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RESEARCH PAPER

A Novel Multi-Sector, Multi-Period DEA Framework for Evaluating Bank Branch Efficiency

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Abstract

From an economic perspective, efficiency refers to the optimal use of resources to produce the maximum possible output, highlighting its critical role in management systems. Managers generally strive to utilize resources effectively to maximize outputs and satisfy all stakeholders. This is particularly important in the Iranian banking industry, where high costs of fund maintenance make efficiency a central concern. In this study, considering the operational structure of Iranian banks, a five-sector model is proposed to illustrate the flow of activities within banks. Based on this structure, a mathematical model using Data Envelopment Analysis (DEA) is developed to evaluate the efficiency of the five sectors. A fuzzy-based approach is then introduced to solve the model. Application of the proposed model to 210 branches of a major Iranian bank indicates that, while the Services and Mobilization of Resources sectors demonstrate relatively high efficiency, the Management sector suffers from significant inefficiency. These findings highlight the need for focused managerial interventions to enhance operational performance in bank branches.

Keywords:
Relative Efficiency, Data Envelopment Analysis (DEA), Multi-Sector Model, Multi-Period Model, Fuzzy Algorithm.

Introduction

Banks are the main providers of financial resources for various economic sectors such as industry, agriculture, and services. Banks as economic agencies aspire to installation, optimum resources, and provide various services to customers so that they can achieve profitability and benefit. Collection and attraction of all types of deposits and their attribution to supply the financial needs of various economic activities are the most important aspects of banking. Banks play a very important and sensitive role in economic systems, and they can highly affect the cash flow in society, regarding the relations and proportions. Banks are considered producers of financial services that use capital and labor as key inputs (Anh, 2025). Bank operations are divided into three sub-processes: (1) Production, (2) Intermediation, and (3) Social welfare (Shi et al., 2023).

In general, the description of the role of banks cannot be simply summarized as “deposit provider and loan provider”; rather, banks simultaneously affect economic development, stability, and financial inclusion by managing available liquidity, reducing information asymmetry, and adapting to digital technologies. Therefore, advanced analyses such as the “multi-component-multi-period model” are an appropriate tool to reveal these complex effects

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and can guide bank managers in policymaking and prioritizing interventions and investments (Breitenlechner et al., 2022; Edo, 2025; Papathomas & Konteos, 2023; Xu, 2025). Recent research shows that the quality of bank development not only determines the quantity of credit but also shapes how banks impact the economic development and sustainability of countries (Xu, 2025).

The crucial importance of banks' activities in economic systems and the resource volumes these units have access to has made efficiency a stable competitive advantage for them. Lionel Robbins, a famous economist, believes that "economic science studies human behavior in relation to scarce resources production to estimate humans' limited material purposes." In this regard, economic science is the study of how to utilize productive resources with maximum conservation to satisfy the most important needs of humans. This conservation is represented in the economic efficiency framework in the best state. In this regard, Robins implies economic efficiency; therefore, economists have mentioned "efficiency" at the center of their studies (Tafazzoli, 2015).

Efficiency is divided into two components: technical efficiency, which refers to a firm's ability to maximize the amount of output from a set of inputs into a system, and allocative efficiency, which is responsible for overseeing the optimal use of inputs based on the price and relevant production technology. Combining the two will lead to economic efficiency (Anh, 2025).

According to the importance of the efficiency issue in economic agencies, there are various methods of measurement. In a general division, there are two classes of parametric and non-parametric methods to measure efficiency. In the parametric method, a certain production function is estimated by statistical techniques, and its efficiency is measured using this function. However, non-parametric methods don't need to estimate the production function. One of the most commonly applied non-parametric methods is data envelopment analysis, which measures the relative efficiency of units in comparison with each other. (Jahanshahlu et al., 2010). DEA is a mathematical model to estimate the efficiency boundary. This method gives a boundary function covering all data, so it is called development or pervasive analysis. (Charnes et al., 1985). After having provided this method, many applications were reported for efficiency evaluation and improvement. The studies of Emruznezhad et al. (2008) and Liu et al. (2013) showed that DEA has the maximum application in the insurance and banking industries. There are various methods for evaluating performance. One of the most popular methods for evaluating performance for data envelopment analysis is the DEA model (Tavassoli, 2025). To provide accurate information to decision makers, the DEA method is proposed (Shi et al., 2023).

DEA helps generate actionable insights for bank managers and policymakers. The aim is to enhance bank performance through a set of targeted operational improvements and strategic supervisory interventions. This method has gained great popularity in the field of bank efficiency due to its ability to manage multiple inputs and outputs (Anh, 2025).

The models of DEA consider evaluation units as a black box, changing inputs to outputs. These models don't focus on structure and input streams of units at all. In general, the traditional DEA model has the following limitations (Anh, 2025):

1. It considers decision-making units as black boxes, which means it ignores the internal structure.
2. It assumes that all outputs are desirable.

Old models are generalizations of these classic models, which consider the input structure of units, too. These models were introduced by Fare and Grosskopf for the first time. In a general view, DEA models can be classified as presented below in Figure 1.

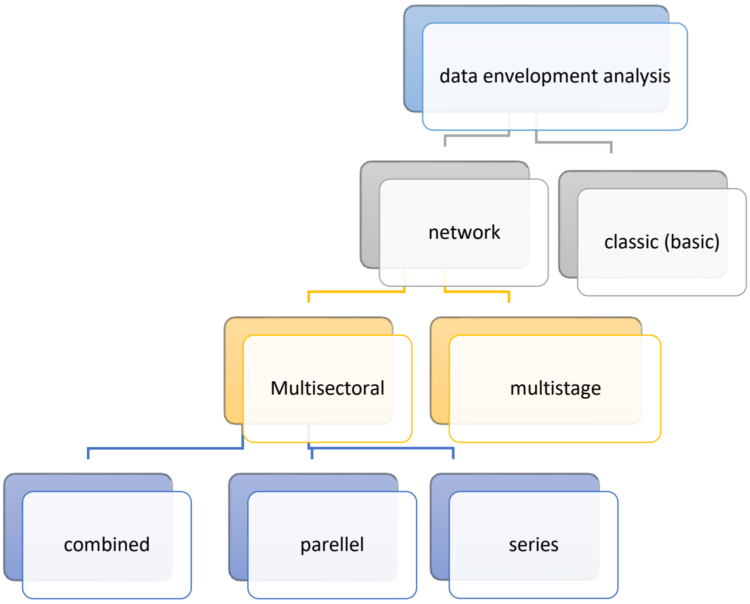

Figure 1. DEA models classifications based on decision-making units' structure
Reference: researcher findings

According to Figure 1, DEA is divided into several-sector and multi-sector classifications. Multi-section models are mostly used for efficiency evaluation of a supply chain consisting of several organizations (Saaei and Najafi, 2013), whereas multi-sector models are related to internal structures of an organization consisting of various sectors.

Regarding classic models, DEA and measuring the efficiency of each decision-making unit are measured only in a definite time, which is actually considered a sectional analysis. However, in real studies, observations are related mostly to a several-day decision-making period, and they are time series data. It is important when the decision-making unit and its changes have to be defined over several time series. The advantage of this task is to define decision-making units in several days and compare its performance in several other time series or other decision-making units. This type of analysis, called DEA, is called multi-duration analysis. (Ramanatan. 2003; Razavi Hajiagha et al., 2015). DEA is becoming increasingly important for measuring efficiency because it does not require assumptions about the relationship between inputs and outputs, and is being exploited in the banking and financial services industry (Shi et al., 2023).

DEA is a mathematical technique that most often uses linear programs to evaluate the efficiency of one or more DMU[†]s (Bolós et al., 2025).In Iran, banks mostly use the classic approach (which has been adopted from the common banking approach) to evaluate their efficiency, measure, and rank without considering internal relationships among units. In the present research, the main objective is to provide a new model based on the Islamic banking pattern to evaluate banks' efficiency by considering the dominant internal structure and the communications inside the branches. Having studied the activity method and working stream on DEA, a model to evaluate this multi-sector model was suggested. Multi-period model is used to study bank branches more accurately to prevent fluctuations as a result of performance in one period. Finally, the application of this model in the branches of Iranian banks is provided.

Literature Review

(Wu et al., 2026) conducted a study to investigate the efficiency of banks in Cambodia, in which

[†] Decision Making Units

a comparison was made between multi-criteria measurement and the DEA approach. At the end, a comprehensive assessment of the competitiveness and efficiency of 52 units was presented. A study by (Gupta et al., 2025a) was conducted to investigate the analysis and prediction of the cost effectiveness of banks in India using a combined approach derived from DEA and SVR. This study, which was conducted through a twin support vector machine (Twin Support Vector Regression (TWSVR)), finally introduces a method to estimate the cost efficiency of decision-making units that combines DDF (Direction Distance Function) with DEA to manage negative data and undesirable outputs.

(Verma & Kashiramka, 2025) conducted a study to investigate the relationship between liquidity creation and cost efficiency of banks with ESG disclosure among 103 commercial banks from 10 emerging banks. In this study, which uses a non-parametric DEA method to determine the cost efficiency of banks, it proves that cost efficiency has a positive effect on liquidity creation, but this relationship is negatively moderated by ESG. Another study titled Predicting and Estimating Bank Profit Efficiency: An Integrated DEA and ML Approach Based on Network DDF was conducted by (Gupta et al., 2025b). This study provides significant insights into the field of bank profitability using a mixed network structure and advanced modeling techniques. (Amirteimoori, 2025) conducted another study on efficiency analysis in DEA with common resources and undesirable outputs, and application in the banking industry. This study, which was obtained from a survey of 40 bank branches, shows that the most important sources of bank inefficiency are related to the interest income and non-performing loans sectors. A study by (R. P. Das et al., 2025) aimed to evaluate financial efficiency at the micro level in India, in which 30 financial institutions in India were evaluated using the DEA method. (Ally, 2025) used data envelopment analysis in a study titled Assessing Sustainability and Efficiency in Domestic and Foreign Banks in Tanzania, referring to different insights from the digital world. (Arora et al., 2025) examined the application of the Slack-based DEA approach to measure the efficiency of banks in the public sector in India despite outstanding debts in the form of assets. Another study by (Uddin et al., 2025) was conducted titled Dynamics of Technical Efficiency in the Banking Industry in India. In this study, a long-term efficiency model of banks in this country was examined using the cross-border DEA approach.

(Obeid, 2025) in his study titled Banking Factors and Various Economic Drivers in the Productivity Sector of the Banking Sector in Jordan and Insights from DEA, SBM, and Logic Models found that the banking industry in Jordan has generally performed well despite economic and geopolitical challenges. In a study conducted by (Boubaker et al., 2025) to examine the balance between non-performing loans and bank profits, various outputs were evaluated using DEA, which led to the division of loans in the manufacturing sector into two categories: good loans and bad loans. (Wang et al., 2025) In a study titled Assessing the Role of Key Factors in Achieving Higher Levels of Efficiency in Turkey, using the CAMELS-DEA approach, they examined the efficiency of banks in Turkey in the period 2010 to 2023, considering the unconventional monetary policies of the central bank, in which a significant relationship can be observed between efficiency, size, and type of bank. (Eraso Cisneros et al., 2025) In a study, they examined comparative evidence in the field of technical efficiency and financial resilience in the banking industry and cooperatives in Ecuador. In this study, which was conducted using the DEA approach and mixed linear models MLM, it was found that cooperatives have a higher technical average and are more flexible than banks. (A. Das et al., 2025) Conducted a study aimed at examining post-merger efficiency in the banking industry in India. In this study, the impact of mergers on the technical efficiency of the banking industry was analyzed using the DEA approach, which revealed that private sector banks experience better improvement than the public sector. A summary of the related literature is shown in Table 1.

Table 1. A summary of related studies

Row	Author/Year	Subject under consideration	Desired extracted result
1	(Wu et al., 2026)	A study on the evaluation of bank efficiency in Cambodia: A comparison between multi-criteria measurement and DEA	Providing a comprehensive assessment using multi-criteria measurement and DEA of bank performance in Cambodia.
2	(Gupta et al., 2025b)	A study of cost-effectiveness analysis and forecasting of Indian banks using a hybrid approach derived from DEA and SVR	Introducing a method to estimate the cost efficiency of decision-making units that combines DDF with DEA.
3	(Gupta et al., 2025a)	Forecasting and estimating bank profit efficiency: An integrated DEA and ML approach based on DDF	Using an integrated approach to provide significant insights into bank profitability using a mixed network structure and advanced modeling techniques.
4	(Verma & Kashiramka, 2025)	Investigating the relationship between liquidity creation and cost efficiency of banks with ESG disclosure among 103 commercial banks	Cost efficiency identified using the DEA approach has a positive impact on liquidity creation, but this relationship is negatively moderated by ESG disclosure.
5	(Amirteimoori, 2025)	Efficiency Analysis in DEA with Common Resources and Adverse Outputs and Application in the Banking Industry	Using DEA to understand that the most important sources of bank inefficiency are related to the interest income and non-performing loan segments.
6	(Ally, 2025)	Investigating and assessing the sustainability and efficiency of domestic and foreign banks in Tanzania, with reference to various insights from the digital world	Using the DEA approach to assess sustainability and efficiency in the digital world.
7	(Arora et al., 2025)	Investigating the application of the Slack-based DEA approach to measure the efficiency of public sector banks in India despite outstanding debts in the form of assets	Focusing on three categories of performance of public sector banks in India using the DEA approach.
8	(Uddin et al., 2025)	Dynamics of technical efficiency in the banking industry in India	A long-run efficiency model of banks in this country was examined using a cross-border DEA approach.
9	(Nguyen et al., 2025)	Examining the efficiency and performance of Islamic banks during the COVID-19 outbreak	Using a two-stage DEA approach, they found that Islamic banks had moderate efficiency and performance during the Covid-19 era.
10	(Obeid, 2025)	Banking Factors and Various Economic Drivers in the Productivity Sector in the Banking Sector in Jordan and Insights from DEA, SBM, and Logic Models	The banking industry in Jordan has generally performed well despite economic and geopolitical challenges.
11	(Boubaker et al., 2025)	Examining the balancing of overdue loans and bank profits	Various outputs were evaluated using DEA, which resulted in loans in the manufacturing sector being divided into two categories: good loans and bad loans.
12	(Wang et al., 2025)	Assessing the role of key factors in achieving higher levels of efficiency in Turkey using the CAMELS-DEA approach	Using the CAMELS-DEA approach, the efficiency of banks in Turkey was examined during the period 2010 to 2023, taking into account the unconventional monetary policies of the Central Bank, in which a significant relationship between efficiency, size, and type of bank is observed.
13	(Eraso Cisneros et al., 2025)	Comparative Evidence Review on Technical Efficiency and Financial Resilience in the Banking Industry and Cooperatives in Ecuador	Using the DEA approach and mixed linear models (MLM), it was found that cooperatives have a higher technical average and are more flexible than banks.
14	(A. Das et al., 2025)	A study of post-merger productivity in the Indian banking industry	Using the DEA approach, the impact of mergers on the technical efficiency of the banking industry was analyzed, and it was found that private sector banks experience better improvement than the public sector.

Determination of the Multi-Sector Structure of Bank Branches

In order to implicit multi-sector structure of bank branches based on the present facts and their executions, bank experts were interviewed. Experts are managers and assistants in three senior, middle, and junior organizational levels, who have more than 10 years of working experience in banking, and who are familiar with most issues in banks, particularly branch activities. Finally, according to librarian studies and the obtained results from experts' interviews, the network general model was obtained based on figure 2 for banks branches activities based on figure (2).

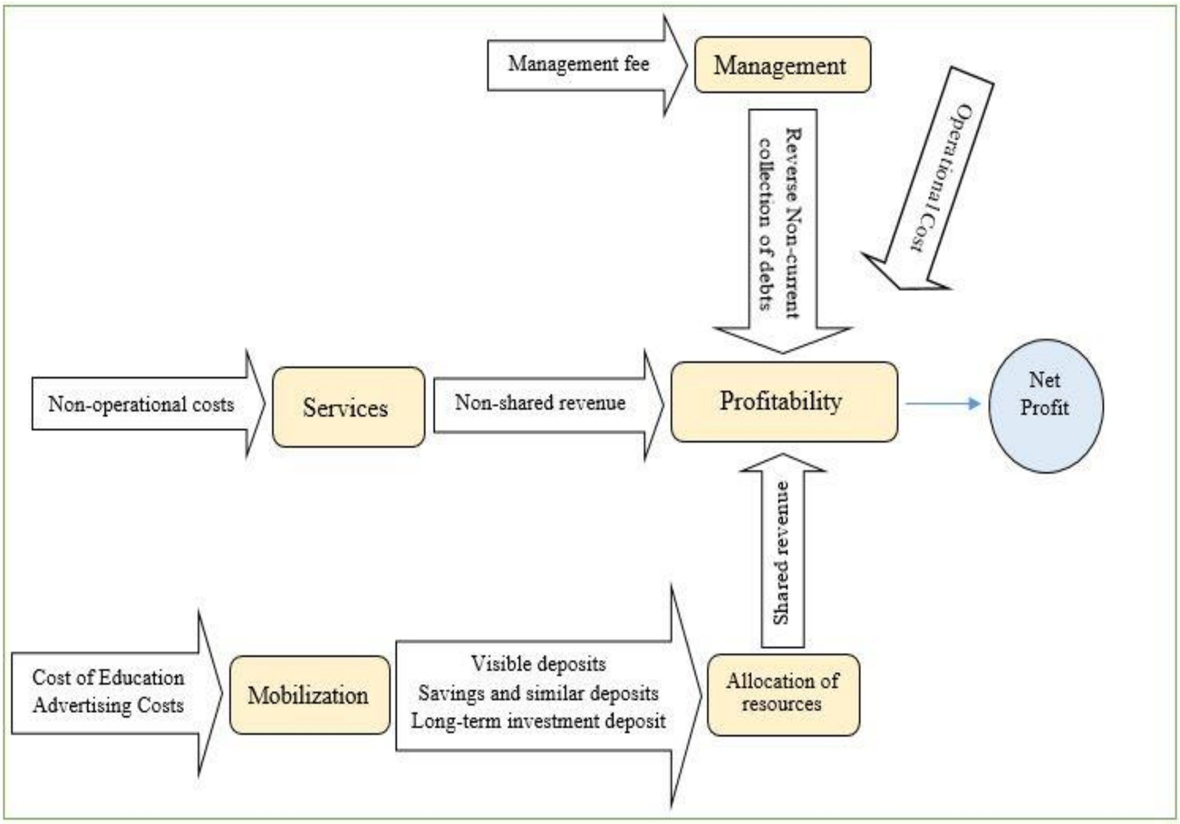

Figure 2. multi-component structure of bank branches

Figure 2 shows the fact that each branch generally consists of 5 sectors in touch with each other through a network. The three sectors: services, mobilization, and allocation of resources are named based on the Islamic banking pattern. (Fadaeinezhad et al., 2015). According to the importance of management performance in resources, allocation, and the non-current collection of debts, and also the important effect of two variables on the bank branches' net profit was emphasized to track and evaluate the two important management and profitability sectors properly.

In the services sector, non-functional costs such as personnel costs, depreciation costs, and administrative costs are input variables, and non-shared revenues such as fees and other incomes are output variables. In mobilization of resources, banks seek to mobilize resources by providing advertising and education. As it is shown in Figure 2, education and advertisement costs are input variables, and all visionary deposits, long-term investment, and savings are considered as output variables in this sector. After resource attraction, banks invest as they are the customers' representatives and the financial intermediary. Consequently, these investments are called non-shared revenues. Since non-current collection negatively affects banks' performances, some costs as fees, are given to the head of branches. The objective is to appraise the heads and the personnel of branches for the collection of debts. Since non-current debts are considered undesirable output, the mentioned problem used Golany and Roll (1989) approach, using the reverse of undesirable indexes. Scheel (2001) showed that this approach has more conservative performance than other confrontation techniques with undesirable data.

Finally, it must be mentioned that most banks' purpose of establishing a branch is to increase net income and profit. Therefore, the profitability sector is significantly important for banks' senior managers for a more accurate evaluation of the bank's performance. In this sector, shared revenues, non-current collection of debts costs, and operational costs are input variables, and net profit is considered the final output variable.

Mathematical Model

By indicating the multi-sector structure and the relationship manner of sectors with each other in Figure 2, it can be claimed that the mentioned model is in combined multi-sector network model classification based on the classification of Figure 1. In this part, the DEA of data design is provided to evaluate this network structure. The used symbols in problem modeling are as follows in Table 2.

Table 2. Input and output variables

Variable output		***Intermediate products***		***Input variables***	
Y_1	Net profit	Z_1	Non-shared revenue	X_1	Non-operational costs
		Z_2	visible deposits	X_2	cost of education
		Z_3	Savings and similar deposits	X_3	Advertising Costs
		Z_4	Long-term investment deposit	X_4	Management fee
		Z_5	Shared revenue	X_5	operational cost
		Z_6	Reverse non-current collection of debts		

Replacing the mentioned indices in Figure 2 makes an equal network structure in Figure 3.

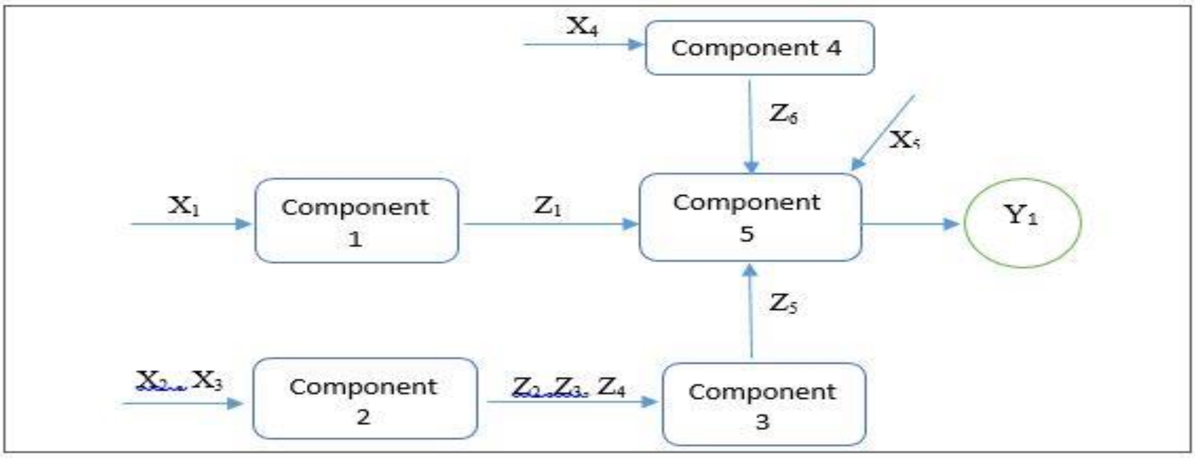

Figure 3. multi-component model of bank branches

According to the bases of DEA, the efficiency of the service sector (first sector) is defined as follows:

$$E_{1j} = \frac{U_1 Z_{1j}}{V_1 X_{1j}}, j = 1,2, \dots, n \quad (1)$$

In this regard, the efficiency of the mobilization of resources (second part), allocation of resources (third part), management (fourth part), and profitability (fifth part) are defined as relations (2) - (5)

$$E_{2j} = \frac{U_2Z_{2j} + U_3Z_{3j} + U_4Z_{4j}}{V_2X_{2j} + V_3X_{3j}}, j = 1,2, \ldots, n \quad (2)$$

$$E_{3j} = \frac{U_5Z_{5j}}{U_2Z_{2j} + U_3Z_{3j} + U_4Z_{4j}}, j = 1,2, \ldots, n \quad (3)$$

$$E_{4j} = \frac{U_6Z_{6j}}{V_4X_{4j}}, j = 1,2, \ldots, n \quad (4)$$

$$E_{5j} = \frac{W_1Y_{1j}}{U_1Z_{1j} + U_6Z_{6j} + U_5Z_{5j} + V_5X_{5j}}, j = 1,2, \ldots, n \quad (5)$$

By considering the mentioned relationships about networks' sectors efficiency, the general efficiency of the system is defined by having the following relation as a convex combination of its sub-systems efficiency:

$$E_j = \alpha_1E_{1j} + \alpha_2E_{2j} + \alpha_3E_{3j} + \alpha_4E_{4j} + \alpha_5E_{5j} \quad (6)$$

In which,

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1 \quad (7)$$

The general research model for each zero-decision-making unit (DMU_0) will be made as follows by considering small logic limits of efficiency smaller than 1(Mehregan, 2011):

$$\begin{aligned}
&\text{Max } Z_0 = \alpha_1E_{10} + \alpha_2E_{20} + \alpha_3E_{30} + \alpha_4E_{40} + \alpha_5E_{50} \\
&E_{1j} = \frac{U_1Z_{1j}}{V_1X_{1j}} \le 1, j = 1,2, \ldots, n \\
&E_{2j} = \frac{U_2Z_{2j} + U_3Z_{3j} + U_4Z_{4j}}{V_2X_{2j} + V_3X_{3j}} \le 1,, j = 1,2, \ldots, n \\
&E_{3j} = \frac{U_5Z_{5j}}{U_2Z_{2j} + U_3Z_{3j} + U_4Z_{4j}} \le 1,, j = 1,2, \ldots, n \\
&E_{4j} = \frac{U_6Z_{6j}}{V_4X_{4j}} \le 1,, j = 1,2, \ldots, n \\
&E_{5j} = \frac{W_1Y_{1j}}{U_1Z_{1j} + U_6Z_{6j} + U_5Z_{5j} + V_5X_{5j}} \le 1, j = 1,2, \ldots, n \\
&\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1 \\
&U_1, U_2, U_3, U_4, U_5, U_6, V_1, V_2, V_3, V_4, V_5, W_1 \ge 0
\end{aligned} \quad (8)$$

Model Solution Algorithm

The provided model in relation (8) is a non-linear fractional programming model that can't be solved by common techniques. Consequently, an algorithm design is needed. They must change into a linear form with mathematical techniques as the first step to simplify the problem. Limitations will change as follows:

$$\begin{aligned}
&U_1Z_{1j} - V_1X_{1j} \le 0, j = 1,2, \ldots, n \\
&U_2Z_{2j} + U_3Z_{3j} + U_4Z_{4j} - V_2X_{2j} - V_3X_{3j} \le 0, j = 1,2, \ldots, n \\
&U_5Z_{5j} - U_2Z_{2j} - U_3Z_{3j} - U_4Z_{4j} \le 0, j = 1,2, \ldots, n \\
&U_6Z_{6j} - V_4X_{4j} \le 0, j = 1,2, \ldots, n \\
&W_1Y_{1j} - U_1Z_{1j} - U_6Z_{6j} - U_5Z_{5j} - V_5X_{5j} \le 0, j = 1,2, \ldots, n
\end{aligned} \quad (9)$$

Now, all limits are multiplied by the corresponding $\alpha_j, j = 1,2,3,4,5$ coefficient. In this case, the limits of relation (9) will change as follows:

$$\begin{aligned}
&\alpha_1 U_1 Z_{1j} - \alpha_1 V_1 X_{1j} \le 0\\
&\alpha_2 U_2 Z_{2j} + \alpha_2 U_3 Z_{3j} + \alpha_2 U_4 Z_{4j} - \alpha_2 V_2 X_{2j} - \alpha_2 V_3 X_{3j} \le 0\\
&\alpha_3 U_5 Z_{5j} - \alpha_3 U_2 Z_{2j} - \alpha_3 U_3 Z_{3j} - \alpha_3 U_4 Z_{4j} \le 0\\
&\alpha_4 U_6 Z_{6j} - \alpha_4 V_4 X_{4j} \le 0\\
&\alpha_5 W_1 Y_{1j} - \alpha_5 U_1 Z_{1j} - \alpha_5 U_6 Z_{6j} - \alpha_5 U_5 Z_{5j} - \alpha_5 V_5 X_{5j} \le 0
\end{aligned} \tag{10}$$

By changing $\alpha_j U_i = U_{ij}, \alpha_j W_i = W_{ij}$, and $\alpha_j V_i = V_{ij}$ for various i and j, the non-linear equation (10) will change to the following linear form: ($j = 1,2, \dots, n$)

$$\begin{aligned}
&\alpha_1 U_1 Z_{1j} - \alpha_1 V_1 X_{1j} \le 0\\
&\alpha_2 U_2 Z_{2j} + \alpha_2 U_3 Z_{3j} + \alpha_2 U_4 Z_{4j} - \alpha_2 V_2 X_{2j} - \alpha_2 V_3 X_{3j} \le 0\\
&\alpha_3 U_5 Z_{5j} - \alpha_3 U_2 Z_{2j} - \alpha_3 U_3 Z_{3j} - \alpha_3 U_4 Z_{4j} \le 0\\
&\alpha_4 U_6 Z_{6j} - \alpha_4 V_4 X_{4j} \le 0\\
&\alpha_5 W_1 Y_{1j} - \alpha_5 U_1 Z_{1j} - \alpha_5 U_6 Z_{6j} - \alpha_5 U_5 Z_{5j} - \alpha_5 V_5 X_{5j} \le 0
\end{aligned} \tag{11}$$

Model objective function (8) is made from the sum of fractional functions. To solve fractional problems whose objective function is produced by several fractional functions, various methods have been provided by Kornbluth and Steuer (1981), Luhandjula (1984), and Dutta et al. (1992). Dutta et al. (1992) method, inspired by Zimmermann's (1978) method, solves fractional planning problems. This method is also used in this article to solve the problem.

Consider the first part of the model objective function (6), we can see that it means $\alpha_1 E_{1j}$, to describe the mentioned method. It will change to the following expression by replacing E_{1j} expression.

$$\alpha_1 E_{1j} = \alpha_1 E_{1j} = \alpha_1 \frac{U_1 Z_{1j}}{V_1 X_{1j}} = \frac{U_{11} Z_{1j}}{V_1 X_{1j}} \tag{12}$$

According to the traits of DEA model, the up limit of each unit efficiency is 1. Assume that the down limit is defined by decision-makers for the system efficiency and its sectors. Consequently, membership function of the numerator (12) will be shown as following:

$$\mu_{11j} = \begin{cases} 0, & U_{11} Z_{1j} \le \theta_l \\ \dfrac{1 - U_{11} Z_{1j}}{1 - \theta_l}, & \theta_l \le U_{11} Z_{1j} \le 1 \end{cases} \tag{13}$$

Similarly, denominator membership function will make as follows by defining the down limit 1 for denominator and undesirable up limit $\phi_u = 1/\theta$ for denominator (12):

$$\mu_{12j} = \begin{cases} 0, & V_1 X_{1j} \ge \phi_u \\ \dfrac{\phi_u - V_1 X_{1j}}{\phi_u - 1}, & 1 \le V_1 X_{1j} \le \phi_u \end{cases} \tag{14}$$

Considering relations (13) and (14), the maximization of the fractional expression (12) will be equal to $\mu_{11j} + \mu_{12j}$ expression. Membership function for nominator and denominator expressions and denominator of 4 other fractions can be obtained by relation (6) with a similar argument.

For the second expression, $\alpha_2 E_{2j}$, membership functions of the denominator and nominator will be as follows after variable exchanging:

$$\mu_{21j}=\begin{cases}0, & U_{22}Z_{2j}+U_{32}Z_{3j}+U_{42}Z_{4j}\le\theta_l\\ \dfrac{1-U_{22}Z_{2j}-U_{32}Z_{3j}-U_{42}Z_{4j}}{1-\theta_l} & ,\theta_l\le U_{22}Z_{2j}+U_{32}Z_{3j}+U_{42}Z_{4j}\le 1\end{cases} \quad (15)$$

$$\mu_{22j}=\begin{cases}0, & V_2X_{2j}+V_3X_{3j}\ge\phi_u\\ \dfrac{\phi_u-V_2X_{2j}-V_3X_{3j}}{\phi_u-1}, & 1\le V_2X_{2j}+V_3X_{3j}\le\phi_u\end{cases} \quad (16)$$

The membership function will be as follows for the third expression, α_3E_{3j}:

$$\mu_{31j}=\begin{cases}0, & U_{53}Z_{5j}\le\theta_l\\ \dfrac{1-U_{53}Z_{5j}}{1-\theta_l}, & \theta_l\le U_{53}Z_{5j}\le 1\end{cases} \quad (17)$$

$$\mu_{32j}=\begin{cases}0, & U_2Z_{2j}+U_3Z_{3j}+U_4Z_{4j}\ge\phi_u\\ \dfrac{\phi_u-U_2Z_{2j}-U_3Z_{3j}-U_4Z_{4j}}{\phi_u-1} & ,1\le U_2Z_{2j}+U_3Z_{3j}+U_4Z_{4j}\le\phi_u\end{cases} \quad (18)$$

The membership function will be as follows for the third expression, α_4E_{4j}:

$$\mu_{41j}=\begin{cases}0, & U_{64}Z_{6j}\le\theta_l\\ \dfrac{1-U_{64}Z_{6j}}{1-\theta_l}, & \theta_l\le U_{64}Z_{6j}\le 1\end{cases} \quad (19)$$

$$\mu_{42j}=\begin{cases}0, & V_4X_{4j}\ge\phi_u\\ \dfrac{\phi_u-V_4X_{4j}}{\phi_u-1}, & 1\le V_4X_{4j}\le\phi_u\end{cases} \quad (20)$$

Finally, membership function will be as follows for the third expression, α_5E_{5j}:

$$\mu_{51j}=\begin{cases}0, & W_{51}Y_{1j}\le\theta_l\\ \dfrac{1-W_{51}Y_{1j}}{1-\theta_l}, & \theta_l\le W_{51}Y_{1j}\le 1\end{cases} \quad (21)$$

$$\mu_{52j}=\begin{cases}0, & U_1Z_{1j}+U_6Z_{6j}+U_5Z_{5j}+V_5X_{5j}\ge\phi_u\\ \dfrac{\phi_u-U_1Z_{1j}-U_6Z_{6j}-U_5Z_{5j}-V_5X_{5j}}{\phi_u-1} & ,1\le U_1Z_{1j}+U_6Z_{6j}+U_5Z_{5j}+V_5X_{5j}\le\phi_u\end{cases} \quad (22)$$

Relations (13) - (22) followed by limit (11) by maximizing total membership functions will make the final multi-component model to evaluate *DMU₀* as follows:

$$\begin{aligned}&\text{Max } \mu_{110}+\mu_{120}+\mu_{210}+\mu_{220}+\mu_{310}+\mu_{320}+\mu_{410}+\mu_{420}+\mu_{510}+\mu_{520}\\ &U_{11}Z_{1j}-V_{11}X_{1j}\le 0\\ &U_{22}Z_{2j}+U_{32}Z_{3j}+U_{42}Z_{4j}-V_{22}X_{2j}-V_{32}X_{3j}\le 0\\ &U_{53}Z_{5j}-U_{23}Z_{2j}-U_{33}Z_{3j}-U_{43}Z_{4j}\le 0\\ &U_{64}Z_{6j}-V_{44}X_{4j}\le 0\\ &W_{15}Y_{1j}-U_{15}Z_{1j}-U_{65}Z_{6j}-U_{55}Z_{5j}-V_{55}X_{5j}\le 0\\ &\forall i,j: U_{ij}\ge 0, V_{ij}\ge 0, W_{ij}\ge 0, U_i\ge 0, V_i\ge 0,\end{aligned} \quad (23)$$

Multi-Component - Multi-Period Model

The offered model in the previous part reflects the multi-sector structure of the problem. In this part, a multi-period model has been studied. Each model variable makes a time series

following an uncertain statistical distribution in a multi-period structure. For example, X_{1j} is replaced with a random vector $\tilde{X}_{1j} = \left(x_{1j}^{1}, x_{1j}^{2}, \dots, x_{1j}^{T}\right)$ in which x_{1j}^{t} is a x_1 variable amount of unit *j* in the *t*th period. Consequently, we face a type of optimization problem. The concept of the mean-variance of Markowitz has been used to solve this multi-period problem.
Model (23) will change as follows by replacing random vectors:

$$\begin{array}{l}
\text{Max } \tilde{\mu}_{110} + \tilde{\mu}_{120} + \tilde{\mu}_{210} + \tilde{\mu}_{220} + \tilde{\mu}_{310} + \tilde{\mu}_{320} + \tilde{\mu}_{410} + \tilde{\mu}_{420} + \tilde{\mu}_{510} + \tilde{\mu}_{520} \\
U_{11}\tilde{Z}_{1j} - V_{11}\tilde{X}_{1j} \le 0 \\
U_{22}\tilde{Z}_{2j} + U_{32}\tilde{Z}_{3j} + U_{42}\tilde{Z}_{4j} - V_{22}\tilde{X}_{2j} - V_{32}\tilde{X}_{3j} \le 0 \\
U_{53}\tilde{Z}_{5j} - U_{23}\tilde{Z}_{2j} - U_{33}\tilde{Z}_{3j} - U_{43}\tilde{Z}_{4j} \le 0 \\
U_{64}\tilde{Z}_{6j} - V_{44}\tilde{X}_{4j} \le 0 \\
W_{15}\tilde{Y}_{1j} - U_{15}\tilde{Z}_{1j} - U_{65}\tilde{Z}_{6j} - U_{55}\tilde{Z}_{5j} - V_{55}\tilde{X}_{5j} \le 0 \\
\forall i, j: U_{ij} \ge 0, V_{ij} \ge 0, W_{ij} \ge 0, U_i \ge 0, V_i \ge 0
\end{array} \tag{24}$$

$\tilde{\mu}_{11j}$ and $\tilde{\mu}_{12j}$ vectors will be as follows, based on random vectors:

$$\begin{array}{l}
\tilde{\mu}_{11j} = \begin{cases} 0, & U_{11}\tilde{Z}_{1j} \le \theta_l \\ \dfrac{U_{11}\tilde{Z}_{1j} - \theta_l}{1 - \theta_l}, & \theta_l \le U_{11}\tilde{Z}_{1j} \le 1 \end{cases} \\
\tilde{\mu}_{12j} = \begin{cases} 0, & V_1\tilde{X}_{1j} \ge \phi_u \\ \dfrac{\phi_u - V_1\tilde{X}_{1j}}{\phi_u - 1}, & 1 \le V_1\tilde{X}_{1j} \le \phi_u \end{cases}
\end{array} \tag{25}$$

Similarly, relations (15) to (22) will change as follows

$$\tilde{\mu}_{21j} = \begin{cases} 0, & U_{22}\tilde{Z}_{2j} + U_{32}\tilde{Z}_{3j} + U_{42}\tilde{Z}_{4j} \le \theta_l \\ \dfrac{U_{22}\tilde{Z}_{2j} + U_{32}\tilde{Z}_{3j} + U_{42}\tilde{Z}_{4j} - \theta_l}{1 - \theta_l} & \\ , \theta_l \le U_{22}\tilde{Z}_{2j} + U_{32}\tilde{Z}_{3j} + U_{42}\tilde{Z}_{4j} \le 1 & \end{cases} \tag{26}$$

$$\mu_{22j} = \begin{cases} 0, & V_2\tilde{X}_{2j} + V_3\tilde{X}_{3j} \ge \phi_u \\ \dfrac{\phi_u - V_2\tilde{X}_{2j} - V_3\tilde{X}_{3j}}{\phi_u - 1}, & 1 \le V_2\tilde{X}_{2j} + V_3\tilde{X}_{3j} \le \phi_u \end{cases} \tag{27}$$

$$\tilde{\mu}_{31j} = \begin{cases} 0, & U_{53}\tilde{Z}_{5j} \le \theta_l \\ \dfrac{U_{53}\tilde{Z}_{5j} - \theta_l}{1 - \theta_l}, & \theta_l \le U_{53}\tilde{Z}_{5j} \le 1 \end{cases} \tag{28}$$

$$\tilde{\mu}_{32j} = \begin{cases} 0, & U_2\tilde{Z}_{2j} + U_3\tilde{Z}_{3j} + U_4\tilde{Z}_{4j} \ge \phi_u \\ \dfrac{\phi_u - U_2\tilde{Z}_{2j} - U_3\tilde{Z}_{3j} - U_4\tilde{Z}_{4j}}{\phi_u - 1} & \\ , 1 \le U_2\tilde{Z}_{2j} + U_3\tilde{Z}_{3j} + U_4\tilde{Z}_{4j} \le \phi_u & \end{cases} \tag{29}$$

$$\tilde{\mu}_{41j} = \begin{cases} 0, & U_{64}\tilde{Z}_{6j} \le \theta_l \\ \dfrac{U_{64}\tilde{Z}_{6j} - \theta_l}{1 - \theta_l}, & \theta_l \le U_{64}\tilde{Z}_{6j} \le 1 \end{cases} \tag{30}$$

$$\tilde{\mu}_{42j} = \begin{cases} 0, & V_4\tilde{X}_{4j} \ge \phi_u \\ \dfrac{\phi_u - V_4\tilde{X}_{4j}}{\phi_u - 1}, & 1 \le V_4\tilde{X}_{4j} \le \phi_u \end{cases} \tag{31}$$

$$\tilde{\mu}_{51j} = \begin{cases} 0, & W_{51}\tilde{Y}_{1j} \le \theta_l \\ \dfrac{W_{51}\tilde{Y}_{1j} - \theta_l}{1 - \theta_l}, & \theta_l \le W_{51}\tilde{Y}_{1j} \le 1 \end{cases} \tag{32}$$

$$\tilde{\mu}_{52j} = \begin{cases} 0, & U_1\tilde{Z}_{1j} + U_6\tilde{Z}_{6j} + U_5\tilde{Z}_{5j} + V_5\tilde{X}_{5j} \geq \phi_u \\ \dfrac{\phi_u - U_1\tilde{Z}_{1j} - U_6\tilde{Z}_{6j} - U_5\tilde{Z}_{5j} - V_5\tilde{X}_{5j}}{\phi_u - 1} \\ , 1 \leq U_1\tilde{Z}_{1j} + U_6\tilde{Z}_{6j} + U_5\tilde{Z}_{5j} + V_5\tilde{X}_{5j} \leq \phi_u \end{cases} \quad (33)$$

Mean and variance of each random vectors can be calculated by common techniques. For example, random vector$\tilde{X}_{1j} = \left(x_{1j}^1, x_{1j}^2, \dots, x_{1j}^T\right)$ will have the mean of:

$$\bar{X}_{1j} = \frac{\sum_{t=1}^{T} x_{1j}^t}{T}$$

And the variance will be:

$$\sigma_{\tilde{X}_{1j}}^2 = \frac{\sum_{t=1}^{T}\left(x_{1j}^t - \bar{X}_{1j}\right)^2}{T}$$

These values can be calculated for other random vectors with similar techniques, whose repetition is avoided. In the mentioned variable, the index and variance are only replaced with $\tilde{X}_{1j}$.

Based on the Markowitz model, the problem is changed to a double-objective problem including (1) maximization objective function mean and (2) minimization objective function variance to evaluate the multi-sector, multi-period structure. Considering the related relations to mean and variance of random vectors, we can obtain the variance and mean of the model objective function elements (24). Therefore, the mean of the objective function is:

$$\bar{\mu}_j = \frac{U_{11}\bar{Z}_{1j} + U_{22}\bar{Z}_{2j} + U_{32}\bar{Z}_{3j} + U_{42}\bar{Z}_{4j} + U_{53}\bar{Z}_{5j} + U_{64}\bar{Z}_{6j} + W_{51}\bar{Y}_{1j} - 5\theta_l}{1 - \theta_l}$$
$$+ \frac{5\phi_u - V_1\bar{X}_{1j} - V_2\bar{X}_{2j} - V_3\bar{X}_{3j} - U_2\bar{Z}_{2j} - U_3\bar{Z}_{3j} - U_4\bar{Z}_{4j} - V_4\bar{X}_{4j} - U_1\bar{Z}_{1j} - U_6\bar{Z}_{6j}}{\phi_u - 1}$$
$$\frac{-U_5\bar{Z}_{5j} - V_5\bar{X}_{5j}}{\phi_u - 1}$$

The following relation is for the variance of the objective function:

$$\sigma_j^2 = \frac{U_{11}^2\sigma_{\tilde{Z}_{1j}}^2 + U_{22}^2\sigma_{\tilde{Z}_{2j}}^2 + U_{32}^2\sigma_{\tilde{Z}_{3j}}^2 + U_{42}^2\sigma_{\tilde{Z}_{4j}}^2 + U_{53}^2\sigma_{\tilde{Z}_{5j}}^2 + U_{64}^2\sigma_{\tilde{Z}_{6j}}^2 + U_{51}^2\sigma_{\tilde{Y}_{1j}}^2}{(1 - \theta_l)^2}$$
$$+ \frac{V_1^2\sigma_{\tilde{X}_{1j}}^2 + V_2^2\sigma_{\tilde{X}_{2j}}^2 + V_3^2\sigma_{\tilde{X}_{3j}}^2 + U_2^2\sigma_{\tilde{Z}_{2j}}^2 + U_3^2\sigma_{\tilde{Z}_{3j}}^2 + U_4^2\sigma_{\tilde{Z}_{4j}}^2 + V_4^2\sigma_{\tilde{X}_{4j}}^2 + U_1^2\sigma_{\tilde{Z}_{1j}}^2}{(\phi_u - 1)^2}$$
$$\frac{U_6^2\sigma_{\tilde{Z}_{6j}}^2 + U_5^2\sigma_{\tilde{Z}_{5j}}^2 + V_5^2\sigma_{\tilde{X}_{5j}}^2}{(\phi_u - 1)^2}$$

Therefore, the model objective function (24) is replaced with the following two objective functions:

$$\begin{cases} Max\bar{\mu}_j \\ Min\sigma_j^2 \end{cases}$$

Now, consider the limitations of model (24). Consider the first limitation of the model, $U_{11}\tilde{Z}_{1j} - V_{11}\tilde{X}_{1j} \leq 0$. Based on Chebyshev's inequality, $\tilde{Z}_{1j}$ random variable with mean $\bar{Z}_{1j}$

and variance $\sigma^2_{\tilde{Z}_{1j}}$ is in distance $\left(\bar{Z}_{1j} - k\sigma_{\tilde{Z}_{1j}}, \bar{Z}_{1j} + k\sigma_{\tilde{Z}_{1j}}\right)$ with a minimum probability of $100(1-\alpha)\%$ in which $k = \sqrt{1/\alpha}$ (Azar and Momeni, 2015). Consequently, the random variables of this limitation will change by being replaced with confidence intervals as follows:

$$U_{11}\left(\bar{Z}_{1j} - k\sigma_{\tilde{Z}_{1j}}, \bar{Z}_{1j} + k\sigma_{\tilde{Z}_{1j}}\right) - V_{11}\left(\bar{X}_{1j} - k\sigma_{\tilde{X}_{1j}}, \bar{X}_{1j} + k\sigma_{\tilde{X}_{1j}}\right) \le 0$$

This relation will change by having some operations on distance numbers as follows:

To have a distance number of the left side smaller than the number of the right side, the upper limit and the mean of this number must be smaller than the number of the right side. (Ayshybvchy and Tanaka, 1990) Consequently, the mentioned inequity will be replaced with the two following inequities:

$$\begin{cases} U_{11}\left(\bar{Z}_{1j} + k\sigma_{\tilde{Z}_{1j}}\right) - V_{11}\left(\bar{X}_{1j} - k\sigma_{\tilde{X}_{1j}}\right) \le 0 \\ U_{11}\bar{Z}_{1j} - V_{11}\bar{X}_{1j} \le 0 \end{cases}$$

Similarly, other limitations of model (24) are replaced with the two equal limitations by replacing Chebyshev distance with random variables, using relationships among distance numbers and relationships among numbers. Consequently, final double- objective model with model (24) will be obtained as follows:

$$\begin{aligned}
&\mathrm{Max}\,\bar{\mu}_j \\
&\mathrm{Min}\,\sigma_j^2 \\
&\mathrm{S.T.} \\
&U_{11}\left(\bar{Z}_{1j} + k\sigma_{\tilde{Z}_{1j}}\right) - V_{11}\left(\bar{X}_{1j} - k\sigma_{\tilde{X}_{1j}}\right) \le 0 \\
&U_{11}\bar{Z}_{1j} - V_{11}\bar{X}_{1j} \le 0 \\
&U_{22}\left(\bar{Z}_{2j} + k\sigma_{\tilde{Z}_{2j}}\right) + U_{32}\left(\bar{Z}_{3j} + k\sigma_{\tilde{Z}_{3j}}\right) + U_{42}\left(\bar{Z}_{4j} + k\sigma_{\tilde{Z}_{4j}}\right) - V_{22}\left(\bar{X}_{2j} - k\sigma_{\tilde{X}_{2j}}\right) - V_{32}\left(\bar{X}_{3j} - k\sigma_{\tilde{X}_{3j}}\right) \\
&\qquad \le 0 \\
&U_{22}\bar{Z}_{2j} + U_{32}\bar{Z}_{3j} + U_{42}\bar{Z}_{4j} - V_{22}\bar{X}_{2j} - V_{32}\bar{X}_{3j} \le 0 \\
&U_{53}\left(\bar{Z}_{5j} + k\sigma_{\tilde{Z}_{5j}}\right) - U_{23}\left(\bar{Z}_{2j} + k\sigma_{\tilde{Z}_{2j}}\right) - U_{33}\left(\bar{Z}_{3j} + k\sigma_{\tilde{Z}_{3j}}\right) - U_{43}\left(\bar{Z}_{4j} + k\sigma_{\tilde{Z}_{4j}}\right) \le 0 \\
&U_{53}\bar{Z}_{5j} - U_{23}\bar{Z}_{2j} - U_{33}\bar{Z}_{3j} - U_{43}\bar{Z}_{4j} \le 0 \\
&U_{64}\left(\bar{Z}_{6j} + k\sigma_{\tilde{Z}_{6j}}\right) - V_{44}\left(\bar{X}_{4j} - k\sigma_{\tilde{X}_{4j}}\right) \le 0 \\
&U_{64}\bar{Z}_{6j} - V_{44}\bar{X}_{4j} \le 0 \\
&W_{51}\left(\bar{Y}_{1j} + k\sigma^2_{\tilde{Y}_{1j}}\right) - U_{15}\left(\bar{Z}_{1j} - k\sigma_{\tilde{Z}_{1j}}\right) - U_{65}\left(\bar{Z}_{6j} - k\sigma_{\tilde{Z}_{6j}}\right) - U_{55}\left(\bar{Z}_{5j} - k\sigma_{\tilde{Z}_{5j}}\right) - V_{55}\left(\bar{X}_{5j} - k\sigma_{\tilde{X}_{5j}}\right) \\
&\qquad \le 0 \\
&W_{51}\bar{Y}_{1j} - U_{15}\bar{Z}_{1j} - U_{65}\bar{Z}_{6j} - U_{55}\bar{Z}_{5j} - V_{55}\bar{X}_{5j} \le 0 \\
&\forall i, j{:}\ U_{ij} \ge 0, V_{ij} \ge 0, W_{ij} \ge 0, U_i \ge 0, V_i \ge 0,
\end{aligned} \tag{34}$$

Model (34) is a non-linear bi-objective model with linear limitations. In this step, a fuzzy approach was used to solve model (34). Assume that the total limitations of model (34) are shown with X, and the decision variables with $x = \left\{\forall i, j{:}\, U_{ij} \ge 0, V_{ij} \ge 0, W_{ij} \ge 0, U_i \ge 0, V_i \ge 0\right\}$. In this regard, model (34) is summarized as follows:

$$\begin{aligned}
&\mathrm{Max}\,\bar{\mu}_j \\
&\mathrm{Min}\,\sigma_j^2 \\
&\mathrm{S.T.}\, x \in X
\end{aligned} \tag{35}$$

To solve this model, two ideal and anti-ideal values were obtained for each $\bar{\mu}_j$ and σ_j^2

functions. The ideal value can be obtained from the solution of the single-objective problem for $\bar{\mu}_j$ function:

$$\begin{aligned} &\mathrm{Max}\bar{\mu}_j \\ &\mathrm{S.T.}\, x \in X \end{aligned} \tag{36}$$

Assume that the optimum solution of this problem is μ_j^*. The anti-ideal value is $\mu_j^- = 0$. Similarly, the ideal value of the variance function will be $\sigma_j^{2*} = 0$ an anti-ideal value will be obtained by solving the following model.

$$\begin{aligned} &\mathrm{Max}\sigma_j^2 \\ &\mathrm{S.T.}\, x \in X \end{aligned} \tag{37}$$

This is an optimization, order 2 problem, and it is simply solved by the existing software. Assume that the anti-ideal value is σ_j^{2-}.

According to these ideal and anti-ideal values, the mean membership function can be made as follows:

$$\lambda(\bar{\mu}_j) = \begin{cases} 0, & 0 \geq \bar{\mu}_j \\ \dfrac{\bar{\mu}_j}{\mu_j^*}, & 0 < \bar{\mu}_j \leq \mu_j^* \end{cases}$$

Actually, if the mean is smaller/equal to zero, its desirability is zero. Then this desirability will increase linearly in $0 < \bar{\mu}_j \leq \mu_j^*$ distance. The membership function relation for the variance function will be as follows:

$$\lambda(\sigma_j^2) = \begin{cases} 0, & \sigma_j^{2-} \leq \sigma_j^2 \\ \dfrac{\sigma_j^{2-} - \sigma_j^2}{\sigma_j^{2-}}, & 0 \leq \sigma_j^2 < \sigma_j^{2-} \end{cases}$$

According to the additivity of two functions $\lambda(\bar{\mu}_j)$ and $\lambda(\sigma_j^2)$, the dual-objective problem of model (34) will change to a single-objective problem as follows:

$$\begin{aligned} &\mathrm{Max}\lambda(\bar{\mu}_j) + \lambda(\sigma_j^2) \\ &\mathrm{S.T.}\, x \in X \end{aligned} \tag{38}$$

Values of decision-making variables will be obtained by solving the mentioned non-linear single-objective model, and variables of the mean efficiency of various sectors can be obtained by replacing these coefficients in the relations E_{1j} ‘E_{2j} ‘E_{3j} ‘$E_{4j,}$ and E_{5j} and using the mean values of input and output indexes.

Case Study

The proposed model in Eq. (38) is a linear programming model that is solved by the present methods. In this part, the mentioned model is used to evaluate the performance of 439 branches of one Iranian bank.

After the collection of the related data to input and output variables, a MATLAB code has been used to solve model (38) for each mentioned branch. Performance histogram of each five-bank sector is shown in Figures 4 to 8.

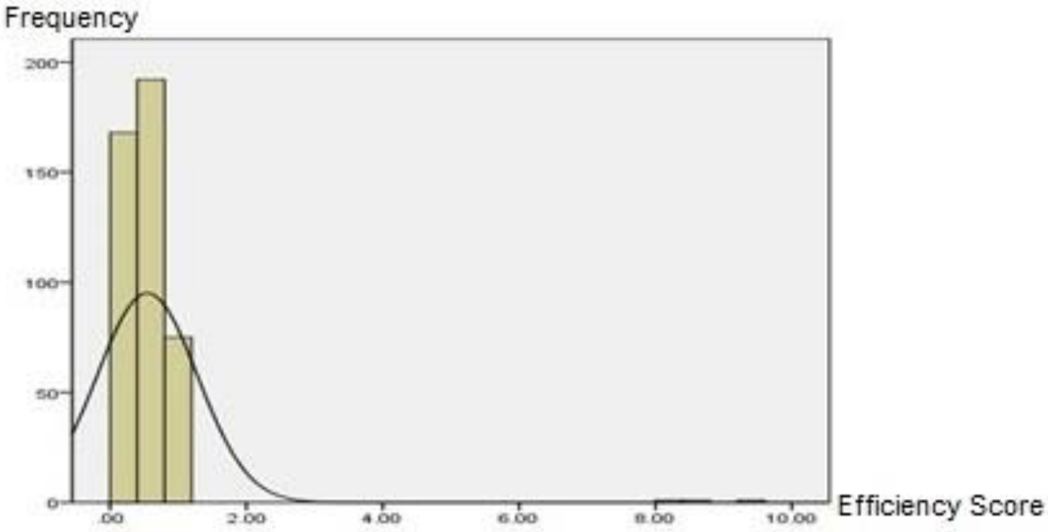

Figure 4. Frequency Distribution of Services Sector Efficiency

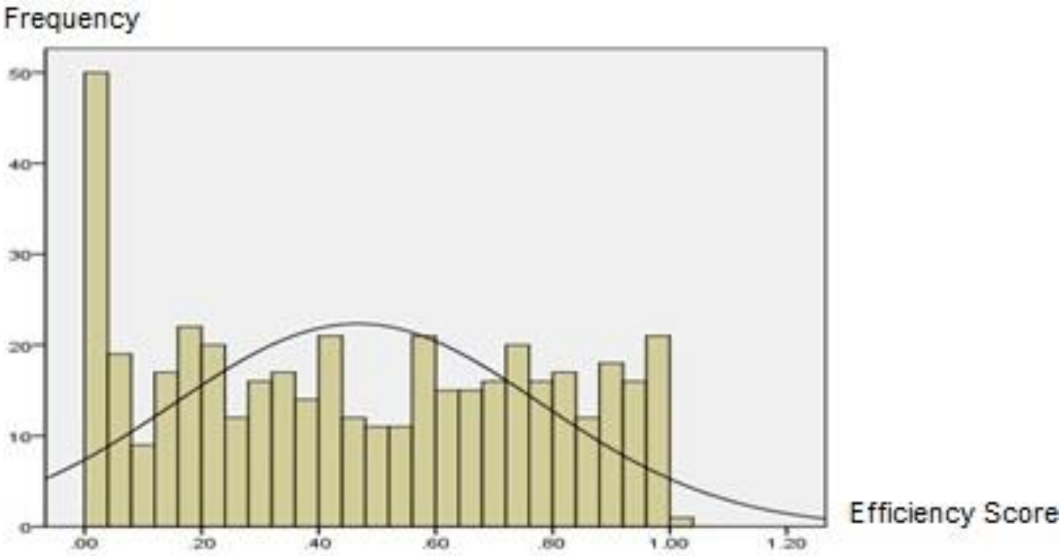

Figure5.Frequency Distribution of Mobilization of Resources Sector Efficiency

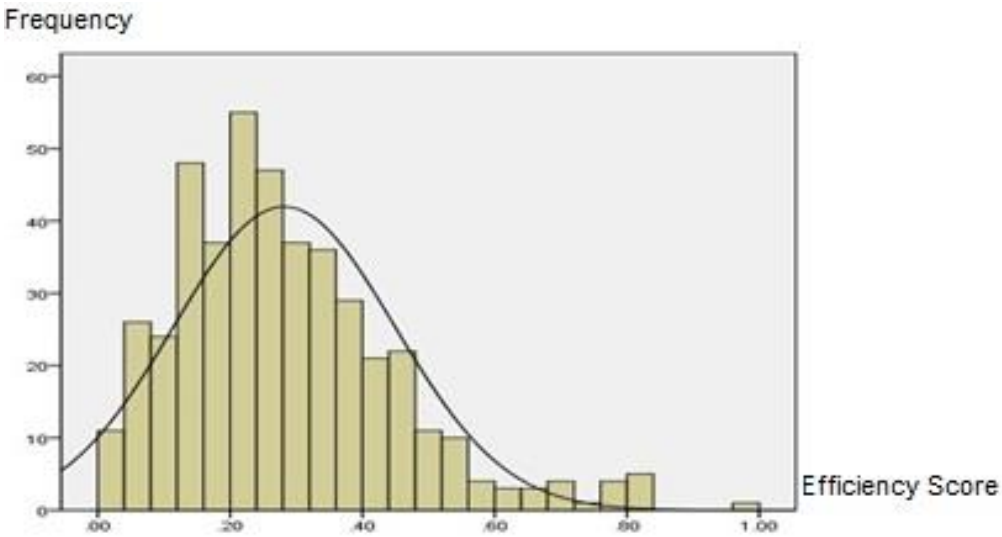

Figure 6. Frequency Distribution of Allocation of Resources Sector Efficiency

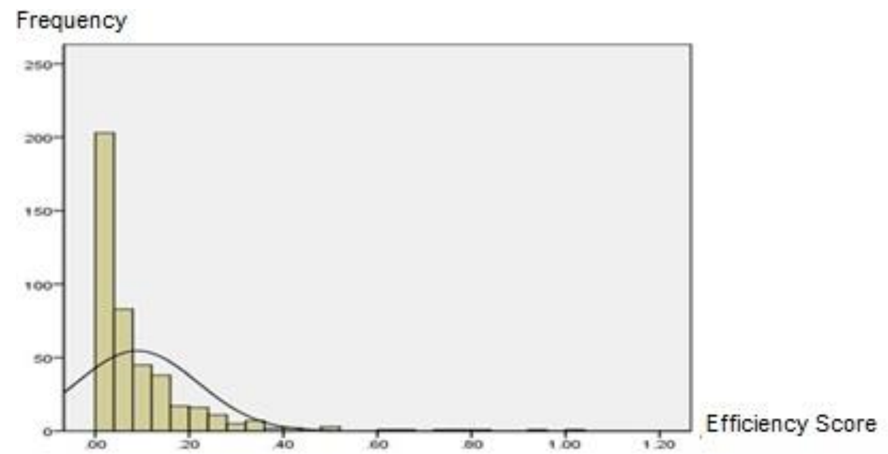

Figure7. Frequency Distribution of Management Sector Efficiency

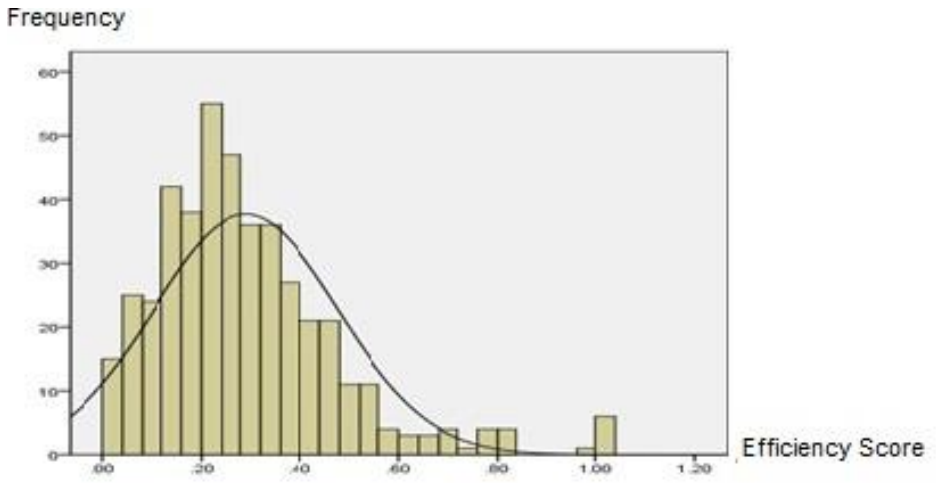

Figure 8. Frequency Distribution of Profitability Sector Efficiency

Variance analysis has been used to more accurately study the bank's various efficiency results, comparing equality of several population means, which means evaluating the lack of difference among efficiency means of five bank structure sectors. The mentioned hypothesis is defined as follows:

$$\begin{cases} H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \\ H_1: \text{at least one equity is not sarisfied.} \end{cases}$$

In which, $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ are mean efficiencies of service, mobilization of resources, allocation of resources, management, and profitability, respectively, which have been obtained from model-solving results. The results of this test are shown in Table 3.

Table 3. One-Way ANOVA for Differences in Efficiency Among Bank Sectors

Source of Variation	Sum of Squares	df	Mean Squares	Statistic F	Sig.(p-value)
Between Sectors	451/55	4	863/13	663/96	000/0
Within Sectors	077/314	2190	143٠/		
Total	528/369	2194			

According to the mentioned table, the P-value was calculated as 0.000. Consequently, H_0 (indifference to various sectors' efficiency) was rejected at 5% confidence level. Paired comparisons of various sectors' mean efficiency are shown in Table 4, using the least significant difference (LSD) test.

Table 4. 95% Confidence Intervals for Efficiency Differences Between Sectors

Sector(I)	Sector(J)	Mean Difference	Std. Error	Sig
Services	Mobilization	٠/07751*	02556٠/	٠/002
	Allocation	٠/26258*	02556٠/	٠/000
	Management	٠/45481*	02556٠/	٠/000
	Profitability	٠/25432*	02556٠/	٠/000
Mobilization	Services	-٠/07751*	02556٠/	٠/002
	Allocation	٠/18507*	02556٠/	٠/000
	Management	٠/37730*	02556٠/	٠/000
	Profitability	٠/17681*	02556٠/	٠/000
Allocation	Services	-٠/26258*	02556٠/	٠/000
	Mobilization	-٠/18507*	02556٠/	٠/000
	Management	٠/19222*	02556٠/	٠/000
	Profitability	-٠/00827	02556٠/	٠/746
Management	Services	-٠/45481*	02556٠/	٠/000
	Mobilization	-٠/37730*	02556٠/	٠/000
	Allocation	-٠/19222*	02556٠/	٠/000
	Profitability	-٠/20049*	02556٠/	٠/000
Profitability	Services	-٠/25432*	02556٠/	٠/000
	Mobilization	-٠/17681*	02556٠/	٠/000
	Allocation	٠/00827	02556٠/	٠/746
	Management	٠/20049*	02556٠/	٠/000

Conclusion

According to the calculated significant level, the mean efficiency of allocation of resources and profitability sectors does not show a significant difference, while the difference between means of other sectors is considerable. The mean efficiency of each sector was 54.55%, for the service sector 46.68%, for the mobilization of resources 28.17%, 29%, and 8.95% for allocation of resources, profitability, and management, respectively, at a 95% significance level.

Evaluating efficiency is one of the most important challenges faced by managers in the dynamic and vital banking industry. A lack of efficiency implies high costs of funds within banks, which in turn leads to increased expenses, reduced profitability, and potential financial crises. Therefore, adopting a deep and rigorous approach to efficiency assessment in banks is of great significance.

Models based on Data Envelopment Analysis (DEA) have been widely used for measuring and evaluating bank efficiency. However, a major shortcoming of classical DEA models is their failure to consider the internal structure and workflow within bank branches and departments. Previous research has attempted to identify and evaluate such internal structures in banking systems.

In the present study, after conducting interviews with banking experts, a localized operational flow typical of Iranian banks was identified. This flow consists of five main components: mobilization of resources, allocation of resources, management, profitability, and services. Based on this structure, an efficiency evaluation model was designed, and a fuzzy algorithm-based approach was developed to solve it.

Applying the proposed model to 210 branches of one of the country’s major banks showed that, although the services and mobilization sectors demonstrate relatively higher efficiency, the management sector suffers from significant inefficiency. This finding highlights the need for greater managerial attention in areas such as loan allocation, customer service quality, and the reduction of financial and non-financial costs, which could help address the observed shortcomings.

Based on the obtained weights for the evaluation indicators and the performance of branches on these indicators, one major reason for the low managerial efficiency appears to be the absence of quantitative and standardized criteria for evaluating the qualitative aspects of branch management. Moreover, increasing resource mobilization by reducing operational costs and boosting joint revenues can improve both overall efficiency and the efficiency of the allocation and profitability sectors. Furthermore, the high level of non-performing loans, as an undesirable output, has contributed to both overall and managerial inefficiency within the bank.

The current study deals with several limitations. first of all, the input and output variables data are considered as exact, while usually they are determined with uncertainty. On the other hand, the problem is formulated in a single time period, while bank branches are performed sequentially over time. Finally, the network components' efficiencies are aggregated using a weighted linear combination, while the network structures can be handled using other methods reviewed by Kao (2014) and Ranter, Shaposhnikov, and Lychev (2023).

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