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RESEARCH PAPER

Fuzzy Multi-Criteria Decision-Making Method F-PSWCA: A Case Study on the Selection and Ranking of Criteria and New Technologies in Dialysis

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Abstract

Selecting appropriate technologies in healthcare systems is a complex decision-making problem involving multiple, often conflicting criteria under uncertainty. In this paper, a novel fuzzy multi-criteria decision-making (MCDM) method, referred to as F-PSWCA, is proposed to enhance the criteria weighting process by incorporating historical performance trends within a fuzzy environment. The proposed method integrates fuzzy regression parameters, including slope, intercept, and coefficient of determination (R^2), to capture both the magnitude and stability of criteria over time. Unlike conventional fuzzy MCDM approaches that rely solely on static expert judgments, F-PSWCA enables dynamic assessment of criteria importance while preserving uncertainty representation. The applicability of the proposed method is demonstrated through a real-world case study on the selection of dialysis water purification technologies, where multiple technical, economic, and operational criteria are considered. Comparative analysis with Fuzzy SAW, Fuzzy TOPSIS, and Fuzzy SECA is conducted to evaluate the robustness and consistency of the results. The findings indicate that while ranking similarities may occur across methods, F-PSWCA provides additional interpretive insights by distinguishing between temporally stable and unstable criteria. The results confirm the effectiveness of the proposed approach as a decision-support tool for technology selection in complex and evolving healthcare environments.

Keywords:
Fuzzy Regression, Fuzzy Decision-Making, Innovation Ranking, Dialysis.

1. Introduction

Decision-making regarding the selection of emerging technologies presents a significant challenge for managers and decision-makers across various industries due to the complexity and multifaceted nature of the process. Often, new technologies are introduced with diverse features and capabilities, each offering specific advantages and disadvantages. Furthermore, selecting a particular technology depends on numerous factors, which may carry different priorities depending on the context. This complexity is especially pronounced in sensitive fields such as medical equipment, where technologies impact on human health.

A primary challenge in technology-related decision-making is the multitude of evaluation

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criteria. These criteria are not always quantifiable; some are qualitative or can only be estimated approximately due to data limitations. Another challenge is the uncertainty and instability of data. Emerging technologies are often in early developmental stages, with limited information available about their real-world performance in operational environments. Furthermore, the environmental and operational conditions of a technology impact on the performance of a system.

Another layer of complexity arises from the interrelationships and mutual influences among criteria. For example, a technology with high initial costs might prove more economical in the long term due to greater efficiency or reduced maintenance requirements. These interactions among criteria often require sophisticated analyses that traditional decision-making methods may not fully address. Additionally, technology selection frequently involves considering the diverse needs of various stakeholders, which can lead to conflicts of interest and further complicate the decision-making process. The need to account for long-term implications also adds to this complexity. A technology that appears efficient in the short term may become less viable over time due to issues such as reduced availability of spare parts or frequent maintenance needs. Therefore, decision-makers must carefully evaluate both short-term benefits and long-term sustainability and costs.

These challenges highlight the necessity for developing and adopting comprehensive tools and methodologies for selecting emerging technologies. Such tools should not only analyze existing data accurately but also effectively manage the uncertainties and complexities inherent in the process, thereby enabling decision-makers to make more informed choices.

MCDM techniques have demonstrated their effectiveness in scenarios involving conflicting objectives, such as selecting suppliers for hospital medical equipment (Chakraborty et al., 2023). These methods have also been employed to prioritize medical waste disposal methods (Beheshtinia et al., 2023) and evaluate supplier platforms for medical equipment (Saleh et al., 2023). In healthcare, MCDM methods have been applied to prioritize population-based nutritional interventions (Aliasgharzadeh et al., 2022) and manage diabetes (Aldaghi & Muzik, 2024). These studies have assessed the efficacy and applicability of MCDM methods, highlighting their potential in decision-making processes within the healthcare industry. Several studies have focused on evaluating emergency departments and aligning medical equipment with existing workforce and infrastructure (Ortíz-Barrios et al., 2023). A systematic review of MCDM and fuzzy MCDM techniques in healthcare underscores their significance in addressing complex decision-making challenges (Chakraborty et al., 2023).

The assessment and ranking of medical equipment suppliers using fuzzy TOPSIS have been explored as a method for evaluating the effectiveness of medical devices in healthcare organizations (Shbool et al., 2021). Additionally, occupational health and safety risks, such as defective medical equipment in workplaces, have been identified using fuzzy MCDM frameworks (Badida et al., 2023). Fuzzy VIKOR and fuzzy AHP methods have also been employed for selecting hospital medical equipment suppliers, underscoring the importance of utilizing diverse decision-making techniques (Chakraborty et al., 2023).

In healthcare management, MCDM methods have been applied for various purposes, including evaluating healthcare system performance (Ortiz-Barrios et al., 2024). Fuzzy TOPSIS method has been used to analyze medical device security and rank suppliers based on effectiveness and safety criteria (Ansari et al., 2020). Additionally, the application of fuzzy TOPSIS to assess medical equipment security through rule-based fuzzy systems has received considerable attention (Yadav et al., 2018).

Hybrid fuzzy MCDM methods have been developed to evaluate the performance of medical equipment manufacturers, highlighting the significance of fuzzy set theory in medical and healthcare fields (Kaya et al., 2022). Furthermore, the integration of various fuzzy MCDM methods has been investigated for prioritizing risks in the healthcare and insurance industries

(Jafarzadeh Ghoushchi et al., 2023). Comparative studies among fuzzy SWARA, VIKOR, and MOORA methods have assessed their effectiveness in weighting criteria and ranking alternatives (Singh et al., 2024). Additionally, hybrid fuzzy decision-making approaches have been proposed to address complex challenges in healthcare supply chain management, underscoring the need for efficient and sustainable solutions (Alamroshan et al., 2022).

The applications of MCDM methods, including fuzzy AHP and ANP, have been reviewed in the context of the COVID-19 pandemic, demonstrating their adaptability in healthcare decision-making (Sotoudeh-Anvari, 2022). Another study by Gao et al., (2024) developed an innovative group decision-making framework using BWM-Entropy-COPRAS to evaluate supply chain sustainability risks. This research used fuzzy sets and applied them in desalination technology selection. Torkayesh et al., (2023) provided an advanced review of evaluation methods based on distance from ideal solutions, applying fuzzy EDAS for medical device selection and hospital site location.

Overall, the existing literature on MCDM applications in healthcare underscores the importance of these methods in enhancing decision-making processes and efficiently prioritizing resources. By utilizing MCDM techniques, healthcare professionals can make more informed decisions in ranking medical and healthcare equipment, ultimately leading to better outcomes for patients and healthcare organizations (Taherdoost & Madanchian, 2023).

Recent advancements in healthcare decision-making have increasingly focused on addressing the dynamic nature of technological performance and data uncertainty. (Goldani & Ishizaka, 2025) developed a hybrid fuzzy BWM-MULTIMOORA framework specifically for selecting healthcare waste treatment technologies, emphasizing the need for consistency in expert judgments within medical settings.

Similarly, Puška et al., (2024) introduced a fuzzy-rough approach for healthcare device selection, highlighting that traditional methods often fail to capture the 'shaky data' inherent in rapid technological transitions. In the realm of weighting mechanisms, Satria et al., (2025) developed a modification of Grey Relational Analysis for dynamic criteria weighting, addressing the need for models that adapt to evolving data patterns over time. Furthermore, Raveena and Umamaheswari (2025) proposed an integrated multi-criteria decision-making framework for sustainable supplier selection in the pharmaceutical industry. Their study incorporates pioneering criteria covering economic, environmental, and social dimensions of sustainability, in addition to traditional evaluation factors. To address uncertainty and vagueness in expert judgments, fuzzy logic was employed, and a hybrid approach combining Fuzzy TOPSIS, Support Vector Regression (SVR), and Grey Relational Analysis (GRA) was applied to evaluate and rank suppliers. The results demonstrate that the proposed framework enhances decision-making accuracy and provides an effective tool for sustainable supplier selection in pharmaceutical supply chains.

Since technological advancements typically depend on historical data that offer insights into growth and development trends, it is essential to carefully analyze these trends in the decision-making process. However, evaluation criteria may not always have assessed values for all alternatives in the past, or the data may contain inadvertent errors. Therefore, it is necessary to develop a method capable of identifying, evaluating, and mitigating these uncertainties and inaccuracies.

In situations where data are limited or ambiguous, relying solely on the results of previous methods is insufficient. Under such conditions, sensitivity analysis plays a crucial role, as changes in data and criteria can significantly affect outcomes. To date, no decision-making method has provided a comprehensive and systematic approach to sensitivity analysis that determines the impact of variations in criteria. Most weighting methods rely primarily on standard deviation or data variations within a decision matrix and may incorporate data correlation or expert opinions as influencing factors. However, when decision-making involves uncertain data, these methods fail to use critical criteria such as temporal changes, technological growth or decline trends, and data accuracy and reliability in the weighting process.

Moreover, the logical and systematic integration of various weighting methods within the framework of existing decision-making approaches has often been overlooked. In many cases, such combinations are made arbitrarily and without relying on logical criteria, ultimately failing to provide coherent and reliable reference weights. Developing and presenting a model that addresses these challenges and aids decision-making under uncertainty is not only a scientific necessity but also a critical step toward enhancing the quality of technological decision-making.

When data are fuzzy, modeling, analysis, and processing procedures become significantly more complex and differ substantially from those used for deterministic data. In fuzzy data, values are not precise and specific but are often associated with uncertainty, ambiguity, or degrees of probability. This characteristic renders traditional decision-making methods, which rely on precise and numerical data, ineffective or even unusable.

Modeling fuzzy data requires tools and techniques capable of transforming ambiguous concepts into quantifiable and analyzable forms. Some recently proposed decision-making methods not only significantly increase computational complexity when handling fuzzy data but may also lead to greater ambiguity in the results, particularly when the number of criteria or decision alternatives is large. However, processing fuzzy data necessitates algorithms and methods that can effectively utilize ambiguous information to generate reliable results.

Our study focuses on decision-making processes that depends on the historical trends of alternatives. This reliance highlights the importance of analyzing and understanding past data. By integrating historical data, sensitivity analysis, and advanced weighting methods, we can develop a comprehensive and effective approach to this issue. Such an approach must handle uncertain and ambiguous data, identify key criteria, and adapt to changes in input information to achieve scientifically valid results. This is the focus of our research, which aims to be both scientifically defensible and practically impactful.

2. Problem Statement

Specifically, this method assists managers and decision-makers in identifying and selecting options that best align with key evaluation criteria from various alternatives. Each criterion involves a degree of uncertainty and relative importance, which are analyzed and evaluated in the proposed method using regression coefficients within a fuzzy framework.

For example, in the field of medical equipment, new technologies may provide limited or inaccurate data regarding their actual performance during the early stages of development. These technologies are often evaluated based on limited information, such as initial tests, early user feedback, or simulations. The proposed method can model this incomplete data using fuzzy variables to evaluate the performance of different alternatives.

One of the key aspects of this method is its emphasis on accurately ranking evaluation criteria. Prioritizing these criteria can be challenging due to their interdependencies or the unpredictable impact of external factors. In selecting medical technologies, criteria such as patient safety, compatibility with existing infrastructure, production costs, and long-term sustainability may vary in importance.

Many criteria for evaluating new technologies are qualitative, such as user satisfaction or compliance with ethical standards. Technologies that have yielded low returns in the past or failed in early tests may be identified as higher-risk options and given less weight in the decision-making process. This feature enables decision-makers to allocate resources more efficiently and focus on options with a higher chance of success. This transparency allows decision-makers to provide well-documented reasons for selecting or eliminating any option, which can be highly valuable in negotiations and communication with stakeholders.

3. Proposed Fuzzy PSWCA Method

The Fuzzy PSWCA method, an extension of PSWCA (Pazhouhandeh & Samouei, 2024), evaluates multi-criteria decision-making problems by simultaneously evaluating the values of decision criteria simultaneously, such that changes over time in option scores over time. Consequently, our objective is not only to determine the current decision-making information matrix but also to incorporate matrices for information from previous time periods.

One of the innovations presented in this paper is the design of a novel decision-making method based on regression, intended for evaluating and ranking options and criteria under fuzzy conditions and uncertainty. This method employs regression coefficients as the primary tool to analyze relationships between criteria and options, enabling a comprehensive review of past technologies and predicting their future performance. Since regression is based on historical data, this method uniquely allows the evaluation of innovations based on the performance of classical predecessors, potentially leading to more accurate and objective decision-making.

Considering that innovations typically bring changes to one or more aspects of existing technologies (except for innovations that result in fundamentally new products), this research, innovation refers to existing practical technologies. Therefore, previous data related to a technology can play a crucial role in providing more precise and efficient evaluations.

Using decision table information and a historical data matrix, a regression equation is derived to obtain the slope, intercept, and coefficient of determination. The normalized values of these parameters are used to calculate a reference weight. This reference weight is then placed into a multi-objective nonlinear mathematical model, which is solved to determine the value of each option. The goal of this mathematical model is to reach the highest decision-making value.

First, we need to define a decision matrix with m options and n criteria. If the matrix $\tilde{X}$ represents the fuzzy evaluation of criteria relative to the options, for each option $i \in (1,2,\dots,m)$ and each criterion $j \in (1,2,\dots,n)$, we have the following equation:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \tag{1}$$

Then, we consider a matrix $\tilde{Y}$ with the same dimensions as the original matrix $\tilde{X}$ to specify the evaluation information of criteria relative to the options at an earlier time. $\tilde{Y}$ provides this information and is expressed by Equation (2).

$$\tilde{Y} = \begin{bmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{y}_{m1} & \cdots & \tilde{y}_{mn} \end{bmatrix} \tag{2}$$

The matrices $\tilde{X}$ and $\tilde{Y}$ must be positive, and their normalization should be performed after determining the initial matrix. The indicators can be either positive or negative; in this case, Equation (3) is used to normalize positive indicators (BC), and Equation (4) is used for normalizing negative indicators (NC). After normalization, we have the normalized matrices $\tilde{X}^N$ and $\tilde{Y}^N$ (expressed in Equations 5 and 6).

$$\tilde{x}_{ij}^N, \tilde{y}_{ij}^N = \left(\frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+}\right), u_j^+ = Max\, u_{ij}\ for\ Benefit\ criteria \tag{3}$$

$$\tilde{x}_{ij}^N, \tilde{y}_{ij}^N = \left(\frac{l_j^-}{u_j}, \frac{l_j^-}{m_j}, \frac{l_j^-}{l_j}\right), l_j^- = Min\, l_{ij}\ for\ Negative\ criteria \tag{4}$$

$$\tilde{X}^N = \begin{bmatrix} \tilde{x}_{11}^N & \cdots & \tilde{x}_{1n}^N \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1}^N & \cdots & \tilde{x}_{mn}^N \end{bmatrix} \tag{5}$$

$$\tilde{Y}^N = \begin{bmatrix} \tilde{y}_{11}^N & \cdots & \tilde{y}_{1n}^N \\ \vdots & \ddots & \vdots \\ \tilde{y}_{m1}^N & \cdots & \tilde{y}_{mn}^N \end{bmatrix} \tag{6}$$

If $\tilde{X}$ and $\tilde{Y}$ are triangular fuzzy numbers, such that $\tilde{X} = (l, m, u)$ and $\tilde{Y} = (a, b, c)$, and $[(\tilde{X}_i)_\alpha^l, (\tilde{X}_i)_\alpha^u]$ and $\left[(\tilde{Y}_i)_\alpha^l, (\tilde{Y}_i)_\alpha^u\right]$ represent the lower and upper bounds of $\tilde{X}$ and $\tilde{Y}$, which are α -cut fuzzy numbers, then the α -cut of these fuzzy numbers is expressed by Equation (7):

$$\widetilde{X_i)}_\alpha = [(m-l)\alpha + l, u - (u-m)\alpha] = [(\tilde{X}_i)_\alpha^l, (\tilde{X}_i)_\alpha^u] \tag{7-1}$$

$$\widetilde{Y_i)}_\alpha = [(b-a)\alpha + a, c - (c-b)\alpha] = \left[(\tilde{Y}_i)_\alpha^l, (\tilde{Y}_i)_\alpha^u\right] \tag{7-2}$$

The coefficients of the regression equation, obtained from the two matrices, $\tilde{X}$ and $\tilde{Y}$ include the slope of the relationship $(\tilde{B}_j)$ the intercept $(\widetilde{B0}_j)$, and the coefficient of determination or explained variance $(\tilde{R}_j^2)$, which are derived from Equations (8), (9), and (10), respectively. These coefficients are obtained through the set of variables $\boldsymbol{x_{ij}}$ and $\boldsymbol{y_{ij}}$, which are the optimal evaluation values used in the above coefficients.

$$\begin{aligned} &\widetilde{\boldsymbol{B_j}} = \left(\boldsymbol{Min}\ (\boldsymbol{B}_j)_\alpha, (\boldsymbol{B}_j)_{\alpha=1} \mathbf{Max}\ (\boldsymbol{B}_j)_\alpha\right), \boldsymbol{B_j} = \frac{\sum_{i=1}^m (\boldsymbol{y_{ij}} - \bar{\boldsymbol{y}})(\boldsymbol{x_{ij}} - \bar{\boldsymbol{x}})}{\sum_{i=1}^m (\boldsymbol{x_{ij}} - \bar{\boldsymbol{x}})^2} \\ &st. \\ &\quad (\tilde{X}_i)_\alpha^l \le \boldsymbol{x_{ij}} \le (\tilde{X}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \\ &(\tilde{Y}_i)_\alpha^l \le \boldsymbol{y_{ij}} \le (\tilde{Y}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \end{aligned} \tag{8}$$

$$\begin{aligned} &\widetilde{\boldsymbol{B0}}_j = \left(\boldsymbol{Min}\ (\boldsymbol{B0}_j)_\alpha, (\boldsymbol{B0}_j)_{\alpha=1} \mathbf{Max}\ (\boldsymbol{B0}_j)_\alpha\right), \boldsymbol{B0_j} = \bar{\boldsymbol{y}} - \boldsymbol{B_j}\bar{\boldsymbol{x}} \\ &st. \\ &(\tilde{X}_i)_\alpha^l \le \boldsymbol{x_{ij}} \le (\tilde{X}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \\ &(\tilde{Y}_i)_\alpha^l \le \boldsymbol{y_{ij}} \le (\tilde{Y}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \end{aligned} \tag{9}$$

$$\begin{aligned} &\widetilde{\boldsymbol{R}_j^2} = \left(\boldsymbol{Min}\ \left(\boldsymbol{R}_j^2\right)_\alpha, \left(\boldsymbol{R}_j^2\right)_{\alpha=1}, \mathbf{Max}\ \left(\boldsymbol{R}_j^2\right)_\alpha\right), \boldsymbol{R}_j^2 \\ &\qquad = \frac{\left(\boldsymbol{m}\left(\sum_{i=1}^m \boldsymbol{x_{ij}y_{ij}}\right) - \left(\sum_{i=1}^m \boldsymbol{x_{ij}}\right)\left(\sum_{i=1}^m \boldsymbol{y_{ij}}\right)\right)^2}{\left[\boldsymbol{m}\sum_{i=1}^m \boldsymbol{x}_{ij}^2 - \left(\sum_{i=1}^m \boldsymbol{x_{ij}}\right)^2\right]\left[\boldsymbol{m}\sum_{i=1}^m \boldsymbol{y}_{ij}^2 - \left(\sum_{i=1}^m \boldsymbol{y_{ij}}\right)^2\right]} \\ &st. \\ &(\tilde{X}_i)_\alpha^l \le \boldsymbol{x_{ij}} \le (\tilde{X}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \\ &(\tilde{Y}_i)_\alpha^l \le \boldsymbol{y_{ij}} \le (\tilde{Y}_i)_\alpha^u \qquad \forall\, i \in (1,2,\dots,m), \forall\, j \in (1,2,\dots,n) \end{aligned} \tag{10}$$

The coefficient of determination is a crucial metric for evaluating the performance of regression models, as it indicates the clarity and explanatory power of the model. It measures the percentage of the variation in the dependent variable (the model's output) that is explained by the independent variables (the inputs or predictors).

In general, the total variation in the dependent variable can be divided into two parts:

- **Variation explained by the regression model** represents the contribution of the independent variables in explaining the variations in the dependent variable.
- **Unexplained variation (residuals)** reflects the influence of factors not accounted for in the model or prediction errors.

  The value of the coefficient of determination always lies between 0 and 1:
- **Close to 0** indicates that the model has very little capability to explain the variations in the

dependent variable, with most variations attributed to other factors or errors.

- **Close to 1** suggests that the model is highly effective in explaining the variations in the dependent variable based on the independent variables.

As a rule of thumb, if the coefficient of determination exceeds 0.6, it can be concluded that the independent variables significantly explain the variation in the dependent variable. This metric is a key indicator for assessing the model's goodness of fit and serves as a measure for evaluating its predictive power.

Since the reference weight is a positive, unitless value less than 1 and it is directly influenced by the regression coefficients, our normalization method follows the approach outlined by Pazhouhandeh & Samouei, (2024).

All coefficients mentioned in Equations (8) to (10) must be normalized. Since these coefficients can take negative values and have different interpretations depending on whether they are positive or negative indicators, it is necessary to defuzzify them first using standard defuzzification methods, such as the one shown in Equation (11). Then, normalization should follow Equations (12-1) to (12-4) for positive indicators and Equations (13-1) to (13-4) for negative indicators.

In these equations, l_j is the coefficient to be normalized. In Equation (13), after multiplying l_j by $-1, h_j$ is calculated such that:

- For positive indicators, Equation (12-2) is used.
- For negative indicators, Equation (13-2) is applied.
- The smallest h_j value is 1.

For both types of indicators (positive and negative):

- H is the largest calculated h_j value.
- l'_j is a positive numerical value, and l'^N_j is the normalized value of l'_j, which lies within the range $(1 \geq l'^N_j > 0)$.

$$If\ \tilde{Z} = (l, m, u) \quad then \quad CertainZ = \frac{l + 4m + u}{6} \tag{11}$$

If for each $j \in (1,2,\dots,n)$, the values are represented as $l_j = \{l_1, l_2, \dots l_n\}$, then:

For a positive indicator (BC):

$$h_j = 1 - \min_j(-l_j) + (-l_j) \mid min\ h_j = 1 \qquad \forall, j \in (1,2,\dots,n) \tag{12-1}$$

$$\max_j h_j = H \tag{12-2}$$

$$l'_j = \frac{H - l_j}{h_j - l_j} \qquad \forall, j \in (1,2,\dots,n) \tag{12-3}$$

$$l'^N_j = \frac{l'_j}{\sum_{j=1}^{m} l'_j} \qquad \forall, j \in (1,2,\dots,n) \tag{12-4}$$

For a negative indicator (NC):

$$h_j = 1 - \min_j(l_j) + l_j \mid \min_j h_j = 1 \qquad \forall, j \in (1,2,\dots,n) \tag{13-1}$$

$$\max_j h_j = H \tag{13-2}$$

$$l'_j = \frac{H - l_j}{h_j - l_j} \qquad \forall, j \in (1,2,\dots,n) \tag{13-3}$$

$$l'^N_j = \frac{l'_j}{\sum_{j=1}^{m} l'_j} \qquad \forall, j \in (1,2,\dots,n) \tag{13-4}$$

Finally, after preparing the initial data including the decision matrix, defuzzified slope coefficients, intercepts, and coefficients of determination—the mathematical model is solved. The following mathematical model, based on the method presented by Pazhouhandeh & Samouei, (2024), operates using independent weights derived from the Fuzzy PSWCA approach. In the enhanced version of the PSWCA method, if external weights (obtained from other methods or human decisions) are to influence the model's solution, the mathematical model below (Equation 13) is used. In the equation below, ${g_j}^N$ represents the normalized values of the external weights:

$$max\ Z = \lambda_a - \beta(\lambda_b + \lambda_c + \lambda_d + \lambda_h) \tag{14-1}$$

$$s.t. \quad \lambda_a \leq \ S_i \qquad \forall\ i \in \{1,2,\dots,m\} \tag{14-2}$$

$$S_i = \sum_{j=1}^{n} w_j \times x_{ij}^N \qquad \forall\ i \in \{1,2,\dots,m\} \tag{14-3}$$

$$\lambda_b = \sum_{j=1}^{n} \left(w_j - B_j^N\right)^2 \tag{14-4}$$

$$\lambda_c = \sum_{j=1}^{m} \left(w_j - B0_j^N\right)^2 \tag{14-5}$$

$$\lambda_d = \sum_{j=1}^{n} \left(w_j - {R_j^2}^N\right)^2 \tag{14-6}$$

$$\lambda_h = \sum_{j=1}^{n} \left(w_j - {g_j}^N\right)^2 \tag{14-7}$$

$$\sum_{j=1}^{n} w_j = 1 \tag{14-8}$$

$$w_j \leq 1 \quad \forall\ j \in \{1,2,\dots,n\} \tag{14-9}$$

$$w_j \geq \varepsilon \quad \forall\ j \in \{1,2,\dots,n\} \tag{14-10}$$

Figure 1 summarizes the steps of the proposed method.

Finally, the key features of the Fuzzy PSWCA method, compared to many other decision-making methods, are summarized in Table 1:

Table 1. Key features of the proposed method (F-PSWCA)

No.	Feature	Description
1	Independent weighting of criteria without the need for integration with other methods.	The F-PSWCA method can independently calculate weights while maintaining the to integrate with other methods, thereby enhancing decision-making accuracy and transparency.
2	Influence of criterion weight on the slope of its fuzzy variations	In a fuzzy environment, sharper changes in criterion values increase their fuzzy weights, emphasizing the importance of criteria that are more sensitive to variations.
3	Influence of criterion weight by the starting point of its fuzzy variations	The initial criteria in a fuzzy environment, depending on whether they are positive or negative, can impact their final weight value, enhancing precision in evaluation and decision-making.
4	Influence of criterion weight by the dispersion and pattern of its fuzzy variations	In the fuzzy model, the dispersion and variation pattern of criteria, based on historical data, affect the final weight. This feature helps reduce the importance of criteria with unstable and unpredictable changes.
5	Sensitivity analysis capability in the decision-making model	The Fuzzy PSWCA model allows for sensitivity analysis. By modifying influential parameters in the fuzzy environment, the impact of changes and criterion sensitivity on final results can be examined.
6	Integration of the decision-making	In the F-PSWCA method, some decision matrix criteria can be

	mathematical model with multi-stage mathematical models	derived by solving portions of more complex fuzzy mathematical models, improving accuracy and consistency in multi-stage analyses.
7	Use of fuzzy regression equations for decision-making	Using regression equations in a fuzzy environment enables a more precise analysis of data variations and decision-making under uncertainty, enhancing the model's capability for accurately predicting future trends.
8	Ability to incorporate external weights from other weighting methods	If the obtained weights are not technically or practically suitable, expert opinions and the influence of other weighting methods can be incorporated.

Figure 1. Flowchart of the Fuzzy PSWCA method steps

4. Case Study

In today's world, technological advancements and innovations across various industries-particularly in the field of medical equipment-are progressing at an unprecedented pace. However, one of the primary challenges faced by managers, researchers, and decision-makers is the optimal selection and ranking of emerging technologies, especially under conditions of uncertainty. Moreover, the precise evaluation of available options requires careful consideration of multiple, often conflicting criteria that can significantly impact decision-making quality.

In the dialysis treatment process, the quality of purified water is of paramount importance, as any malfunction in water purification systems can directly affect patient health. Dialysis water purification devices play a crucial role in ensuring water quality, making the optimal selection of these devices from the available market options a complex, multi-criteria challenge.

Various companies have developed dialysis water purification systems with different technologies and features. Products such as **Aquaboss1, Aquaboss2, Aquaboss3, AradX, AradX2, F1, F2, F3, N1, and N2** exemplify these systems, each with its own capabilities and limitations. Consequently, selecting the most suitable device requires a comprehensive analysis of various criteria that directly or indirectly influence the performance and efficiency of these systems.

In this context, a group of experts and specialists has identified a set of key criteria for evaluating and ranking these devices, based on relevant standards and expert opinions (Table 2).

Table 2. Key Criteria Derived from Standards and Expert Opinions

Criterion	Definition	Sources of Uncertainty in Data
Device's Disinfection Capability	The ability to eliminate microorganisms and ensure water safety.	The effectiveness of the device in removing microorganisms may vary under different conditions (e.g., water temperature, pressure, microorganism type).
Dead Space in the Device	Areas where water flow may be stagnant, potentially leading to contamination.	Can vary based on hospital requirements, such as the addition of extra filters, or areas where the manufacturer has not effectively minimized dead space.
Piping Quality	The materials and design of the piping, which affect water quality and system stability.	Depends on the materials used, design, and environmental conditions, making precise assessment challenging. Non-compliance with standards can introduce ambiguity.
Rate of Failures Leading to Complete Dialysis Interruption	The frequency of critical failures that disrupt dialysis operations.	Insufficient historical data. Some failures may be minor and go unnoticed, while others may be critical.
Pump Quality	The durability and efficiency of pumps as a key component of the device.	Directly related to hospital maintenance practices, which may lead to inaccurate data.
Salt Removal Efficiency of Filters	The ability of filters to remove salts and ensure purified water quality.	The quality of input water affects filter performance. Measured efficiency data may be inaccurate due to factors such as acid washing and improper maintenance.
Number of Remote-Control Parameters	The extent of remote-control capabilities for better monitoring and management.	Devices may support various parameters, but the exact definition (e.g., number of supported sensors for temperature, pressure, or system status monitoring) may be unclear or incomplete.
Manufacturer's Recommended On-Site Service Interval	The recommended time interval for on-site servicing by technical teams.	Suggested intervals may not align with real-world device usage. Service intervals may vary depending on usage

		intensity and environmental conditions.
Quality of Electromechanical Components	The level of quality and durability of electronic and mechanical parts.	May vary between devices, over time, or due to supply chain restrictions in certain countries.
Availability of Spare Parts	The ease and speed of obtaining spare parts for repairs.	Affected by both manufacturer and customer maintenance practices. Availability depends on market conditions and supplier production.
		Access to parts may differ by region.
Product Lifespan	The durability of the device under operational conditions.	Product lifespan depends on usage, environmental conditions, and maintenance.
		Reliable real-world lifespan data may be insufficient.
Average Water Recovery Efficiency	The ratio of usable water to total consumed water.	Efficiency depends on the quality of input water, which can vary.
		Operational factors such as pressure and temperature, also impact efficiency.

These criteria, due to their complexity, make the evaluating of options a challenging task. Some criteria are quantitative nature (such as salt removal percentage or average water recovery efficiency), while others are qualitative (such as piping quality or electromechanical component quality). However, certain information is only available in a fuzzy or approximate manner due to uncertainty or data limitations. For instance, the salt removal range of a device can be estimated based on the type of membrane filter used, making it a fuzzy value. Similarly, the lifespan of a device can be roughly determined through research on the time until a major overhaul is required due to failure (rather than an upgrade to meet new standards). However, evaluating a device's disinfection capability requires a precise examination of its design. Because this product (where design depends on environmental conditions and customer requirements), can sometimes lead to incorrect or incomplete decision tables.

For problems which require the simultaneous management of complexity, ambiguity, and uncertainty, traditional decision-making methods are insufficient. Therefore, it is necessary to adopt a novel approach capable of evaluating multiple criteria while considering uncertainty and historical data. This approach should be able to assess the decision table in a way that accounts for data dispersion, ambiguity, temporal trends toward convergence, and the complexity and sensitivity of the data.

4.1. Steps for Solving the Case Study

To address the presented case study, the necessary steps are detailed as follows:

Steps 1 and 2: Constructing the Decision Matrix and Historical Data Matrix

At this stage, a decision matrix is created to evaluate various decision-making options against specific criteria. This matrix, also referred to as the decision matrix, organizes information related to criteria and alternatives. Typically, the evaluation criteria are placed in the columns, while decision alternatives are listed in the rows.

To construct this matrix, data related to the criteria were extracted from the database of Diba Ebtekar Mehrad Iran and from shared customer feedback across different manufacturers. Additionally, a separate matrix containing historical data records was compiled to facilitate a more precise comparison and analysis.

In this decision-making process, products from one of the world's leading manufacturers of hemodialysis water purification systems, Diba Ebtekar Mehrad, and five products from two reputable Iranian manufacturers were selected. However, due to data disclosure limitations, the names and models of the Iranian brands have been omitted. Table 3 presents the names of the selected products, while Table 4 outlines the evaluation criteria and their corresponding substitute terms used in the decision-making model.

Table 3. Names and Abbreviations of Selected Products in the Decision-Making Model

J	I	H	G	F	E	D	C	B	A	Alternative Term
N2	N1	F3	F2	F1	AradX2	AradX	Aquaboss3	Aquaboss2	Aquaboss1	Product Model

Table 4. Abbreviations and Corresponding Evaluation Criteria

Alternative Term	Evaluation Criteria
AA	Device's Ability to Disinfect
AB	Amount of Dead Space in the Device
AC	Quality of Piping
AD	Rate of Failures Leading to Complete Dialysis Shutdown
AE	Quality of the Pump
AF	Salt Removal Efficiency of the Filter
AH	Recommended Manufacturer Service Interval
AJ	Quality of Electromechanical Components
AI	Availability of Spare Parts
AL	Number of Parameters Controllable Remotely
AM	Product Lifespan
AN	Average Water Efficiency

The criteria for the decision-making table are based on standard requirements (ISO23500:2019) and features that enhance the operational reliability. Eliminating dead space in vessels, membranes, and piping is considered a key requirement. Dead space includes areas where water stagnates or moves at a velocity below the permissible limit. These spaces pose significant challenges due to bacterial growth, excessive water and energy consumption during cleaning, and inefficiencies in disinfection, ultimately reducing system performance.

Additionally, an automatic cleaning system for the pre-reverse osmosis (RO) filtration pathway prevents water stagnation and inhibits bacterial growth. A timed cleaning system for all pipelines up to the dialysis machine prevents water accumulation and contamination. Semi-automatic chemical disinfection systems capable of disinfecting all parts of the device without requiring a technician's presence and ensuring that filters are not bypassed are also crucial for ensuring device performance.

The recycling of the device's wastewater should be calculated scientifically, considering the quality of the input water and pre-treatment systems. Pre-treatment is particularly critical, as it removes chlorine, turbidity, organic materials, and heavy metals, thereby improving water quality and prolonging the life of membranes and other device components. Using inert materials for components in the inlet and outlet pathways of membranes helps preserve water quality and prevents potential contamination.

The device must prevent substandard or out-of-range water from entering the dialysis machine. In such cases, an audible alarm should activate and remain operational for at least three minutes, ensuring it is audible from a distance of three meters. Additional essential criteria include using silent pumps with high efficiency and durability, incorporating dual carbon beds to minimize the risk of excessive chloramine levels, and installing UV systems and endotoxin removal filters to enhance reliability, especially in devices with dead space. Maintenance should be conducted preventively, predictively, and correctively to ensure the long-term health and efficiency.

Table 5. Fuzzy decision matrix based on evaluation criteria

	A	B	C	D	E	F	G	H	I	J
AA	(4,5,6)	(3.2,4,4.8)	(2.4,3,3.6)	(2.4,3,3.6)	(2.4,3,3.6)	(0.8,1,1.2)	(0.8,1,1.2)	(1.6,2,2.4)	(0.8,1,1.2)	(1.6,2,2.4)
AB	(1.8,3,4.2)	(1.8,3,4.2)	(1.8,3,4.2)	(2.4,4,5.6)	(2.4,4,5.6)	(3.6,6,8.4)	(3.6,6,8.4)	(3,5,7)	(3.6,6,8.4)	(3,5,7)

AC	(5,10,15)	(5,10,15)	(5,10,15)	(3.5,7,10.5)	(3.5,7,10.5)	(3.5,7,10.5)	(4,8,12)	(4,8,12)	(4,8,12)	(4,8,12)
AD	(1.875,2.5,3.125)	(1.875,2.5,3.125)	(1.5,2,2.5)	(1.875,2.5,3.125)	(1.5,2,2.5)	(2.25,3,3.75)	(2.25,3,3.75)	(1.5,2,2.5)	(2.25,3,3.75)	(1.5,2,2.5)
AE	(3,5,7)	(3,5,7)	(3,5,7)	(2.4,4,5.6)	(3,5,7)	(1.2,2,2.8)	(1.2,2,2.8)	(1.2,2,2.8)	(3,5,7)	(3,5,7)
AF	(99.301,99.5,99.699)	(99.301,99.5,99.699)	(99.301,99.5,99.699)	(98.303,98.5,98.697)	(98.303,98.5,98.697)	(96.806,97,97.194)	(98.303,98.5,98.697)	(98.303,98.5,98.697)	(98.303,98.5,98.697)	(96.806,97,97.194)
AH	(12,15,18)	(10.4,13,15.6)	(8,10,12)	(9.6,12,14.4)	(11.2,14,16.8)	(4,5,6)	(8,10,12)	(9.6,12,14.4)	(8,10,12)	(8,10,12)
AJ	(2.4,3,3.6)	(2.4,3,3.6)	(2.4,3,3.6)	(0.8,1,1.2)	(1.6,2,2.4)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)
AI	(4,5,6)	(4,5,6)	(4,5,6)	(2.4,3,3.6)	(3.2,4,4.8)	(1.6,2,2.4)	(2.4,3,3.6)	(3.2,4,4.8)	(3.2,4,4.8)	(3.2,4,4.8)
AL	(1.3,2,2.7)	(1.3,2,2.7)	(1.3,2,2.7)	(1.95,3,4.05)	(2.6,4,5.4)	(1.95,3,4.05)	(2.6,4,5.4)	(1.95,3,4.05)	(2.6,4,5.4)	(1.3,2,2.7)
AM	(13,20,27)	(9.75,15,20.25)	(6.5,10,13.5)	(6.5,10,13.5)	(7.8,12,16.2)	(7.15,11,14.85)	(6.5,10,13.5)	(7.8,12,16.2)	(7.8,12,16.2)	(7.8,12,16.2)
AN	(72.75,75,77.25)	(87.3,90,92.7)	(92.15,95,97.85)	(63.05,65,66.95)	(82.45,85,87.55)	(48.5,50,51.5)	(58.2,60,61.8)	(72.75,75,77.25)	(72.75,75,77.25)	(72.75,75,77.25)

Table 5 presents the evaluation of criteria in relation to the existing and newly introduced technologies by manufacturers.

Table 6. Fuzzy decision matrix of historical criteria for each product

	A	B	C	D	E	F	G	H	I	J
AA	(4,5,6)	(3.2,4,4.8)	(2.4,3,3.6)	(1.6,2,2.4)	(2,2.5,3)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(1.6,2,2.4)	(0.4,0.5,0.6)	(0.8,1,1.2)
AB	(1.8,3,4.2)	(1.8,3,4.2)	(1.8,3,4.2)	(2.4,4,5.6)	(2.4,4,5.6)	(4.8,8,11.2)	(4.8,8,11.2)	(4.8,8,11.2)	(4.2,7,9.8)	(3.6,6,8.4)
AC	(4.5,9,13.5)	(4.5,9,13.5)	(4.5,9,13.5)	(3,6,9)	(3.5,7,10.5)	(2.5,5,7.5)	(3,6,9)	(3.5,7,10.5)	(4.5,9,13.5)	(4.5,9,13.5)
AD	(2.25,3,3.75)	(2.25,3,3.75)	(1.5,2,2.5)	(3,4,5)	(1.5,2,2.5)	(4.5,6,7.5)	(3.75,5,6.25)	(3,4,5)	(1.5,2,2.5)	(1.5,2,2.5)
AE	(2.4,4,5.6)	(2.82,4.7,6.58)	(2.82,4.7,6.58)	(2.4,4,5.6)	(2.4,4,5.6)	(1.8,3,4.2)	(1.8,3,4.2)	(1.8,3,4.2)	(3,5,7)	(3,5,7)
AF	(98.303,98.5,98.697)	(98.303,98.5,98.697)	(98.303,98.5,98.697)	(96.806,97,97.194)	(96.806,97,97.194)	(96.806,97,97.194)	(96.806,97,97.194)	(96.806,97,97.194)	(98.303,98.5,98.697)	(98.303,98.5,98.697)
AH	(8,10,12)	(5.6,7,8.4)	(4,5,6)	(4,5,6)	(4,5,6)	(1.6,2,2.4)	(4,5,6)	(4,5,6)	(1.6,2,2.4)	(1.6,2,2.4)
AJ	(1.6,2,2.4)	(1.6,2,2.4)	(1.6,2,2.4)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)	(0.8,1,1.2)
AI	(4,5,6)	(4,5,6)	(4,5,6)	(2.4,3,3.6)	(2.4,3,3.6)	(1.6,2,2.4)	(1.6,2,2.4)	(1.6,2,2.4)	(3.2,4,4.8)	(3.2,4,4.8)
AL	(2.6,4,5.4)	(2.6,4,5.4)	(2.6,4,5.4)	(3.25,5,6.75)	(3.25,5,6.75)	(3.25,5,6.75)	(3.25,5,6.75)	(3.25,5,6.75)	(1.95,3,4.05)	(1.95,3,4.05)
AM	(7.8,12,16.2)	(6.5,10,13.5)	(6.5,10,13.5)	(5.2,8,10.8)	(6.5,10,13.5)	(5.2,8,10.8)	(5.2,8,10.8)	(6.5,10,13.5)	(6.5,10,13.5)	(6.5,10,13.5)
AN	(72.75,75,77.25)	(72.75,75,77.25)	(72.75,75,77.25)	(48.5,50,51.5)	(63.05,65,66.95)	(48.5,50,51.5)	(48.5,50,51.5)	(63.05,65,66.95)	(48.5,50,51.5)	(63.05,65,66.95)

Table 6 presents the historical records of these technologies, including their older versions (2021)

Step 3: Normalizing the Decision Matrix

To ensure alignment and comparability of criteria, data normalization or scaling is performed. This process is essential because various criteria may have different units of measurement or scales. Depending on the problem, normalization methods may include linear normalization, division by the maximum or minimum value, or other standardization techniques. In this study, Equations (3) and (4) are used to normalize the data in the decision matrix and historical records.

After applying the normalization formulas, the evaluation data for the criteria and decision-making options are organized into a new matrix that presents the criteria in a normalized format. To enhance evaluation, the historical data related to the criteria are also normalized and displayed in a separate matrix.

Table 7. Normalized Fuzzy Decision Matrix Based on Evaluation Criteria

	A	B	C	D	E	F	G	H	I	J
AA	(0.67,0.83,1)	(0.53,0.67,0.8)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.13,0.17,0.2)	(0.13,0.17,0.2)	(0.27,0.33,0.4)	(0.13,0.17,0.2)	(0.27,0.33,0.4)
AB	(0.43,0.6,1)	(0.43,0.6,1)	(0.43,0.6,1)	(0.32,0.45,0.75)	(0.32,0.45,0.75)	(0.21,0.3,0.5)	(0.21,0.3,0.5)	(0.26,0.36,0.6)	(0.21,0.3,0.5)	(0.26,0.36,0.6)
AC	(0.33,0.67,1)	(0.33,0.67,1)	(0.33,0.67,1)	(0.23,0.47,0.7)	(0.23,0.47,0.7)	(0.23,0.47,0.7)	(0.27,0.53,0.8)	(0.27,0.53,0.8)	(0.27,0.53,0.8)	(0.27,0.53,0.8)
AD	(0.48,0.6,0.8)	(0.48,0.6,0.8)	(0.6,0.75,1)	(0.48,0.6,0.8)	(0.6,0.75,1)	(0.4,0.5,0.67)	(0.4,0.5,0.67)	(0.6,0.75,1)	(0.4,0.5,0.67)	(0.6,0.75,1)
AE	(0.43,0.71,1)	(0.43,0.71,1)	(0.43,0.71,1)	(0.34,0.57,0.8)	(0.43,0.71,1)	(0.17,0.29,0.4)	(0.17,0.29,0.4)	(0.17,0.29,0.4)	(0.43,0.71,1)	(0.43,0.71,1)
AF	(1,1,1)	(1,1,1)	(1,1,1)	(0.99,0.99,0.99)	(0.99,0.99,0.99)	(0.97,0.97,0.97)	(0.99,0.99,0.99)	(0.99,0.99,0.99)	(0.99,0.99,0.99)	(0.97,0.97,0.97)
AH	(0.67,0.83,1)	(0.58,0.72,0.87)	(0.44,0.56,0.67)	(0.53,0.67,0.8)	(0.62,0.78,0.93)	(0.22,0.28,0.33)	(0.44,0.56,0.67)	(0.53,0.67,0.8)	(0.44,0.56,0.67)	(0.44,0.56,0.67)
AJ	(0.67,0.83,1)	(0.67,0.83,1)	(0.67,0.83,1)	(0.22,0.28,0.33)	(0.44,0.56,0.67)	(0.22,0.28,0.33)	(0.22,0.28,0.33)	(0.22,0.28,0.33)	(0.22,0.28,0.33)	(0.22,0.28,0.33)
AI	(0.67,0.83,1)	(0.67,0.83,1)	(0.67,0.83,1)	(0.4,0.5,0.6)	(0.53,0.67,0.8)	(0.27,0.33,0.4)	(0.4,0.5,0.6)	(0.53,0.67,0.8)	(0.53,0.67,0.8)	(0.53,0.67,0.8)
AL	(0.24,0.37,0.5)	(0.24,0.37,0.5)	(0.24,0.37,0.5)	(0.36,0.56,0.75)	(0.48,0.74,1)	(0.36,0.56,0.75)	(0.48,0.74,1)	(0.36,0.56,0.75)	(0.48,0.74,1)	(0.24,0.37,0.5)
AM	(0.48,0.74,1)	(0.36,0.56,0.75)	(0.24,0.37,0.5)	(0.24,0.37,0.5)	(0.29,0.44,0.6)	(0.26,0.41,0.55)	(0.24,0.37,0.5)	(0.29,0.44,0.6)	(0.29,0.44,0.6)	(0.29,0.44,0.6)
AN	(0.74,0.77,0.79)	(0.89,0.92,0.95)	(0.94,0.97,1)	(0.64,0.66,0.68)	(0.84,0.87,0.89)	(0.5,0.51,0.53)	(0.59,0.61,0.63)	(0.74,0.77,0.79)	(0.74,0.77,0.79)	(0.74,0.77,0.79)

The results of this stage include two final matrices, presented in Tables 7 and 8. These matrices serve as inputs for subsequent stages of analysis and decision-making. Specifically, these tables prepare the information in a comparable format, enabling the evaluation of different options based on specified criteria.

Table 8. The Fuzzy Unscaled Decision Matrix Based on Evaluation Criteria

	A	B	C	D	E	F	G	H	I	J
AA	(0.67,0.83,1)	(0.53,0.67,0.8)	(0.4,0.5,0.6)	(0.27,0.33,0.4)	(0.33,0.42,0.5)	(0.07,0.08,0.1)	(0.07,0.08,0.1)	(0.27,0.33,0.4)	(0.07,0.08,0.1)	(0.13,0.17,0.2)
AB	(0.43,0.6,1)	(0.43,0.6,1)	(0.43,0.6,1)	(0.32,0.45,0.75)	(0.32,0.45,0.75)	(0.16,0.23,0.38)	(0.16,0.23,0.38)	(0.16,0.23,0.38)	(0.18,0.26,0.43)	(0.21,0.3,0.5)
AC	(0.33,0.67,1)	(0.33,0.67,1)	(0.33,0.67,1)	(0.22,0.44,0.67)	(0.26,0.52,0.78)	(0.19,0.37,0.56)	(0.22,0.44,0.67)	(0.26,0.52,0.78)	(0.33,0.67,1)	(0.33,0.67,1)
AD	(0.4,0.5,0.67)	(0.4,0.5,0.67)	(0.6,0.75,1)	(0.3,0.38,0.5)	(0.6,0.75,1)	(0.2,0.25,0.33)	(0.24,0.3,0.4)	(0.3,0.38,0.5)	(0.6,0.75,1)	(0.6,0.75,1)
AE	(0.34,0.57,0.8)	(0.4,0.67,0.94)	(0.4,0.67,0.94)	(0.34,0.57,0.8)	(0.34,0.57,0.8)	(0.26,0.43,0.6)	(0.26,0.43,0.6)	(0.26,0.43,0.6)	(0.43,0.71,1)	(0.43,0.71,1)
AF	(1,1,1)	(1,1,1)	(1,1,1)	(0.98,0.98,0.98)	(0.98,0.98,0.98)	(0.98,0.98,0.98)	(0.98,0.98,0.98)	(0.98,0.98,0.98)	(1,1,1)	(1,1,1)
AH	(0.67,0.83,1)	(0.47,0.58,0.7)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.13,0.17,0.2)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.13,0.17,0.2)	(0.13,0.17,0.2)
AJ	(0.67,0.83,1)	(0.67,0.83,1)	(0.67,0.83,1)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)	(0.33,0.42,0.5)
AI	(0.67,0.83,1)	(0.67,0.83,1)	(0.67,0.83,1)	(0.4,0.5,0.6)	(0.4,0.5,0.6)	(0.27,0.33,0.4)	(0.27,0.33,0.4)	(0.27,0.33,0.4)	(0.53,0.67,0.8)	(0.53,0.67,0.8)
AL	(0.39,0.59,0.8)	(0.39,0.59,0.8)	(0.39,0.59,0.8)	(0.48,0.74,1)	(0.48,0.74,1)	(0.48,0.74,1)	(0.48,0.74,1)	(0.48,0.74,1)	(0.29,0.44,0.6)	(0.29,0.44,0.6)
AM	(0.48,0.74,1)	(0.4,0.62,0.83)	(0.4,0.62,0.83)	(0.32,0.49,0.67)	(0.4,0.62,0.83)	(0.32,0.49,0.67)	(0.32,0.49,0.67)	(0.4,0.62,0.83)	(0.4,0.62,0.83)	(0.4,0.62,0.83)
AN	(0.94,0.97,1)	(0.94,0.97,1)	(0.94,0.97,1)	(0.63,0.65,0.67)	(0.82,0.84,0.87)	(0.63,0.65,0.67)	(0.63,0.65,0.67)	(0.82,0.84,0.87)	(0.63,0.65,0.67)	(0.82,0.84,0.87)

Step 4: Trimming Decision Matrices and Records with α =0.8 and Calculating Fuzzy Regression Coefficients

At this stage, to conduct a more precise analysis and establish mathematical constraints for model optimization, the decision matrices and records are trimmed using a specific membership level called alpha (α).

The **α** level is a criterion for determining the minimum acceptable membership in a fuzzy set. Here, the value **α = 0.8** has been selected, meaning that only values with a membership level of 0.8 or higher are considered in the calculations. This trimming process transforms each matrix element into an interval (upper and lower bounds), which serves as mathematical constraints for the model.

Step5: Determining the Upper and Lower Bounds for Regression Coefficients

The upper and lower constraints derived from matrix trimming are incorporated into the optimization process to compute the regression coefficients. This process allows us to define an interval for the regression coefficients, reflecting possible variations based on the selected membership level. The lower and upper bounds of the regression coefficients are determined based on the mathematical models presented in Equations 8 to 10, while the midpoint corresponds to the value obtained when α = 1. Using the obtained results, the regression coefficients are defined as a Triangular Fuzzy Number (TFN). The triangular fuzzy numbers in Tables 5 and 6 were derived from a combination of technical records from the 'Diba Ebtekar Mehrad' database and expert elicitation. For qualitative criteria, a 1-9 linguistic scale was converted to fuzzy numbers, while for quantitative ones (like Salt Removal), the fuzzy range was determined based on the variance in operational performance data.

Table 9. Trimmed Decision Matrix with Coefficient (α = 0.8) Based on Evaluation Criteria

	A	B	C	D	E	F	G	H	I	J
AA	[0.8,0.86]	[0.64,0.7]	[0.48,0.52]	[0.48,0.52]	[0.48,0.52]	[0.16,0.18]	[0.16,0.18]	[0.32,0.34]	[0.16,0.18]	[0.32,0.34]
AB	[0.57,0.68]	[0.57,0.68]	[0.57,0.68]	[0.42,0.51]	[0.42,0.51]	[0.28,0.34]	[0.28,0.34]	[0.34,0.41]	[0.28,0.34]	[0.34,0.41]
AC	[0.6,0.74]	[0.6,0.74]	[0.6,0.74]	[0.42,0.52]	[0.42,0.52]	[0.42,0.52]	[0.48,0.58]	[0.48,0.58]	[0.48,0.58]	[0.48,0.58]
AD	[0.58,0.64]	[0.58,0.64]	[0.72,0.8]	[0.58,0.64]	[0.72,0.8]	[0.48,0.53]	[0.48,0.53]	[0.72,0.8]	[0.48,0.53]	[0.72,0.8]
AE	[0.65,0.77]	[0.65,0.77]	[0.65,0.77]	[0.52,0.62]	[0.65,0.77]	[0.27,0.31]	[0.27,0.31]	[0.27,0.31]	[0.65,0.77]	[0.65,0.77]
AF	[1,1]	[1,1]	[1,1]	[0.99,0.99]	[0.99,0.99]	[0.97,0.97]	[0.99,0.99]	[0.99,0.99]	[0.99,0.99]	[0.97,0.97]
AH	[0.8,0.86]	[0.69,0.75]	[0.54,0.58]	[0.64,0.7]	[0.75,0.81]	[0.27,0.29]	[0.54,0.58]	[0.64,0.7]	[0.54,0.58]	[0.54,0.58]
AJ	[0.8,0.86]	[0.8,0.86]	[0.8,0.86]	[0.27,0.29]	[0.54,0.58]	[0.27,0.29]	[0.27,0.29]	[0.27,0.29]	[0.27,0.29]	[0.27,0.29]
AI	[0.8,0.86]	[0.8,0.86]	[0.8,0.86]	[0.48,0.52]	[0.64,0.7]	[0.32,0.34]	[0.48,0.52]	[0.64,0.7]	[0.64,0.7]	[0.64,0.7]
AL	[0.34,0.4]	[0.34,0.4]	[0.34,0.4]	[0.52,0.6]	[0.69,0.79]	[0.52,0.6]	[0.69,0.79]	[0.52,0.6]	[0.69,0.79]	[0.34,0.4]
AM	[0.69,0.79]	[0.52,0.6]	[0.34,0.4]	[0.34,0.4]	[0.41,0.47]	[0.38,0.44]	[0.34,0.4]	[0.41,0.47]	[0.41,0.47]	[0.41,0.47]
AN	[0.76,0.77]	[0.91,0.93]	[0.96,0.98]	[0.66,0.66]	[0.86,0.87]	[0.51,0.51]	[0.61,0.61]	[0.76,0.77]	[0.76,0.77]	[0.76,0.77]

Tables 9 and 10 present the values obtained from the **α -cut (α = 0.8)** of the fuzzy decision matrix and the fuzzy historical data matrix.

Table 10. Trimmed Decision Matrix with Coefficient (α = 0.8) for Product Historical Data Based on Evaluation Criteria

	A	B	C	D	E	F	G	H	I	J
AA	[0.8,0.86]	[0.64,0.7]	[0.48,0.52]	[0.32,0.34]	[0.4,0.44]	[0.08,0.08]	[0.08,0.08]	[0.32,0.34]	[0.08,0.08]	[0.16,0.18]
AB	[0.57,0.68]	[0.57,0.68]	[0.57,0.68]	[0.42,0.51]	[0.42,0.51]	[0.22,0.26]	[0.22,0.26]	[0.22,0.26]	[0.24,0.29]	[0.28,0.34]
AC	[0.6,0.74]	[0.6,0.74]	[0.6,0.74]	[0.4,0.49]	[0.47,0.57]	[0.33,0.41]	[0.4,0.49]	[0.47,0.57]	[0.6,0.74]	[0.6,0.74]
AD	[0.48,0.53]	[0.48,0.53]	[0.72,0.8]	[0.36,0.4]	[0.72,0.8]	[0.24,0.27]	[0.29,0.32]	[0.36,0.4]	[0.72,0.8]	[0.72,0.8]
AE	[0.52,0.62]	[0.62,0.72]	[0.62,0.72]	[0.52,0.62]	[0.52,0.62]	[0.4,0.46]	[0.4,0.46]	[0.4,0.46]	[0.65,0.77]	[0.65,0.77]
AF	[1,1]	[1,1]	[1,1]	[0.98,0.98]	[0.98,0.98]	[0.98,0.98]	[0.98,0.98]	[0.98,0.98]	[1,1]	[1,1]
AH	[0.8,0.86]	[0.56,0.6]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]	[0.16,0.18]	[0.4,0.44]	[0.4,0.44]	[0.16,0.18]	[0.16,0.18]
AJ	[0.8,0.86]	[0.8,0.86]	[0.8,0.86]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]	[0.4,0.44]
AI	[0.8,0.86]	[0.8,0.86]	[0.8,0.86]	[0.48,0.52]	[0.48,0.52]	[0.32,0.34]	[0.32,0.34]	[0.32,0.34]	[0.64,0.7]	[0.64,0.7]
AL	[0.55,0.63]	[0.55,0.63]	[0.55,0.63]	[0.69,0.79]	[0.69,0.79]	[0.69,0.79]	[0.69,0.79]	[0.69,0.79]	[0.41,0.47]	[0.41,0.47]
AM	[0.69,0.79]	[0.58,0.66]	[0.58,0.66]	[0.46,0.53]	[0.58,0.66]	[0.46,0.53]	[0.46,0.53]	[0.58,0.66]	[0.58,0.66]	[0.58,0.66]
AN	[0.96,0.98]	[0.96,0.98]	[0.96,0.98]	[0.65,0.65]	[0.84,0.85]	[0.65,0.65]	[0.65,0.65]	[0.84,0.85]	[0.65,0.65]	[0.84,0.85]

Steps 6 to 8: Calculating Regression Coefficients of the Decision Matrix and Its Normalization

Based on **Equations 8 to 10**, the regression coefficients have been calculated using the information from the historical data matrix and the decision matrix. These coefficients are used to determine the reference weight.

The calculations related to these coefficients are presented in **Tables 11 and 12**.

Table 11. Fuzzy Coefficients of Slope, Intercept, and Determination Coefficient for Evaluation Criteria

Criteria	Fuzzy Regression Coefficient			Defuzzified Regression Coefficients			Normalized Regression Coefficients		
	Slope	Intercept	Determination Coefficient	Slope	Intercept	Determination Coefficient	Slope	Intercept	Determination Coefficient
AA	(0.74,0.84,0.965)	(0.083,0.12,0.155)	(0.881,0.95,0.981)	0.844	0.12	0.944	0.51	0.81	1
AB	(0.417,0.78,1.287)	(-0.059,0.12,0.274)	(0.648,0.96,1)	0.804	0.116	0.915	0.36	1	0.3
AC	(-0.164,0.53,1.686)	(-0.353,0.23,0.636)	(0,0.55,1)	0.607	0.201	0.533	0.35	0.85	0.33
AD	(0.076,0.28,0.505)	(0.371,0.46,0.602)	(0.022,0.26,0.572)	0.284	0.469	0.272	1	0.75	1
AE	(0.803,1.56,2.868)	(-1.049,-0.3,0.11)	(0.313,0.81,1)	1.652	-0.357	0.759	1	0.62	0.48
AF	(0.336,0.3,0.454)	(0.539,0.69,0.657)	(0.061,0.08,0.107)	0.332	0.659	0.081	0.24	1	0.05
AH	(0.435,0.57,0.73)	(0.322,0.38,0.451)	(0.383,0.58,0.768)	0.574	0.382	0.579	0.35	0.91	0.36
AJ	(0.997,1.23,1.521)	(-0.347,-0.18,-0.067)	(0.82,0.89,0.95)	1.24	-0.189	0.888	0.76	0.69	0.56
AI	(0.475,0.66,0.88)	(0.15,0.26,0.359)	(0.49,0.7,0.853)	0.666	0.258	0.691	0.4	0.87	0.43
AL	(-0.241,0.48,1.247)	(-0.222,0.21,0.675)	(0,0.13,0.564)	0.488	0.216	0.181	0.29	0.85	0.11
AM	(0.194,1.12,2.838)	(-1.167,-0.2,0.21)	(0.023,0.59,0.995)	1.252	-0.293	0.563	0.74	0.66	0.35
AN	(0.698,0.79,0.819)	(0.105,0.13,0.197)	(0.575,0.64,0.688)	0.78	0.137	0.637	0.48	0.82	0.67

Table 12. Deterministic Coefficients of Slope, Y-Intercept, and Determination Coefficient for Evaluation Indicators

Index	Deterministic Coefficients			Normalized Deterministic Coefficients		
	Slope	**Y-Intercept**	**Determination Coefficient**	**Slope**	**Y-Intercept**	**Determination Coefficient**
Device's Disinfection Capability	0.844	0.12	0.944	0.51	0.81	1
Dead Space in the Device	0.804	0.116	0.915	0.36	1	0.3
Piping Quality	0.607	0.201	0.533	0.35	0.85	0.33
Failures Leading to Complete Dialysis Shutdown	0.284	0.469	0.272	1	0.75	1
Pump Quality	1.652	-0.357	0.759	1	0.62	0.48
Salt Removal Percentage of Filter	0.332	0.659	0.081	0.24	1	0.05
Number of Remotely Controllable Parameters	0.574	0.382	0.579	0.35	0.91	0.36
Manufacturer’s Recommended On-Site Service Interval	1.24	-0.189	0.888	0.76	0.69	0.56
Electromechanical Component Quality	0.666	0.258	0.691	0.4	0.87	0.43
Availability of Spare Parts	0.488	0.216	0.181	0.29	0.85	0.11
Product Lifespan	1.252	-0.293	0.563	0.74	0.66	0.35
Average Water Efficiency	0.78	0.137	0.637	0.48	0.82	0.67

Step 5: Solving the Nonlinear Mathematical Model

After solving the mathematical model from relations 1-14 to 10-14, the decision variable of the problem, i.e., the reference weight and the score for each option, are calculated based on that. In cases where the value of β ensures the stability of the reference weights (in this example, $\beta = 4$), the reference weight value is obtained as shown in Table 13. Since the decision matrix's cutting values have both an upper and lower bounds, a single value can be used for the decision-making table, and then model 14 can be solved. Alternatively, model 14 can be solved twice, once for the lower bound and once for the upper bound of the decision matrix, and the weights and scores of the options are then combined into a single value. This single value will be the average of the two bounds.

Table 13. Reference weights obtained for each criterion based on the Fuzzy PSWCA method

Index	Lower Bound PSWCA Fuzzy Weight	Upper Bound PSWCA Fuzzy Weight	SECA Fuzzy Weight
Device's Disinfection Capability	0.1600	0.1600	0.078
Dead Space in the Device	0.1680	0.1720	0.064
Piping Quality	0.0001	0.0001	0.066
Failures Leading to Complete Dialysis Shutdown	0.0810	0.0830	0.094
Pump Quality	0.1980	0.2010	0.087
Salt Removal Percentage of Filter	0.0001	0.0001	0.072
Number of Remotely Controllable Parameters	0.0240	0.0170	0.070
Manufacturer’s Recommended On-Site Service Interval	0.1900	0.1910	0.096
Electromechanical Component Quality	0.0630	0.0590	0.070
Availability of Spare Parts	0.0001	0.0001	0.151
Product Lifespan	0.0580	0.0640	0.076
Average Water Efficiency	0.0580	0.0540	0.076

Step9 and 10: Stability in Ranking Analysis

Based on Figure 2, which shows the ranking results of options based on various methods, the Aquaboss1 option ranks first in all methods, indicating its high performance. Since it has performed the best in all six methods, it can be concluded that this option likely has outstanding features across all evaluation criteria. Options F1 and F2 consistently rank at the bottom (positions 9 and 10) in all methods, indicating their overall weakness in the evaluated criteria. This stability in low ranking may point to the inefficiency of these products in the targeted areas. The N1 option has received different rankings in various methods. For instance, in the Fuzzy SECA (FSECA) method, it is ranked 5th, while in other methods, it ranks 8th. This discrepancy suggests that the criteria emphasized in the FSECA method are more suitable for this option, but its performance is lower in other methods. The AradX option consistently ranks lower (positions 6 or 7) in all methods. Although the ranking differences are not large, this may indicate that its performance is below average compared to the other options.

4.2. Method Performance Analysis

If the results of the fuzzy PSWCA method are presented as two bounds, i.e., [L-FPSWCA-U-FPSWCA], their average value would be M-FPSWCA. As shown in Figure 2, the M-FPSWCA and fuzzy SAW methods provide nearly identical results. While the ranking results of F-PSWCA and fuzzy SAW show similarity in this specific case study, the fundamental advantage of F-PSWCA lies in its dynamic weighting mechanism. Unlike fuzzy SAW, which relies on static weights, F-PSWCA incorporates temporal variations and historical data through fuzzy regression coefficients.

For most options, except for Aquaboss2, the rankings are the same, which may indicate the use of similar criteria or weightings in these two methods. A relative similarity between the Fuzzy TOPSIS and M-FPSWCA methods is also observed, although there are differences in certain options, such as N1. The Fuzzy SECA method, compared to other methods, provides different rankings in more cases (such as Aquaboss3, N1, and N2). It appears that this method focuses on specific criteria that have resulted in different outcomes.

The fuzzy SECA method determined the weights based on standard deviation and correlation, and the obtained weights were compared with those derived from our proposed method. Furthermore, we followed a model that maximized fuzzy data using before implementing them in the mathematical model. Additionally, fuzzy TOPSIS and fuzzy SAW methods were selected as alternative approaches to verify the accuracy of the results.

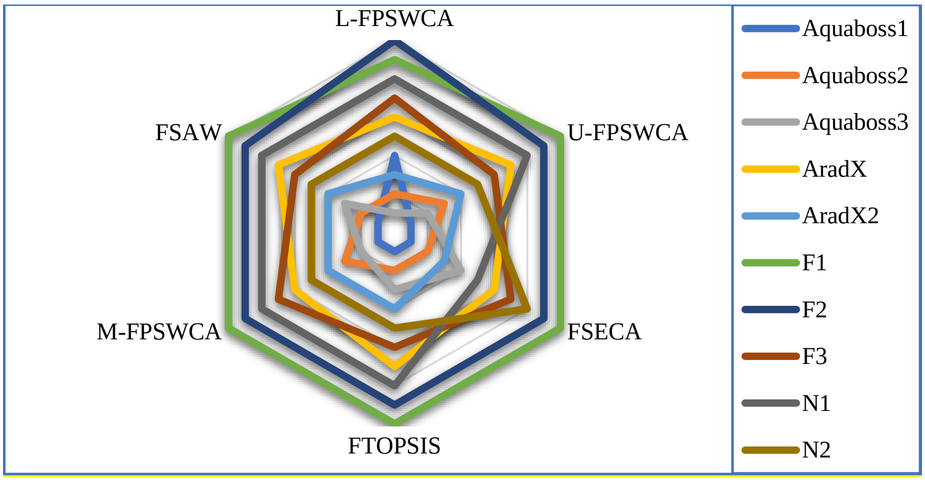

Figure 2. Comparison of Solution Results with Other MCDM Methods The weights of "quality of pipelines," "percentage of filter salts removal," and "availability of spare parts" are very low (0.0001). Possible reasons for these results include:

- **Low Coefficient of Determination (R^2):** These criteria may have low R^2 values in regressions, indicating that they have limited ability to explain variations in the results. For example, "percentage of filter salts removal" has an R^2 of 0.09, which is very low and indicates a weak relationship between this criterion and the overall results.
- **Low Slopes:** The slopes of some criteria are low, suggesting that changes in these criteria do not significantly impact the overall results. For instance, "quality of pipelines" has a slope of 0.35, which is weaker than that of more important criteria.

Criteria with R^2 values closer to 1 (such as "device disinfecting capability" and "dead space in the device") appear to have a stronger role in explaining variations in results, which gives them higher weights. Criteria with higher slopes, such as "breakdowns leading to complete dialysis stoppage" and "pump quality," have a major role in determining the final results. These criteria significantly influence the model's output, indicating their high importance in decision-making.

Criteria with higher intercepts, like "dead space in the device" and "filter salts removal percentage," show that even when other criteria are constant, these factors have a high initial impact. However, the low R^2 values for some of these criteria result in their lower final weight.

The "pump quality" and "manufacturer's recommended service interval" criteria, due to their higher weights (0.1995 and 0.1905), indicate that technical and maintenance factors of the devices are crucial in decision-making. This suggests the operational importance and performance of the devices over time.

If criteria with low weights are practically and technically significant, alternative weighting methods should be used, and sensitivity analysis should be performed to determine how changes in each criterion affect the final weight.

4.3. Discussion and Analysis

This study introduces a novel method for weighting criteria within the F-PSWCA framework. The method is mathematically formulated to maximize the overall performance of options, considering the temporal variations in the criteria in the decision matrix. By using the F-PSWCA method, it becomes possible to more accurately evaluate options and assign appropriate weights to the criteria. This research evaluates options based on past data compared to the present decision matrix.

Additionally, the F-PSWCA method investigates the relationships between criteria and directly incorporates the impact of their positive or negative changes in the weighting process. The coefficients affecting the weighting are scaled using a heuristic method that considers the slope of changes and the initial values of the criteria. The three key parameters—slope, intercept, and R^2—help decision-makers make better predictions with more complete information and data over time. Increasing the number of options can also improve decision-making accuracy.

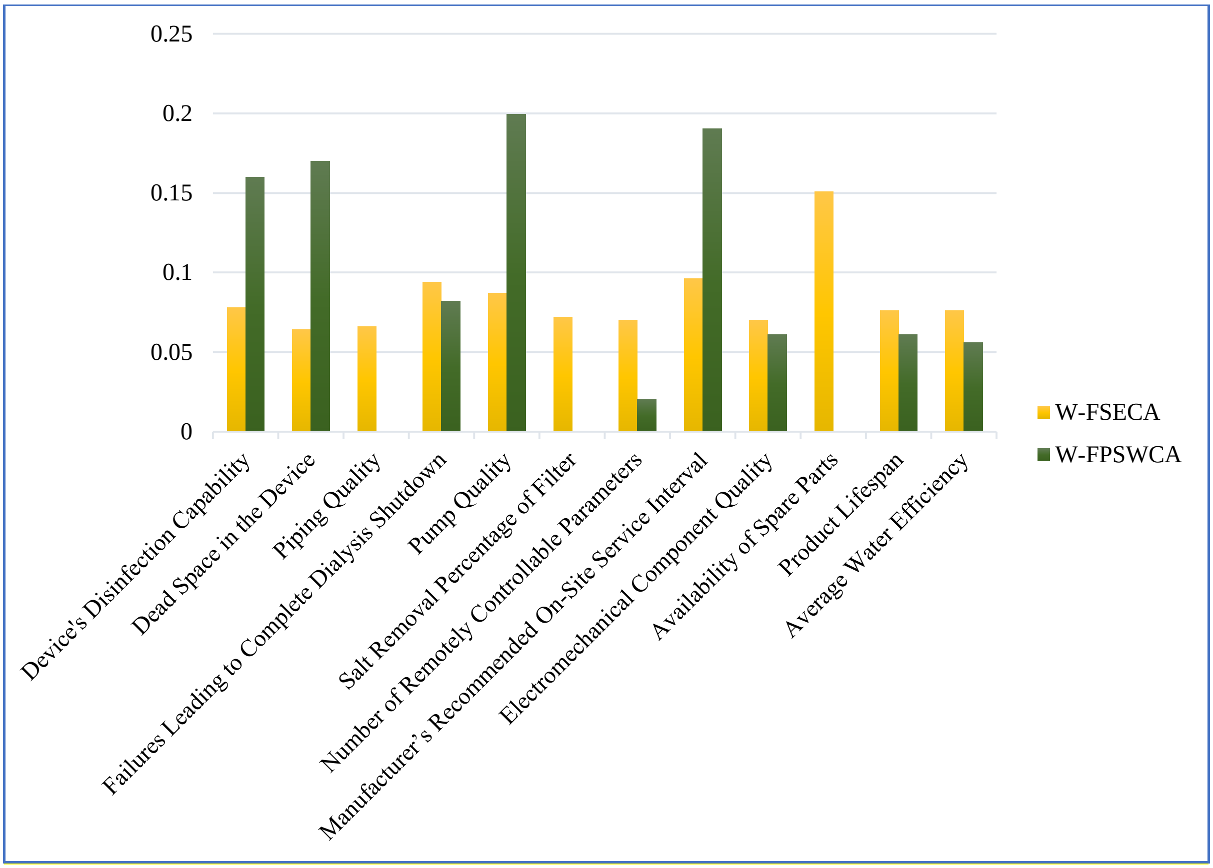

Figure 3. Comparison of Index Weights Obtained by the Fuzzy SCA and PSWCA Methods

The F-PSWCA method has advantages for decision-making processes where criteria undergo significant temporal changes. This technique uses sensitivity analysis of decision-makers' opinions and meaningful changes in criteria to enhance decision-making. Moreover, the method can incorporate expert opinions and other weighting methods, with the effect possibly influenced by adding new constraints to the model.

The near-zero weights observed for certain criteria, such as AI (Availability of Spare Parts), do not imply their insignificance in a practical or clinical context. In the proposed F-PSWCA model, criteria weights are dynamically extracted from the statistical behavior of data over time. A low weight indicates a low level of statistical significance, meaning that the historical data for these specific criteria lacked a consistent or predictable trend across the alternatives.

As discussed in Section 3 (Equation 14-1 to 14-10), the model is designed to handle cases where a criterion is strategically vital but receives a low weight due to historical data patterns. The F-PSWCA framework facilitates the integration of 'External Weights' (derived from expert elicitation or alternative weighting methodologies) with the data-driven results. This hybrid approach ensures that the final weighting and the ranking of alternatives are executed in a single, integrated step, balancing empirical evidence with strategic priorities.

In summary, the F-PSWCA method, by integrating temporal information and historical data, provides better evaluations, helping decision-makers make informed choices based on influencing factors. This method is adaptable to increasing numbers of options and criteria without complicating the overall model structure. This feature makes it valuable for applications where new criteria are added over time, differentiating it from other methods that may become complex when dealing with large numbers of criteria and options. These advantages make the F-PSWCA method more suitable than other fuzzy MCDM methods for decision-making in contexts where criteria and data fluctuate over time and require more accurate predictions.

5. Conclusion

The F-PSWCA method is an innovative approach to multi-criteria decision-making that offers more precise criteria weighting by leveraging temporal analysis and past data. This method addresses the weaknesses of traditional decision-making approaches by incorporating time-related information, improving decision accuracy, and prediction capabilities. Among its notable advantages are sensitivity analysis, flexibility in combination with other weighting methods, and the use of fuzzy regression equations to analyze future trends. These features make F-PSWCA an invaluable tool for studies that require accurate, predictable analysis. Its ability to examine changes over time and its detailed criterion analysis make it effective in selecting new technologies, innovating medical devices, risk management, strategic decision-making, optimizing manufacturing and service industries, and evaluating human resources and employee performance. Furthermore, in environmental and sustainable management research, it can analyze the impacts of various variables over time. The method is also useful in financial and investment analyses and supplier and supply chain evaluations. Given that F-PSWCA can accurately analyze time-dependent criteria and predict changes, it is a powerful tool in any field requiring sensitivity analysis, trend prediction, and improved decision-making.

5.1. Future Studies

To further improve the F-PSWCA method, several suggestions can be made to enhance its accuracy, flexibility, and applicability:

- **Integration with Machine Learning Algorithms:** Machine learning algorithms can improve the weighting process and identify complex patterns in the data. This would allow the F-PSWCA model to provide more accurate decisions by analyzing past data and uncovering hidden trends.
- **Development of Dynamic Sensitivity Analysis Capabilities:** Instead of static sensitivity analysis, dynamic models can be developed to continuously evaluate the impact of changes in criteria and parameters over time. This would help decision-makers identify the strengths and weaknesses of criteria under various conditions.
- **Using Multi-Objective Fuzzy Optimization:** Multi-objective fuzzy optimization methods can help the PSWCA model achieve optimal weights for criteria based on multiple simultaneous objectives. This approach would be beneficial for solving more complex, multidimensional problems.
- **Incorporation of Scenario Analysis and Simulation:** Creating and simulating different scenarios in the F-PSWCA model can help decision-makers assess the potential impacts of unforeseen changes or events on the final results. This capability would enhance the flexibility and comprehensiveness of the method.
- **Development of Big Data Decision-Making Models:** With the increasing volume of data available, developing a version of F-PSWCA capable of analyzing big data could improve decision-making accuracy and reliability.
- **Improvement of Fuzzy Scaling Methods:** New, more accurate fuzzy scaling methods can help reduce errors caused by data dispersion, leading to better results in criteria weighting.
- **Development of Practical Software Tools:** Advanced, user-friendly software based on the

F-PSWCA model, capable of interacting with users, analyzing various data, and providing comprehensive, visual reports, could make the method more applicable in real-world scenarios.

Declarations:

Ethics approval and consent to participate

Not applicable.

Consent for publication

We, the undersigned, give our consent for the publication of identifiable details, which can include
photograph(s) and/or videos and/or case history and/or details within the text to be published in the above
journal and article. We confirm that we have seen and been given the opportunity to read both the material and the article.

Availability of data and material

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors have no relevant financial or non-financial interests to disclose.

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