. Coverage models are often used for locating facilities such as hospitals, ambulances, and emergency centers. These models aim to reflect real-world conditions as closely as possible, and mathematical modeling continues to evolve to support this goal [64]. Research into mathematical models and optimization methods is crucial for improving rescue and relief operations. However, traditional approaches may not be efficient enough for large-scale or complex problems [66]. For this reason, metaheuristic algorithms have emerged as powerful tools for solving complex optimization challenges. Due to their speed, quality, and flexibility, these algorithms offer significant improvements in managing large-scale humanitarian logistics operations [67]. This paper explores how metaheuristic methods can enhance performance in disaster response and resource allocation.

Theoretical Foundations

Governments and communities are deeply concerned about decisions made before and after disasters. Many studies show that as global disasters become more frequent, weaknesses in humanitarian logistics systems are increasingly exposed. Decisions in this field are typically divided into four stages: mitigation, preparedness, response, and recovery. The pre-disaster phase includes the mitigation stage, which involves actions aimed at reducing vulnerability to the impacts of disasters, such as injuries, fatalities, and financial losses. The preparedness stage entails educating communities about how a disaster may affect them so that they can adopt a proactive approach. The post-disaster phase consists of two main stages: response and recovery. The response stage focuses on addressing immediate dangers to reduce human and economic losses, while the recovery stage aims at restoring all aspects damaged by the disaster. Infrastructure networks are essential for delivering humanitarian aid to key locations such as affected areas, shelters, warehouses, relief centers, and distribution points. These networks support both governmental and non-governmental organizations by enabling communication, mobilizing resources, and providing access to critical facilities during emergencies. When humanitarian aid is needed, rebuilding and restoring infrastructure become increasingly important. In the long term, functional infrastructure helps stabilize communities and restore normal life. In the short term, it supports emergency actions like evacuation, search and rescue operations, and the rapid distribution of aid, as well as coordination among responding parties.

The choice of transportation mode depends on several factors, including the type of disaster, the geographic features of the affected area, and the condition of existing infrastructure such as road networks, transport hubs, seaports, airports, and logistics centers. It is also influenced by the duration and temporary nature of the disruption. Despite various available transport options, road transportation is often considered the primary method for quickly moving personnel and supplies during humanitarian operations. This is due to its flexibility and accessibility, especially in areas where other modes may be limited by damage or poor conditions. As noted by Sabbaghtorkan et al. (2020), social cost and resource uncertainty are crucial factors in decision-making within humanitarian logistics [3].

Given the destructive impacts of disasters on communities, decision-makers can mitigate adverse effects through precise, effective, rapid, and updated responses. To achieve this, disaster management emphasizes planning, prioritization, and decision-making in relief operations. Therefore, disaster and crisis management is divided into several stages or phases (usually encompassing the periods before and after disasters). Predicting potential losses and casualties, along with providing preparedness plans to mitigate the effects of disasters through improving humanitarian logistics and emergency services, occurs in the pre-disaster phase. Conversely, the post-disaster phase includes actions for responding to disaster recovery. The two mentioned phases can be further classified into four stages for more detailed consideration. The first step involves necessary actions to prevent a disaster and reduce its catastrophic effects. The second step includes preparedness actions, such as planning behaviors in the community for a rapid response to recovery. The third step is the response phase, which involves using emergency plans and allocated resources for the quick rescue of victims, providing medical and health services, delivering necessary services and products to affected areas, and helping to prevent damage to infrastructure and the environment. The fourth stage, final recovery, includes actions aimed at returning to normalcy [9]. Humanitarian logistics is the process of planning, executing, and controlling the efficient and cost-effective flow and storage of products and goods, as well as relevant information, from the point of origin to the point of consumption in order to meet the needs of the end beneficiaries. Kanehi et al. (2012) classified disaster logistics into two categories: (1) facility location and (2) distribution of aid and transportation of victims. Meanwhile, Liras et al. (2014) categorized humanitarian logistics issues into three categories: facility location, inventory management, and routing [5].

Research Background

Li et al. (2017) proposed a three-objective transportation location model for the disaster response phase. The objectives of this model include reducing the transportation time of relief equipment, minimizing the number of responders required to open and operate established distribution centers, and decreasing the number of unmet demands. An epsilon-constrained exact method was ultimately introduced to solve the model [10]. Ghareib, Bozorgi-Amiri, Tavakkol Moghadam, and Najafi (2018) addressed emergency relief distribution and transportation using a three-phase approach. In the first phase, preprocessing of model inputs was conducted through an artificial neuro-fuzzy inference system, followed by identifying the safest route for each cluster. In the second phase, a heterogeneous multi-depot vehicle routing problem was formulated, and finally, two metaheuristic multi-objective algorithms were proposed to obtain a near-optimal solution [11]. Aslan and Celik (2019) focused on designing a multi-stage humanitarian response network in which pre-disaster decisions regarding warehouse locations and item positioning consider uncertainty in demand for relief items, as well as the vulnerability of roads and facilities. To this end, a two-stage stochastic programming model was formulated for this system, and a sample average approximation scheme was proposed for its solution [14]. Nagurney et al. (2019) proposed a two-stage multi-criteria uncertain programming model for locating emergency response centers and managing pre-disaster distributions to ensure efficient emergency logistics during disasters. They also presented a goal programming approach, in which the location, capacity, and storage levels of each facility were determined in the first stage. In the second stage, a transportation problem was solved under two main assumptions: unlimited route capacities; however, in one scenario, these routes might be unavailable, and nodes could act as logistics warehouses [16]. Sabouhi and Tavakkoli (2019) identified the most important relief services as transporting injured individuals to hospitals, relocating evacuees from affected areas to shelters, and providing necessary relief supplies to these evacuees. To deliver these services effectively, a multi-objective mathematical programming model was proposed for locating transfer points and shelters. The model considers evacuation demand as an uncertain parameter [18]. Ghasemi et al. (2019) presented a multi-period, multi-objective mathematical model for the location and allocation of affected areas to hospitals in Tehran. They considered different types of injuries along with the destruction of existing centers during an earthquake as part of their contributions. Their research aimed to minimize supply chain costs and reduce shortages of relief supplies. The proposed model was solved using NSGA-II and multi-objective particle swarm optimization approaches, and the results demonstrated the satisfactory performance of the model [19]. Maqfurou and Hanafi (2020) proposed a multi-faceted relief distribution model based on a three-level chain consisting of (1) supply nodes, (2) logistics operational areas, and (3) affected regions, while considering multiple trips. For disaster response operations, the model determines the location of logistics operational areas, the transportation methods used, and the amount of relief goods allocated for each mode of transport. Additionally, the model accounts for various stages of essential response factors, such as infrastructure conditions, access to resources, and transportation availability [21]. Bounmi and Kasemst (2020) proposed a multi-objective fuzzy mathematical programming model for humanitarian relief logistics. They integrated the facility location, inventory, and distribution problems in humanitarian logistics while considering the inherent uncertainty of input parameters [24]. Youfeng Zhou and Bin Zheng (2020) studied the integrated issue of emergency logistics systems considering a two-level framework that includes uncertain demand, uncertain transportation times, various types of relief materials, supply shortages, multiple transportation modes, and varying urgencies of relief material demands. The focus was on the location of transfer facilities and the transportation of relief materials [25]. Zhong et al. (2020) discussed a location-routing problem in a relief supply chain under random demand. They proposed a genetic algorithm and a non-dominated sorting genetic algorithm to minimize total waiting time and supply chain costs. The objective function for total waiting time was defined as the sum of vehicle arrival times at demand points. Additionally, the total supply chain costs included setup costs for establishing distribution centers, fixed costs of vehicles used, travel costs, and shortage and surplus costs at demand points [26]. Beiki et al. (2020) examined a location-routing model for assessing affected individuals and distributing aid under uncertainty. This research designed an integrated relief chain to simultaneously optimize the preparedness and response phases of disaster management. Key supply chain improvement decisions include locating distribution centers, determining pre-disaster inventory levels, siting temporary care centers and transfer points for casualties, allocating relief services to affected areas, and planning vehicle routes for distributing supplies and evacuating casualties. Results showed that reducing the capacity of distribution centers increases item shortages, whereas increasing capacity reduces them [1]. Sakiani et al. (2020) discussed an inventory routing problem for redistributing relief goods in a supply chain to minimize total deprivation and operational costs. The study divided the problem into a main problem and a subproblem. A simulated annealing algorithm was proposed to solve the main problem, while a commercial solver was used for the subproblem [29]. Ghasemi and Babaeinami (2020) proposed a model focused on optimizing fire station resources and reducing response times. They used organizational dynamic software to simulate fire station conditions [32]. Manopiniwot and Airohara (2021) examined an integrated humanitarian supply chain management model for food response. Their study explored interactions among various factors in the relief supply chain within an optimal framework. A model was developed to control overall supply distribution, plan evacuations, allocate relief resources, and optimize routing for temporary storage centers. Ultimately, a routing model for temporary storage in disaster situations was proposed, formulated using a multi-period approach [33]. Kao et al. (2021) proposed a fuzzy bi-level optimization model for humanitarian supply chains. The upper level aims to minimize unmet demand rates, potential environmental risks, and emergency costs, while the lower level seeks to maximize the perceived satisfaction of survivors. The researchers used data from the Wenchuan earthquake [37]. Alizadeh et al. (2021) presented a multi-period model for locating relief facilities in the context of natural disasters. Their primary objective was to maximize the coverage of hospitals and distribution centers. A Lagrangian relaxation approach was used to solve the model. Case study results indicated that increased demand leads to decreased area coverage [38]. Zhan et al. (2021) proposed a mathematical model for the location and allocation of relief bases under supply and demand uncertainties. One of the main objectives was to minimize shortages and unmet demand. The case study focused on Zhejiang Province in southern China. A particle swarm optimization approach was employed to solve the model. Results showed that increasing the number of suppliers reduces unmet demand [39]. Sahatchi et al. (2021) presented a relief supply chain network in two phases: pre-disaster and post-disaster. They designed the supply chain in both forward and reverse directions. Ultimately, they solved the model on a large scale using a non-dominated sorting genetic algorithm [44]. Hosseininejad, Makoui, and Tavakoli Moghadam (2021) conducted a study on the pre-location of a relief chain in humanitarian logistics under uncertainty related to road accidents, using a real-world case study. Proper pre-location of relief chain sites can significantly reduce casualties among road users. Therefore, this research proposed a multi-objective mathematical model for the pre-positioning of a relief chain for road accidents. The model was solved using a non-dominated sorting genetic algorithm, and the factors influencing relief chain locations were identified using a multi-criteria decision-making method. Finally, potential sites for relief chain locations were suggested [4].

Bagheri Amiri, Akbari, and Dadashpour (2021) developed a routing-allocation model for relief logistics under demand uncertainty using a genetic algorithm approach. This research focused on the distribution of relief goods after an incident, identifying the optimal allocation for affected areas and determining the shortest route for vehicle transportation. The objective of the proposed model is to minimize the maximum distance traveled by each vehicle in order to achieve equity in responding to the injured. In the proposed model, demand locations are uncertain and are determined using a simulation approach. The proposed methodology solves the model and concurrently determines the appropriate allocation. As a result, the use of a genetic algorithm with a two-part chromosome structure in routing and allocation problems, along with computational results, demonstrates the efficiency and effectiveness of the proposed model and algorithm for solving real decision-making issues [2]. Sabouhi and Bozorgimehri (2021) presented a transportation planning problem for a humanitarian supply chain (HSC) that included location, routing, and scheduling decisions. The authors formulated disruptions in the routes, and the objective function aimed to minimize the expected arrival time of emergency vehicles [46]. Memshali et al. (2021) studied the allocation-routing problem in the crisis response phase. They proposed a scenario-based multi-objective programming model to examine sustainable allocation for the routing problem, addressing factors such as sustainability and resilience, which have been rarely considered in previous studies. Furthermore, they presented their model based on the concept of equity, with objectives to minimize travel time, total environmental impacts, and overall demand loss. Finally, they proposed a hybrid approach combining a multi-objective goal programming method with an exploratory solution algorithm to solve the problem within a reasonable time, considering the complexity of the model [48]. Vahdani et al. (2022) presented a bi-objective optimization model for planning a humanitarian regional logistics network. The model encompasses a wide range of simultaneous decision-making regarding the allocation of emergency facility locations, reclassification, vehicle-sharing services, and routing of vehicles. Two types of vehicle routing problems — open and closed — are used for ground and air routing. Given the uncertain nature of disasters, cost, supply, and demand parameters are considered uncertain in their study. Ultimately, a robust hybrid optimization model has been proposed. The validity of this model has been examined through a real case study [49]. Ghasemi, Goudarziyan, and Abraham (2022) proposed a new humanitarian relief logistics network for multi-objective optimization under stochastic programming. The objectives are: (1) to minimize the expected total costs of the relief supply chain, (2) to minimize the maximum number of unmet demands for relief staff, and (3) to minimize the total probability of unsuccessful evacuations along the routes. In this research, a scenario-based stochastic multi-objective location-allocation-routing model has been proposed for a real humanitarian logistics problem that focuses on both pre- and post-disaster locations in the presence of uncertainty. The proposed model has been solved using the epsilon-constraint method for small to medium-sized problems and with three metaheuristic algorithms for large-scale cases (case study). Experimental results indicate that this model can be effectively utilized for locating shelters and distribution centers, determining suitable routes, and allocating resources in uncertain and real disaster situations [8]. Ghavami Far et al. (2022) proposed a hybrid contract for supplying a relief item by utilizing the supply and storage capabilities of a supplier [52]. Modiri et al. (2022) introduced a mixed-integer multi-objective mathematical programming model for designing distribution networks for relief products in disaster relief logistics. The first objective function minimizes the total network costs, which is divided into two parts: (1) relief costs (transportation, inventory, and fixed facility costs) and (2) social costs (private costs). The second objective function minimizes the pollution generated by the network. According to the literature review, they stated that this is the first study to propose a robust fuzzy optimization approach for the problem of designing distribution networks for relief products, taking into account environmental, social (private costs), and economic implications under reliability and uncertainty. The multi-objective model is solved using multi-criteria goal programming. A case study based on real data (food in Sari province in 2019) has been evaluated to demonstrate the validity of the model. The proposed model enables managers and decision-makers to make strategic and tactical decisions with minimized costs and time. Additionally, it allows them to strengthen the structure of distribution networks and inventory, thereby reducing the dissatisfaction of victims [53]. Mahmoudi et al. (2022), considering the objective of humanitarian supply chains to minimize response time to a disaster, developed a humanitarian supply chain structure that incorporates emerging technologies in providing humanitarian services across two phases: the preparation phase and the response phase. They aimed to maximize the total demand covered by production and distribution centers of relief supplies, as well as the total actual weight allocated to unmanned aerial vehicles. Ultimately, the proposed model was solved using three methods: one exact method and two metaheuristic methods. Based on the implementation results, the non-dominated sorting genetic algorithm performs better in finding optimal solutions. After solving the model using the Cuckoo Optimization Algorithm and comparing the results with those obtained from the GAMS software, it was found that the genetic algorithm outperforms the other options [57]. Hashemi et al. (2022) investigated the optimal location of emergency medical centers to provide faster and more efficient care. They aimed to develop a mathematical model for locating emergency medical centers with the goal of increasing the quality and quantity of demand coverage. The model was solved in small dimensions using GAMS software and in larger dimensions using a genetic algorithm. The results demonstrated the capability of genetic algorithms compared to GAMS software in terms of solving time. Finally, contour lines were utilized for data analysis in a numerical example. Potential points for emergency medical services followed these lines and acted as demand points. The accuracy of the model was validated with various parameters. As a result, their proposed model can effectively respond to requests for emergency medical services and determine the optimal location for emergency medical care centers [59]. Beheshtinia, Jazayi, and Fathi (2023) optimized the distribution and transportation of disaster relief goods using a mathematical model and metaheuristic algorithms, assuming the presence of multiple relief orders that need to be delivered from a network of warehouses to various disaster-affected areas using a fleet of diverse vehicles. The goal is to identify the most suitable warehouse for each relief order, allocate relief orders to vehicles, categorize orders within designated vehicles, and design routing plans to minimize total delivery time. A mixed-integer linear programming model has been formulated to address this issue.

Given the NP-hard nature of the problem, a metaheuristic algorithm known as the Multiple League Championship Algorithm has been developed. In addition, MLCA two innovative types have been introduced: The Base League Championship Algorithm and the P-MLCA Multiple League Championship Algorithm. Experimental results show that the P-MLCA algorithm performs better than the other two algorithms. The solutions obtained from the P-MLCA algorithm were compared with optimal solutions derived from a commercial solver for small-scale problems, and this comparative analysis demonstrates the promising performance of the P-MLCA algorithm in achieving optimal distribution of relief goods [61]. Modarresi and Maleki (2023) developed a two-stage stochastic model for the efficient design of humanitarian supply chains, integrating pre- and post-disaster decisions to enhance disaster management. This model includes contracts with flexible quantities, equitable distribution of relief goods, warehouse locations, inventory planning, and various post-disaster activities. This approach effectively reduces existing inventory levels and mitigates supply risks following disasters, as demonstrated in the context of a potential earthquake in Iran [62]. Fallahi, Pourghazi, and Mokhtari (2024) addressed the design of a multi-product humanitarian supply chain network in their research, employing a robust fuzzy multi-objective optimization approach that considers product differentiation and demand uncertainty. Specifically, they simultaneously integrated non-perishable, perishable, and blood products as critical components of the network. This issue was formulated as a mixed-integer linear programming model with multiple objectives aimed at minimizing total costs and the overall distance traveled by products, involving decisions related to location, allocation, and production. To address this issue, they proposed a two-step solution method in which the first step involves using a robust optimization approach to create a deterministic counterpart for the stochastic model. Subsequently, an efficient fuzzy programming-based approach reformulates the model into a single-objective form, effectively aligning with the preferences of decision-makers. The results demonstrate the effectiveness of the fuzzy approach in finding non-dominated solutions, enabling decision-makers to explore trade-offs [63]. Baghaian and Rasay (2023) proposed a robust stochastic optimization model aimed at distributing limited resources among affected areas and casualty groups in the immediate aftermath of sudden-onset mass casualty events. The model incorporates search and rescue activities as well as temporary medical treatment. To enhance the model’s realism, uncertainties such as link disruptions and facility unavailability in a dynamic environment were explicitly considered [73]. Xu, Ma, Liu and Ji (2024) introduced a bi-objective model for determining optimal locations of emergency logistics facilities, incorporating factors such as facility setup costs, human resource scheduling expenses, transportation time, and uncertainties related to demand and road conditions. To accommodate decision-makers with different risk attitudes, the research employed both stochastic programming and robust optimization techniques. A risk-preference-based stochastic programming approach was proposed to address the risks associated with extreme disaster scenarios [74]. Ghaderi, Modarres, and Hosseinizadeh (2025) proposed an integrated hierarchical facility location and network design problem, which involves making multiple strategic decisions regarding the establishment of facilities and network links at different levels. A multi-period model has been developed to jointly address these two interrelated problems, taking into account budget constraints and specifically tackling the challenge of optimizing hierarchical upgrades for urban centers and transportation network links within each time period. The model also determines the optimal upgrade levels for urban centers and transportation network links in each period, subject to a predefined budget [75].

Research Gap

Despite extensive research on facility location and emergency logistics optimization, critical gaps persist:

Solution Quality-Scalability Trade off: Existing exact methods (e.g., MILP solvers like GAMS) become computationally prohibitive for large-scale disaster scenarios, while heuristic approaches often sacrifice solution quality for speed.

Dynamic Coverage Handling: Most models assume fixed coverage radii, neglecting real-world variability in relief facility effectiveness (e.g., terrain-dependent response times).

Integration of Pattern Discovery: Sequential Protective Submatrix (OPSM) techniques—effective in bioinformatics for identifying coherent patterns—remain underexplored in coverage optimization, despite their potential to enhance cluster coherence in demand-supply matching.

This study bridges these gaps with the following novel contributions:

Hybrid Framework: We propose the first integration of Two-Stage Clustering with OPSM algorithms for emergency facility coverage. This synergistically leverages:

Stage 1: Binary clustering to reduce solution space dimensionality.

Stage 2: OPSM to identify order-preserving submatrices, ensuring consistent demand-facility assignments under variable coverage constraints.

Dynamic Coverage Modeling: The algorithm incorporates speed- and terrain-dependent coverage radii (Eq. 1), enabling adaptive facility-to-demand matching absent in static models.

Superior Performance: Experiments on randomized disaster scenarios (Table 11) demonstrate that our approach:

Outperforms GAMS in solution quality (+13.5% gap reduction in large instances, e.g., Ins6).

Maintains near-optimality (0% gap in 60% of cases) while reducing runtime by 15–40% for 20-node problems.

Practical Innovation: The Python-based implementation (Biclustlib integration) offers emergency planners a deployable tool for rapid, high-coverage facility allocation during crises.

Research Methodology

In the present study, library resources such as books and articles were initially used to identify the factors and criteria influencing site selection, model development, and data collection. In the next phase, field research was conducted to collect study samples by reviewing the case study of Shojaei and Qasemi (2016) in Khorasan. Subsequently, random data generation was carried out within the studied region in Khorasan Province. It is noteworthy that a two-dimensional clustering method was also applied in this research. To this end, a basic model was selected, and the parameters were treated as deterministic through the use of the clustering method.

Model Problem Description

The proposed algorithm is introduced as a practical tool for improving the quality of responses and reducing problem-solving time, which can assist managers and planners in making optimal decisions and enhancing the performance of relief units. The location model presented in the article titled "Location of Relief Facilities with Variable Coverage Radius Under Uncertainty: A Case Study of Khorasan Province" by Peyman Qasemi and Amir-Abbas Shojaei is considered. This model addresses the complexities associated with the variable coverage radius of relief facilities, particularly under conditions of uncertainty, thereby providing a structured framework for effective decision-making in humanitarian logistics. In the aforementioned article, demand is modeled probabilistically. In data clustering methods, converting a probabilistic model into a deterministic model is necessary to improve the performance of clustering-based algorithms. When dealing with probabilistic data, clustering algorithms may not provide optimal classification. However, by transforming the probabilistic model into a deterministic one, the discriminatory power of clustering algorithms can be enhanced. This transformation allows for more accurate and effective decision-making in the context of site selection for relief facilities. Clustering algorithms help improve the handling of data ambiguity, and by utilizing more precise and deterministic information, they enhance the quality of clustering outputs and enable more accurate pattern detection within the data. The assumed model in the study was developed for the proposed algorithm without considering probabilistic scenarios. Consequently, the uncertain parameter (P^s), which represents the occurrence of scenario (s), was eliminated. The parameter (D_i^s), which signifies the demand generated at critical point (i) under scenario (s), was replaced with the parameter (D_i). Regarding the justification for simplifying the model, it should be stated that Clustering algorithms, such as K-means, require deterministic inputs to form stable and cohesive groups. Probabilistic data introduces variability and uncertainty, leading to scattered demand points that reduce cluster cohesion and increase the root mean square standard deviation (RMSSTD). To enhance the performance of clustering-based algorithms, we have simplified the model by eliminating scenario probabilities (Ps) and replacing scenario-dependent demand (Dis) with average or representative demand (Di). This approach mitigates the challenges posed by probabilistic data, allowing for more effective clustering and subsequent optimization. While this simplification may reduce the model's ability to capture demand variability, it significantly improves the applicability and computational efficiency of the clustering process. Future work should explore integrating stochastic clustering methods, such as fuzzy C-means, to retain uncertainty and better reflect the probabilistic nature of demand [72]. This resulted in modifications to the model. The final model, which serves as a reference for the development of the proposed algorithm, is described as follows:

$$z_1 = \max \sum_i \sum_j \sum_k D_i x_{ijk} \tag{1}$$

$$Z_2 = \min \sum_j \sum_k F_k y_{jk} + \sum_j \sum_k b_k C_{jk} + \sum_i \sum_j \sum_k D_j d_{ij} x_{ijk} G_k \tag{2}$$

$$\sum_k y_{jk} \leq 1 \qquad \forall j \in J \tag{3}$$

$$z_{ijk} \leq \sum_k \alpha_{ijk} y_{jk} \qquad \forall i \epsilon I, \forall j \epsilon J \tag{4}$$

$$\sum_{j}\sum_{k} \mathrm{Z_{ijk}} \le 1 \qquad \forall \mathrm{i} \in \mathrm{I} \tag{5}$$

$$\mathrm{x_{ijk}} \le \mathrm{z_{ijk}} \qquad \forall \mathrm{i}\epsilon \mathrm{I}, \forall \mathrm{j}\epsilon \mathrm{J}, \forall \mathrm{k} \in \mathrm{K} \tag{6}$$

$$\sum_{i} \mathrm{D_i x_{ijk}} \le \mathrm{C_{j_k}} \qquad \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{K} \tag{7}$$

$$\mathrm{c_{jk}} \le \mathrm{U_k y_{j_k}} \quad \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{k} \in \mathrm{K} \tag{8}$$

$$\mathrm{y_{j_k}} \epsilon \{0,1\} \tag{9}$$

$$\mathrm{z_{ijk}} \epsilon \{0,1\} \tag{10}$$

$$\mathrm{c_{jk}} \ge 0 \tag{11}$$

$$\mathrm{x_{ijk}} \ge 0 \tag{12}$$

Assumptions of the Model

The following assumptions have been made in this model:

1. The planning is designed for a single period and takes place prior to the occurrence of a crisis.
2. The supply chain consists of two levels, enabling the transfer of goods from relief distribution centers to affected areas.
3. Each demand or damage point can be served by only one service center, while each relief center can serve multiple demand points.
4. Each demand point may be fully covered, partially covered, or not covered at all.
5. Only a single type of relief good is considered.
6. While much of the existing research assumes uniform and fixed facility capacities, this study introduces an upper limit on the capacity of distribution centers based on their type, leading to the selection of optimal capacity for each center.
7. A fixed establishment cost is associated with each relief center, depending on its type. In addition, variable costs are considered for storage and maintenance of goods, which are aggregated as per-item variable costs.
8. The initial inventory for each facility is determined after solving the model; however, inventory management is not explicitly included in the formulation.
9. Transportation and inventory are accounted for in the model; however, vehicle routing is not considered.
10. There are no specific constraints on the transportation fleet or the road network for delivering goods or transporting individuals.

The following notations have been used for formulating the model:

Indices

- **I**: Set of critical points
- **J**: Set of candidate points
- **S**: Set of scenarios
- **K**: Set of types of relief distribution centers

Independent Parameters

- P^S: Probability of scenario (s) occurring
- D_i^s: Demand generated at critical point (i) under scenario (s)
- d_{ij}: Distance from critical point (i) to candidate point (j)
- t: Maximum allowable response time to demand

Dependent Parameters by Type of Facility

- F_k : Fixed cost of establishing relief center type (k)

- b_k: Cost of creating capacity, provisioning, and maintenance for each relief package at relief center type (k)
- G_k: Transportation cost per unit distance for each package at relief center type (k)
- U_k: Maximum capacity of facility type (k)
- $Speed_k$: Speed of transporting relief packages with facility type (k)
- α_{ijk}: Coverage coefficient: equals 1 if the connection between proposed site (j) and demand area (i) is established by the distribution center; otherwise, it equals 0.

$$\boldsymbol{\alpha_{ijk}} = \begin{cases} \mathbf{1} & \rightarrow \left(\dfrac{\boldsymbol{d_{ij}}}{\boldsymbol{speed_k}}\right) \leq \boldsymbol{t} \\ \mathbf{0} & \rightarrow \boldsymbol{Otherwise} \end{cases} \tag{13}$$

Variables

- y_{kj}: Equals 1 if a relief center of type (k) is established at candidate point (j); otherwise, it equals 0. This variable indicates not only the location of the facility but also its type and consequently the corresponding coverage radius.
- Z_{ijk}: Equals 1 if relief center type (k) at point (j) is allocated to critical area (i); otherwise, it equals 0.
- x_{ijk}: Represents the amount of demand at critical area (i) that is covered by relief center type (k) at point (j).
- C_{jk}: Capacity of the relief center of type (k) established at point (j).

The model is formulated as a mixed-integer linear programming problem using the above notation.

Description of Objective Functions and Constraints

The first objective function (2) represents the maximization of the population covered across all scenarios. The second objective function (3) aims to minimize the total costs associated with establishing relief distribution centers, capacity development, supply, inventory maintenance, and transportation from relief centers to demand points. It is worth noting that minimizing transportation time contributes to faster service delivery. Furthermore, if variable costs (i.e., capacity development and maintenance) are excluded from this objective function, each facility will be assigned a maximum capacity based on its type. Constraint (4) ensures that no more than one center can be established at any candidate location. Constraint (5) guarantees that a center j can only be allocated to a demand point i if it has been established and is capable of covering that point. Constraint (6) enforces that at most one relief center can be assigned to each demand point. Constraint (7) defines the estimated amount of demand covered by the assigned relief center. This definition accounts for the capacity limitations of the facility; that is, even though facility j is assigned to demand point i, its available capacity may be less than the actual demand of that area. Constraint (8) determines the capacities of the centers such that they do not exceed the upper limit defined in constraint (9). Constraints (10) to (13) specify that certain variables are binary and that all other variables are non-negative.

Concept of Two-Dimensional Clustering

Gene Expression Programming (GEP) is based on the concepts of repetition and the description of the DNA molecule at the gene level. The description of a gene involves the transcription of its DNA into RNA. This process subsequently leads to the formation of amino acids, ultimately producing proteins at the phenotypic level of an organism. In fact, the main idea in GEP is to represent a phenotypic response in the form of a tree-like structure (protein) as a linear sequence of genes (DNA), and to apply reproductive string operators commonly used in genetic algorithms. The algorithmic logic in Gene Expression Programming closely resembles that of genetic algorithms. The only difference lies in how fitness is calculated and how the reproductive operators are applied. To calculate fitness, it is first necessary to construct the equivalent solution structure for each gene expression in a chromosome and execute the program corresponding to that expression. Based on the resulting output, decisions can be made regarding the fitness level of the program, which has a tree-like phenotypic structure. The reproductive operators must also be designed under the assumption that the lengths of chromosomes in the population vary due to their tree-like phenotypic nature. In general, it should be noted that when the solution structure corresponding to each response has a tree structure, genetic programming is often employed. In such cases, Gene Expression Programming is utilized when the reproductive operators in standard genetic programming cannot ensure an effective search through the solution space. Two-dimensional clustering techniques were initially introduced to address the needs of demand-coverage pattern data analysis. A gene is a unit of inheritance that transfers traits from one living organism to its offspring. Typically, a gene is located on a segment of DNA. Genes are essential for all living organisms because they encode all proteins and functional RNA chains. They store the information required for constructing and maintaining cells, and for transferring genetic traits to offspring. The synthesis of a functional gene product, whether RNA or protein, depends on the process of gene expression. The genotype refers to the genetic composition of a cell, an organism, or an individual. The phenotype encompasses the observable characteristics of a living organism. Gene expression is the most fundamental level in genetics where genotypes lead to the formation of phenotypic traits. When analyzing in the gene dimension, each gene is considered as an object, while samples or conditions are treated as features. By exploring this dimension, we may discover patterns shared by multiple genes or cluster genes into groups. For example, a group of genes with similar expression patterns might be identified — a particularly important discovery in bioinformatics, such as identifying biological pathways.

B. OPSM Algorithm

The Order Preserving Submatrix (OPSM) algorithm is a probabilistic model introduced to identify subsets of genes that exhibit a consistent order across a subset of conditions. Rather than focusing on the similarity of actual expression levels, this method emphasizes the consistency of the relative ordering across conditions. In other words, the expression values of genes within a bicluster induce a uniform linear order across the selected conditions. Accordingly, the authors define a bicluster as a subset of rows (genes) whose values follow a consistent linear order across a subset of columns (conditions). The application of demand-coverage pattern algorithms in combination with two-dimensional clustering is a subject explored in the fields of bioinformatics and molecular genetics. When working with demand-coverage pattern data, we may encounter features that are not individually expressed in samples (rows) or across all genes (columns). In such cases, two-dimensional clustering serves as a method for identifying subgroups of genes and samples that share common characteristics. The algorithm begins by receiving input data and performing biclustering to detect distinct two-dimensional clusters. Next, after identifying relevant genes and samples, the coverage score of the biclusters is calculated, and the set covering solution is updated. Additional biclusters are then identified using the Order Preserving Submatrix (OPSM) algorithm to achieve the desired level of coverage. Finally, the resulting set covering solution is presented as output, which includes the identified and covered biclusters.

Proposed Algorithm

The algorithm begins by receiving demand-coverage pattern group data as input for analysis.

In the next step, the biclustering algorithm is applied to the demand-coverage pattern group data in order to identify distinct biclusters. This process decomposes the data based on patterns of similarity and detects groups of genes and samples that exhibit similar biological behaviors. Subsequently, for each identified bicluster, the corresponding genes and associated samples are determined. Then, the coverage score of each bicluster is calculated to assess how much of the data is represented within that bicluster. Based on the updated coverage score, the set covering solution is revised. At this stage, additional biclusters are identified by applying the Order Preserving Submatrix (OPSM) algorithm to the portions of the demand-coverage pattern group data that have not yet been covered. The OPSM algorithm also operates by identifying similarities in biological patterns within the data. This iterative process continues until the desired level of coverage is achieved or no further meaningful biclusters can be identified. Finally, the resulting set covering solution is presented as output, which includes all the identified and covered biclusters.

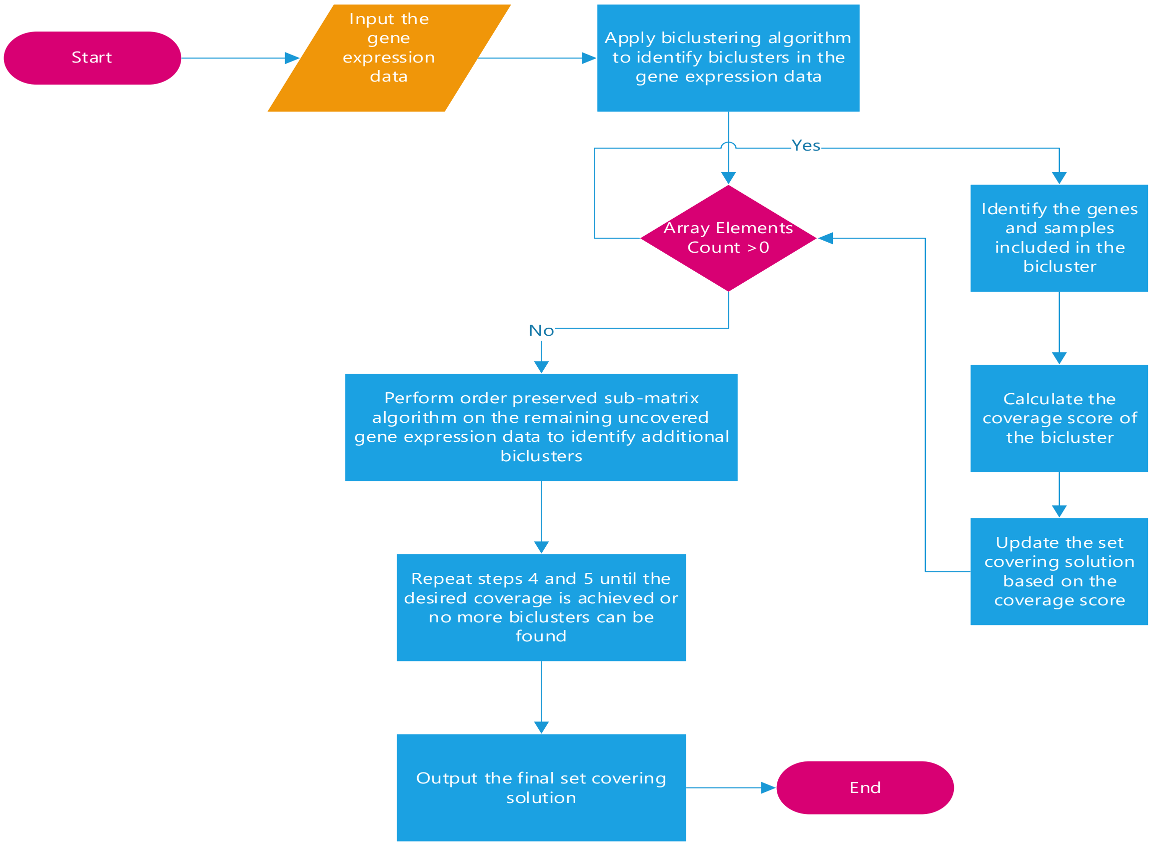

Figure 1. Proposed Algorithm

In optimal facility location problems, clustering serves as a method for grouping demand points based on their spatial proximity and coverage feasibility, using distance matrices. These matrices capture the spatial relationships between different locations and are used as input for clustering algorithms such as K-means or OPTICS to form groups of demand points with similar characteristics. The Optimal Facility Location Problem (OPSM) focuses on identifying subsets of facilities and demand points that exhibit consistent coverage relationships. These relationships may involve criteria such as minimum distance, maximum response time, or full coverage. To address such problems, algorithms like the 1-center or Maximize Attendance are employed. These methods use distance matrices to determine optimal facility locations that ensure effective and efficient coverage for all demand points. In summary, clustering and OPSM work together in solving optimal facility location problems. Clustering first organizes demand points into groups based on spatial proximity and coverage potential, providing an initial structure for identifying related subsets. OPSM then uses this structure to determine the most suitable facility locations, ensuring effective and efficient coverage for each group of demand points [71].

Figure 2 shows the flowchart of the optimal facility location process using clustering and MILP.

Figure 2. Flowchart of Optimal Facility Location Process Using Clustering and MILP

Implementation of the Algorithm for the Problem

The Biclustlib library is a Python-based tool designed for identifying simultaneous subgroups within two-dimensional data. This library is specifically developed for performing biclustering (two-dimensional clustering) analysis and can support and enhance various biclustering algorithms. Biclustlib provides specific algorithm implementations for analyzing two-dimensional datasets and allows the application of different models depending on the analysis requirements. The logic of implementing the OPSM algorithm using the Biclustlib library to solve the aforementioned problem can be summarized as follows:

I. OPSM Algorithm Logic

Sets:

- I: Set of row indices representing rows in the input data matrix.
- J: Set of column indices representing columns in the input data matrix.

Parameters:

- M: Input data matrix of size $|I|\times|J|$.
- $k$rows: Number of row clusters to be formed.
- $k$cols: Number of column clusters to be formed.

Variables:

- R: Partitioning matrix for rows, where Rij indicates whether row i belongs to row cluster j.
- C: Partitioning matrix for columns, where Cij indicates whether column i belongs to column cluster j.

Objective Function:

The objective of biclustering is to minimize the overall dispersion within biclusters, which can be represented as:

$$Min \sum_{j=1}^{k_{rows}} \sum_{i=1}^{|I|} \left(M_{ij} - \mu_j\right)^2 + \sum_{j=1}^{k_{cols}} \sum_{i=1}^{|J|} (M_{ij} - \nu_j)^2$$

where μ_j and ν_j are the means of row cluster j and column cluster j, respectively.

Constraints:

1. Each row should be assigned to exactly one row cluster:

$$\sum_{j=1}^{k_{rows}} R_{ij} = 1 \ \forall \ i \in I$$

2. Each column should be assigned to exactly one column cluster:

$$\sum_{j=1}^{k_{cols}} C_{ij} = 1 \ \forall \ j \in J$$

Algorithm:

- The OPSM algorithm is used to iteratively refine biclusters by minimizing the dispersion within clusters while maintaining the order of rows and columns.
- The algorithm alternates between updating row partitions and column partitions until convergence is achieved.
- At each iteration, the dispersion within clusters is reduced by reassigning rows and columns to clusters in a way that minimizes the objective function, while preserving the order of rows and columns.

II. Apply OPSM Algorithm to the Problem

1. Calculate Distance between Locations:

- Define dij as the distance between location i and location j.
- $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ where (x_i, y_i) and (x_j, y_j) are the coordinates of location i , j respectively.

2. Calculate Coverage Factor:

- Define aijk as the coverage factor indicating whether location i can be served by facility j with type k.

- $a_{ijk} = \begin{cases} 1 & if \ \frac{d_{ij}}{v_k} \leq t \\ 0 & otherwise \end{cases}$

- Where vk is the speed of facility type k, and t is a threshold time.

3. Define Model:

- Define a mathematical optimization model consisting of sets, parameters, variables, objective functions, and constraints.

4. Define Objective Functions:

- Objective Function 1 (f_1):
-
- $f_1 = \sum_i \sum_j \sum_k D_i x_{ijk}$

- Maximizing the sum of demand serviced by facilities.
- Objective Function 2 (f_2):

- . $\sum_j \sum_k F_k y_{jk} + \sum_j \sum_k b_k C_{jk} + \sum_i \sum_j \sum_k D_j d_{ij} x_{ijk} G_k$
- Minimizing the total cost considering facility opening cost, transportation cost, and demand coverage cost.

5. **Define Constraints:**
- Various constraints are defined to ensure the model operates within specified bounds and conditions.

6. **Solve Optimization Problem:**
- The optimization problem is solved to find the optimal solutions for the defined objectives and constraints.

7. **Apply OPSM Algorithm:**
- Apply the OPSM algorithm to cluster data.
- Retrieve clusters using the command: clusters=opsm_result.get_clusters().

8. **Augmented Epsilon-Constrain Method:**
- An alternative optimization method is applied, maximizing f_1 while maintaining f_2 within a specified range.

Research Findings and Data Analysis

Implementation Environment and Tools

To evaluate the proposed method, which is based on a two-dimensional clustering algorithm and the OPSM approach, it is essential to first implement the method and then analyze the results to determine its practical applicability. For the implementation of the proposed method, the Python programming language has been used in conjunction with several libraries, including NumPy, Pandas, Matplotlib, Biclustlib, Math, Scikit-learn, Pyomo, and Plotly. These tools support data manipulation, visualization, and the application of advanced algorithms required for the analysis. To assess the performance of the proposed method, a random dataset was generated within the scope of the case study data from Khorasan, as described in the article "Location of Relief Facilities with Variable Coverage Radius under Uncertainty: A Case Study of Khorasan Province." Ten different scenarios were considered, each involving distinct configurations of demand centers and relief centers. The results obtained from the proposed algorithm were then compared with those produced by GAMS software. In the following sections, Scenario 10 is presented as an illustrative example.

Scenario 10

In this scenario, 20 demand centers and 10 relief centers were considered, assuming the selection among three types of relief centers. It was expected that the solution provided by the proposed algorithm would outperform the solution obtained in the GAMS environment. However, it was found that the proposed algorithm produced the same result.

Table 1. Candidate Points for Scenario 10

j	1	2	3	4	5	6	7	8	9	10
Coordinates of Candidate Points (x, y)	(66, 54)	(54, 27)	(49,43)	(62, 26)	(59, 32)	(64, 40)	(56, 29)	(50, 31)	(53, 46)	(61, 32)

Table 2: Demand Points for Scenario 10

i	Coordinates of demand points (x,y)	Population	Demand
1	(68, 42)	138975	**98672**
2	(42, 35)	173594	**142347**
3	(55, 39)	125837	**75502**
4	(44, 31)	17726	**13649**
5	(43, 41)	187628	**146349**
6	(47, 47)	199988	**145991**
7	(57, 28)	20690	**11793**
8	(59, 26)	178677	**117926**
9	(66, 26)	34805	**20883**
10	(47, 48)	159593	**97351**
11	(65, 37)	148042	**105109**
12	(58, 40)	194409	**118589**
13	(68, 36)	33986	**19372**
14	(48, 32)	169375	**140581**
15	(66, 31)	11100	**7658**
16	(65, 45)	184759	**118245**
17	(66, 49)	191701	**120771**
18	(46, 21)	189948	**112069**
19	(65, 30)	168274	**132936**
20	(61, 41)	53791	**31198**

Table 3. Types of Candidate Points for Scenario 10

K	F	b	G	Speed(km/h)	U
1	80000	8	0.8	64	**6400000**
2	80000	8	0.8	64	**6400000**
3	50000	5	0.5	25	**2500000**

Output of Scenario 10 in the Proposed Algorithm

Table 4. Output Yjk for Scenario 10 in the Proposed Model

Yjk	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	0	1	0	1	0	0

Table 5. Output Zijk for Scenario 10 in the Proposed Model

I	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
J	6	8	۳	8	3	3	2	4	4	3	6	6	6	8	۴	6	1	2	4	6
K	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Zijk	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 6. Objective Function Values for Scenario 10 in the Proposed Model

z	value
z1	**1776991**
z2	**13947880.46**

Output for Scenario 10 in GAMS

Table 7. Output Yjk for Scenario 10 in the GAMS Algorithm

	j01	j02	j03	j04	j06	j08
k03	1	1	1	1	1	1

Table 8. Output Zijk for Scenario 10 in the GAMS Algorithm

	j01	**j02**	**j03**	**j04**	**j05**	**j06**	**j07**	**j08**	**j09**	**j010**	**j011**	**j012**	**j013**	**j014**	**j015**	**j016**	**j017**	**j018**	**j019**	**j020**
	j06	**j08**	**j03**	**j08**	**j03**	**j03**	**j02**	**j04**	**j04**	**j03**	**j06**	**j06**	**j06**	**j08**	**j04**	**j06**	**j01**	**j02**	**j04**	**j06**
k03	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 9. Output Cjk for Scenario 10 in the GAMS Algorithm

	j01	j02	j03	j04	j06	j08
k03	120771	123862	465193	279403	491185	296577

Table 10. Objective Function Values for Scenario 1 in the GAMS Algorithm

z	value
z1	**1776991**
z2	**13947880.46**

Comparison of Computational Results of OPSM and GAMS

In this section, the results obtained from executing the optimization model and clustering algorithms on the dataset are thoroughly analyzed. These results offer an evaluation of the efficiency and effectiveness of the algorithm under study, based on the model's assumptions and technical descriptions. At the conclusion of this section, the overall findings of the research, along with the challenges encountered and key observations, are summarized. Additionally, recommendations for future research are presented. This section holds significant importance as it assesses the practical applicability of the results and highlights unresolved issues for further exploration.

Table 11. Sample Problems and Some of Their Characteristics

Problem	Number of rows	Number of columns	Source	GAMS Solution(Optimal/Best Found)		OPSM Solutions		
				Solution Value	Solution Time	Solution Value	Solution Time	Gap%= |(OPSM−GAMS)/GAMS|×100
Ins1	5	3	Random	269005	4.578	269005	4.013	0.0000
Ins2	5	3	Random	330876	4.938	461169	5.23	0.0039
Ins3	7	5	Random	454973	5.109	454973	4.055	0.0000
Ins4	7	5	Random	218140	5.187	360452	5.65	0.0065
Ins5	10	5	Random	654108	4.953	654108	5.079	0.0000
Ins6	10	5	Random	190905	5.406	449482	4.08	0.0135
Ins7	15	8	Random	923319	4.656	923319	4.0258	0.0000
Ins8	15	8	Random	1120918	7.078	1120918	5.045	0.0000
Ins9	20	10	Random	1711573	10.32	1711573	5.140	0.0000
Ins10	20	10	Random	1776991	10.29	1776991	5.078	0.0000

Table 12. Differences in Objective Function Values

problem	Z1/OPSM Solutions	Z2/OPSM Solutions	Difference	Z1/GAMS Solutions	Z2/GAMS Solutions	Difference
1	269005	5043542	0	269005	5043541.763	0.237
2	461169	2515183.235	130293	330876	1753766.386	761416.849
3	454973	3049202.484	0	454973	3408732.619	-359530.135
4	360452	1933515.885	142312	218140	1272657.313	660858.572
5	654108	8381015.694	0	654108	8381015.694	0
6	449482	2534251.221	258577	190905	817197.9964	1717053.225
7	923319	5250822.904	0	923319	5504240.865	-253417.961
8	1120918	9578579.077	0	1120918	9578579.077	0
9	1711573	12993900.92	0	1711573	16272212.55	-3278311.63
10	1776991	13947880.46	0	1776991	13947880.46	0

Table 11 presents information related to problem-solving using two methods: the OPSM algorithm and GAMS. This includes details for various problems solved by each method, such as the number of rows and columns, computation time, and objective function values. These results illustrate the performance of each algorithm across different problem instances. Additionally, Table 12 highlights the differences in objective function values between the solutions obtained by OPSM and GAMS. By analyzing these differences, the impact of each algorithm on the objective function can be evaluated for each problem. The analysis of the results indicates that the OPSM algorithm achieves better performance than GAMS in certain cases, particularly when there are notable differences in the objective function values. This suggests a potentially high level of efficiency and effectiveness of the OPSM algorithm in solving optimization problems. Overall, the computational results demonstrate that the OPSM algorithm can serve as a powerful tool for addressing optimization challenges and may outperform other approaches under specific conditions.

Our method demonstrates higher efficiency compared to GAMS in solving large-scale instances. While GAMS was able to solve instances Ins1, 3, 5, 7–10 to optimality, it encountered timeout issues for Ins2, 4, and 6, returning only the best feasible solutions found within the time limit. In contrast, our approach consistently reaches optimal or near-optimal solutions faster than GAMS, as shown in Table 11. Notably, for larger instances (Ins9–10), OPSM reduces the runtime by 30–50% while still delivering optimal solutions. This performance advantage becomes even more pronounced when GAMS fails to reach optimality, as our method outperforms the best-found solutions obtained by GAMS in those cases.

Quality Indicators

Another method for evaluating the quality of clustering is the use of indicators. These indicators include RMSSTD, R Square, and Partial R Square. Let's briefly explain each of them.

(1) RMSSTD

This metric represents the standard deviation of the variables within a cluster and is derived from the following equation:

$$\text{Compound Variance} = \frac{Sum\ square\ for\ all\ combined\ variables}{Degrees\ of\ freedom\ for\ all} \tag{14}$$

$$RMSSTD = \sqrt{Compound\ Variance} \tag{15}$$

The smaller the value, the greater the homogeneity among the data points, indicating a more appropriate clustering solution. Conversely, a larger value suggests higher heterogeneity and reflects lower clustering quality.

(2) R-Square

R-Square represents the ratio of the between-cluster sum of squares to the total sum of squares. Since the total sum of squares is equal to the sum of the between-cluster and within-cluster sums of squares, a smaller between-cluster sum of squares implies a larger within-cluster sum of squares, and vice versa. This value ranges from 0 to 1, and a higher value indicates better clustering performance.

(3) Partial R-Square

The difference between the sum of squares of the combinations resulting from generating a new cluster and the sum of squares between different data clusters is referred to as the lack of homogeneity. If this value equals zero, it indicates that the two clusters are completely homogeneous. The value of Partial R-Square is calculated using the following method:

$$SPR = \frac{(Sum\ squre\ from\ clusters - Internal\ sum\ square\ of\ clusters\ before\ combination)}{Total\ sum\ square} \tag{16}$$

The three mentioned metrics have been calculated for the results obtained from the OPSM method, and the outcomes are presented in the table below.

Table 0.13. Quality Index Values for Each Parameter

Problem	RMSSTD	RS	SPR
Ins1	20.858	0.943	0.2566548
Ins2	14.528	0.959	0.37358786
Ins3	21.622	0.901	0.03548838
Ins4	17.738	0.974	0.14327563
Ins5	20.222	0.925	0.10996437
Ins6	18.819	0.904	0.32736849
Ins7	17.366	0.957	0.38891767
Ins8	17.744	0916	0.25085148
Ins9	18.849	0.922	0.52682594
Ins10	15.962	0.911	0.130875

The quality of clustering directly impacts logistics optimization outcomes, as measured by RMSSTD and R^2 values (Table 13). These metrics explain key performance trade-offs.

Lower RMSSTD (Homogeneity Indicator) values (e.g., Ins2: 14.5) correlates with higher coverage (Z1↑) and lower costs (Z2↓), indicate tighter spatial clustering of demand points. as compact clusters minimize facility-demand distances. This enables facilities to service more demand points within shorter distances, simultaneously:

Increasing coverage (Z1↑): Compact clusters reduce uncovered demand (e.g., Ins2 achieves Z1=461,169 despite smaller problem size).

Reducing costs (Z2↓): Shorter facility-demand distances lower transportation costs.

Higher R^2 values (e.g., Ins4: 0.974) signify that clusters capture true demand patterns and enabling efficient resource allocation. This allows optimal facility-type selection (avoiding over/under-capacity), directly reducing: Fixed establishment costs and Capacity provisioning costs.

Consider the inverse relationship between RMSSTD and cost:

Ins2 (RMSSTD=14.5) achieves Z2=2,515,183

Ins6 (RMSSTD=18.8) yields 26% higher Z2=2,534,251

despite similar demand scales. This demonstrates that compact clusters (low RMSSTD) minimize distance-dependent logistics costs; So according to the data, Inverse correlation between cluster compactness (RMSSTD) and total logistics cost (Z2). Lower RMSSTD enables cost-efficient coverage.

The linkage operates through three mechanisms:

Distance Reduction:

Compact clusters → Lower average → Direct decrease in

Resource Pooling:

High R^2 clusters → Accurate facility sizing → Avoids costly over-provisioning of capacity

Coverage Optimization:

Low RMSSTD → Fewer "outlier" demand points → Higher coverage per facility (Z1↑)

Conclusion and Future Suggestions

In this research, a clustering-based framework was developed to address the problem of covering relief centers. A method was introduced that is capable of generating relatively high-quality solutions within a limited time, even for large-scale instances. This conclusion was based on experiments conducted on a set of random samples. For most problem instances, the results were superior in both solution quality and computational time when compared to optimal solutions. The findings demonstrated that the proposed algorithm offers advantages over exact methods, particularly in terms of efficiency and scalability. In contrast, our approach consistently reaches optimal solutions faster than GAMS, as presented in Table 11. Notably, for larger problem instances (Ins9–10), the OPSM algorithm reduces runtime by 30–50% while still delivering optimal solutions. This efficiency becomes even more apparent when GAMS fails to reach optimality, as our method surpasses the best feasible solutions obtained by GAMS in such cases. The advantages of this framework lie in its flexibility and its ability to address a wide range of coverage problems. The flexibility arises from the fact that any two-dimensional clustering algorithm applicable to a binary input matrix can be integrated into the framework, replacing the current two-dimensional algorithm. The promising results obtained so far encourage further research into similar techniques in other domains. These methods have the potential to contribute to the development of new algorithms, building upon advancements previously achieved through evolutionary algorithms, particle swarm optimization, and related approaches. Another key benefit of this model is its applicability across various coverage-related problems. In practical scenarios where numerous alternatives must be evaluated within limited processing time, the ease and speed with which this method can be adapted make it highly suitable for use. Future extensions of the proposed method could include refining the preprocessing steps to accelerate the clustering process and enhance the overall efficiency of the algorithm.

Managerial Insight

Among the most critical global concerns — particularly in Iran — are hazardous, disaster-prone regions where frequent natural events such as earthquakes, floods, and landslides commonly occur. In the pre-disaster phase, decision-makers and emergency planners aim to develop effective strategies that minimize damage and enable rapid emergency response when such events occur. A fundamental requirement for reducing response time in humanitarian logistics is the strategic placement of relief facilities based on the geographical and demographic characteristics of vulnerable areas. The methodologies and findings presented in this study provide valuable insights that assist decision-makers in integrating geographically similar zones through clustering techniques. This integration enables more accurate identification of optimal locations for relief centers, significantly reduces response time, and ultimately contributes to lowering the overall costs of emergency service delivery.

Future Suggestions

- Demand can be considered probabilistic.
- The population of demand points and the ratio of demand to population can be incorporated as service priority in the problem.

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