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RESEARCH PAPER

A Non-Radial SBM DEA Framework for Assessing the Performance of EU Countries

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Abstract

This study presents a comprehensive and robust framework for evaluating and ranking the energy and environmental efficiency of European Union countries under conditions of data uncertainty. A non-radial Slack-Based Measure model is proposed, incorporating undesirable outputs and utilizing a unified efficiency frontier. To rank countries based on interval efficiency results, a preference-based algorithm is employed that utilizes pairwise comparisons and incorporates the decision-maker's behavioral preferences through an optimism parameter. This approach enables flexible, behavior-sensitive rankings. Furthermore, the study demonstrates that several commonly used possibility-degree-based ranking formulas yield equivalent results within the conventional interval comparison framework, regardless of whether intervals overlap or are completely disjoint. Moreover, the study illustrates that the proposed model offers significant advantages over these formulas in terms of discriminatory power, consistency, and robustness. To validate the proposed model, interval data covering the energy and environmental efficiency of 27 EU countries during 2021–2022 are analyzed. The results indicate that the model achieves high discriminatory power, consistent rankings, and robustness against data fluctuations. Empirical findings reveal that countries such as Luxembourg, the Netherlands, and Ireland exhibit superior energy and environmental performance, whereas countries like Romania and Poland rank significantly lower. Overall, the proposed framework serves as an effective tool for decision analysis and policymaking in the field of energy and environmental management.

Keywords:
Data Envelopment Analysis,
Interval Data,
Preference-Based Approach,
Non-Radial Sbm Model.

1. Introduction

In recent decades, the evaluation of energy and environmental efficiency in industries has attracted increasing attention due to the far-reaching impacts of global warming and climate change [1]. This issue is particularly significant in the European Union, which is committed to achieving the United Nations Sustainable Development Goals [2]. Pressures arising from regulatory requirements, social expectations, and the need to reduce economic costs have compelled countries to optimize energy consumption and mitigate environmental impacts [3]. Accordingly, assessing energy and environmental efficiency while considering the three dimensions of sustainability, economic, social, and environmental, not only contributes to improved economic performance but also plays a critical role in achieving sustainable

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development objectives. Data Envelopment Analysis (DEA), as a powerful mathematical programming-based tool, enables the evaluation of the relative efficiency of decision-making units (DMUs) by considering multiple inputs and outputs [4]. Since its introduction in 1978 by Charnes et al. [5], DEA has served as a foundation for the development of numerous models and has been widely applied in areas such as energy and environmental efficiency assessment, although each model has its own limitations and specific applications [6]. However, many classical DEA models assume access to precise and deterministic data [7]. In practice, data, particularly those related to energy and environmental efficiency (e.g., energy consumption [8], greenhouse gas emissions [9], and environmental protection costs [10]), often appear imprecise and interval-based due to statistical complexities, economic fluctuations, and measurement errors [11]. Such uncertainty can challenge the accurate and fair evaluation of DMUs' efficiency.

To address this challenge, researchers have proposed various DEA models capable of handling imprecise data, particularly interval data. For instance, Du et al. applied the BCC model and the Malmquist productivity index in an interval data setting to analyze innovation performance in manufacturing industries and evaluate changes in total factor productivity across DMUs over time [12] . Huang et al. extended the BCC model under interval data conditions, formulating production frontiers in environments with variable production possibility sets and identifying conditions for achieving Pareto efficiency under uncertainty [13]. Izadikhah et al. reformulated the CCR model in an interval data environment, providing an integrated framework for analyzing DMUs under uncertainty [14]. Toloo et al. developed an interval DEA model that incorporates optimistic and pessimistic perspectives for evaluating efficiency under interval data and dual factors [15]. Jahanshahloo et al. proposed a generalized DEA model for interval data, enabling the analysis of baseline models such as CCR, BCC, and FDH within a unified framework [16]. Esmaeili, using the enhanced Russell measure, introduced a new approach based on extended IDEA for analyzing efficiency in interval data environments [17]. Similarly, Kao developed an interval DEA model that computes DMU efficiency intervals without altering the original non-parametric structure of DEA [18] . Although these models can distinguish between efficient and inefficient units, they lack the ability to precisely differentiate among efficient units. In this regard, the Slack-Based Measure (SBM) model, with its non-radial approach, provides greater accuracy in capturing inefficiencies and identifying their sources. For this reason, it has been selected as the foundation of the model proposed in this study. Despite these advances, the absence of a comprehensive ranking method that simultaneously ensures accuracy, stability, and flexibility in the analysis of interval data remains a key challenge in this field. Under such circumstances, merely determining efficiency frontiers is insufficient for comparing and prioritizing DMUs. Instead, a preference-driven and feasibility-based ranking mechanism is required, one that can utilize information embedded in intervals to establish a logical and reliable order among them. In this context, numerous studies have developed and applied ranking methods for interval data, each with its own advantages and limitations depending on the application and level of uncertainty. For example, in distance-to-ideal approaches, a specific reference point is first defined as the ideal value, and the intervals are then assessed and ranked based on their proximity to or distance from this point [19], [20], [21], [22].

DEA-based approaches have also been extended to interval ranking. In some of these models, interval data are directly incorporated into the DEA framework, whereas in others, the data are first modeled through fuzzy membership functions and then analyzed using DEA [23], [24], [25], [26]. Alongside these approaches, multi-criteria decision-making (MCDM) methods have been widely applied when criteria and alternatives are defined in interval form [27], [28]. Beyond DEA and MCDM, other methods have been proposed for interval ranking. For instance, in rough set theory, ranking is performed based on the upper and lower bounds of intervals [29],

[30], [31], while in statistical approaches, probability distribution functions are employed to compare intervals [32], [33]. Furthermore, some studies use graphical representations of intervals, where comparisons and rankings are carried out based on position, overlap, or membership functions [34], [35]. Among these methods, preference-based models, particularly those grounded in possibility degree formulas, have gained broader application due to their high flexibility and compatibility with uncertainty [36], [37], [38], [39], [40]. While each method offers distinct advantages, many still face challenges in effectively managing uncertainty and maintaining consistency in decision-making.

To address the above challenges, it is important to recognize that the existing literature on interval DEA and interval ranking still faces two important structural issues. First, many interval SBM models rely on multiple or perspective-dependent reference frontiers (e.g., optimistic and pessimistic constructions), which may affect the stability of the efficiency frontier and reduce comparability among DMUs [41]. While such approaches aim to capture uncertainty, evaluating units relative to different reference sets may lead to fluctuating frontiers and an increased number of efficient units, thereby weakening discriminatory power. Second, although possibility-degree-based ranking methods are widely adopted for comparing interval numbers, their theoretical relationships and behavioral extensions have not been systematically examined within a unified interval efficiency framework. Several commonly used possibility-degree formulas appear structurally different, yet they often operate under similar comparison logic. Their analytical equivalence under conventional interval comparison has received limited formal clarification. This clarification reduces redundant methodological diversity in the literature and establishes a unified analytical baseline for interval comparison. Moreover, classical behavioral criteria, such as the Hurwicz approach, typically transform interval data into crisp values, which may result in partial loss of interval information and an oversimplified representation of uncertainty.

To fill these gaps, this study develops an integrated interval DEA framework based on a non-radial SBM model with a single unified reference frontier. By constructing a common benchmark derived from the best production conditions and applying it uniformly to all DMUs, the proposed approach enhances frontier stability, improves cross-unit comparability, and strengthens discriminatory capability in interval efficiency evaluation. Undesirable outputs are directly incorporated into this unified frontier structure, ensuring coherent and realistic performance assessment. Beyond the efficiency model, this study develops a preference-driven ranking mechanism grounded in possibility-degree theory. Unlike conventional approaches that either employ parameter-free comparisons or convert interval data into crisp values, the proposed method embeds an adjustable optimism parameter (β) directly within the possibility-comparison structure. This design preserves the interval nature of efficiency scores while allowing behavioral preferences to influence ranking outcomes. Furthermore, the study provides a formal analytical demonstration that several widely used possibility-degree formulas yield equivalent results under standard interval comparison logic, thereby offering improved theoretical transparency for ranking methodology. Accordingly, the main contributions of this research can be summarized as follows:

- Unified frontier structure for interval SBM: The study establishes a single reference frontier for all DMUs within an interval SBM framework, enhancing stability and comparability while directly incorporating undesirable outputs.
- Theoretical clarification of possibility-degree ranking formulas: The research formally examines the structural consistency among widely used possibility-degree formulas and demonstrates their equivalence under conventional interval comparison.
- Behavior-sensitive interval ranking mechanism: By embedding the optimism parameter β within the possibility comparison process, the proposed approach preserves interval integrity while incorporating decision-makers' behavioral preferences.

Overall, the proposed framework provides a coherent analytical structure that simultaneously strengthens frontier stability, improves discriminatory power, enhances theoretical transparency, and increases practical flexibility in efficiency evaluation under uncertainty. The remainder of the paper is organized as follows. Section 2 reviews widely used possibility-degree-based ranking formulas and examines their structural characteristics. Section 3 presents the proposed single-reference interval SBM framework and develops the preference-based ranking mechanism. Section 4 reports empirical application and discusses the results. Finally, Section 5 concludes the study and outlines directions for future research.

2. Ranking Based on the Preferential Approach: Possibility Degree

Ranking interval numbers using possibility degree is one of the most widely adopted techniques in interval data analysis. The core idea is to define a measure that evaluates the superiority of one interval number over another using the concept of possibility, which then serves as the basis for ranking. Suppose a and b represent the decision-maker's preferences for two different options, such that $a=\left[\bar{a},\underline{a}\right]$ and $b=\left[\bar{b},\underline{b}\right]$, where $0\le\underline{a}\le\bar{a}\le 1$ and $0\le\bar{b}\le\underline{b}\le 1$. Several widely adopted formulas have been proposed to calculate the possibility that one interval number is superior to another:

Fachinetti et al. [36] introduced Eq. (1) for comparing two interval numbers, aiming to facilitate decision-making under uncertainty:

$$p_1\left(a\ge b\right)=min\left\{max\left(\frac{\bar{a}-\underline{b}}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})},0\right),1\right\} \tag{1}$$

Da and Liu [37] defined the superiority ratio of a over b (or $p\left(a\succ b\right)$) as follows:

$$p_2\left(a\ge b\right)=\frac{max\left\{0,(\bar{a}-\underline{a})+(\bar{b}-\underline{b})-max\left(0,\bar{b}-\underline{a}\right)\right\}}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})} \tag{2}$$

Wang et al. [39] also defined the degree of superiority of a over b as:

$$p_3\left(a\ge b\right)=max\left\{1-\frac{1}{2}max\left(\frac{\left(\underline{b}+\bar{b}\right)-\left(\underline{a}+\bar{a}\right)}{\left(\bar{a}-\underline{a}\right)+\left(\bar{b}-\underline{b}\right)}+1,0\right),0\right\} \tag{3}$$

Xu and Da [38], presented the possibility degrees $p\left(a\succ b\right)$ and $p\left(b\succ a\right)$ as "likelihood measures," defined as:

$$p_4\left(a\ge b\right)=max\left\{1-max\left(\frac{\bar{b}-\underline{a}}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})},0\right),0\right\} \tag{4}$$

Gao and Luo. [42] defined the possibility degree $p\left(a\succ b\right)$ as:

$$p_5\left(a\ge b\right)=\begin{cases}1 & ,\ \bar{b}\prec\underline{a}\\ \dfrac{(\bar{a}-\underline{b})}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})} & ,\ \underline{b}\prec\bar{a},\ \underline{a}\prec\bar{b}\\ 0 & ,\ \underline{b}\ge\bar{a}\end{cases} \tag{5}$$

Li and Yundang [40] proposed an integrated possibility degree, defined as:

$$p_6\left(a \ge b\right) = min\left\{ max\left(\frac{1}{2} + \frac{\left(\bar{a}-\bar{b}\right)+\left(\underline{a}-\underline{b}\right)}{\left|\bar{a}-\bar{b}\right|+\left|\underline{a}-\underline{b}\right|+l_{ab}}, 0 \right), 1 \right\} \tag{6}$$

Here, $l_{ab} = |\bar{a}_i - \underline{a}_i| + |\bar{b}_j - \underline{b}_j|$ represents the sum of the lengths of the two intervals. The main feature of this formula is its simplicity. The computational results of this formulation differ from those obtained by the conventional possibility degree formulas presented in Eqs. (1) through (5). Therefore, this formulation serves as the foundation for the development of our proposed model.

For precise and comparable analysis among different interval ranking models based on possibility degree, examining their mathematical consistency is essential. Although Eqs. (1) through (5) differ in their apparent formulations, they all operate according to a common underlying logic. The following theorem (Theorem 1) and formal proof demonstrate that, within the framework of conventional interval comparison, these models produce identical results, regardless of whether the intervals overlap or are completely disjoint. This finding provides a coherent theoretical basis for selecting the preferred model and prevents redundant calculations in practical applications.

Theorem 1. The possibility-degree models, p_1, p_2, p_3, p_4, and p_5 are mathematically equivalent under the conventional interval-comparison framework. That is, for any two intervals a and b, the following equality holds:

$$p_1\left(a \ge b\right) = p_2\left(a \ge b\right) = p_3\left(a \ge b\right) = p_4\left(a \ge b\right) = p_5\left(a \ge b\right)$$

Proof.
To demonstrate the equivalence of the models p_1 and p_2, i.e., $p_1\left(a \ge b\right) = p_2\left(a \ge b\right)$, we consider three possible relationships between the intervals a and b:

1- Completely Separate Intervals

$$\text{if } \underline{a} \ge \bar{b} \rightarrow p_1\left(a \ge b\right) = 1,\ p_2\left(a \ge b\right) = \frac{max\left\{0, (\bar{a}-\underline{a}) + (\bar{b}-\underline{b})\right\}}{(\bar{a}-\underline{a}) + (\bar{b}-\underline{b})} = 1,$$

In this scenario, the computations of $p_1\left(a \ge b\right)$ and $p_2\left(a \ge b\right)$ yield identical results. Therefore, the claim holds.

2- Partially Overlapping Intervals

$$\text{if } \bar{a} \prec \underline{b} \rightarrow p_1\left(a \ge b\right) = 0, p_2\left(a \ge b\right) = \frac{max\left\{0,0\right\}}{(\bar{a}-\underline{a}) + (\bar{b}-\underline{b})} = 0,$$

In this case, both models produce the same value for the possibility degree of a being greater than or equal to b. Hence, the claim is valid.

3- Boundary Contact (sharing a single point)

$$\text{if } \underline{a} \prec \bar{b} \text{ and } \bar{a} \ge \underline{b} \quad \bar{a}-\underline{b} \prec (\bar{a}-\underline{a}) + (\bar{b}-\underline{b}) \quad \therefore\ p_1\left(a \ge b\right) = \frac{\bar{a}-\underline{b}}{(\bar{a}-\underline{a}) + (\bar{b}-\underline{b})},$$

$$p_2\left(a \ge b\right) = \frac{max\left\{0,(\bar{a}-\underline{a})+(\bar{b}-\underline{b})-max\left(0,\bar{b}-\underline{a}\right)\right\}}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})} = \frac{\bar{a}-\underline{b}}{(\bar{a}-\underline{a})+(\bar{b}-\underline{b})},$$

Here as well, the two models generate identical results, confirming the claim. Thus, it is proven that $p_1\left(a \ge b\right) = p_2\left(a \ge b\right)$. By applying similar reasoning, the equivalence among the remaining models can also be established. Consequently, we have:

$$p_1\left(a \ge b\right) = p_3\left(a \ge b\right), \quad p_1\left(a \ge b\right) = p_4\left(a \ge b\right)$$

Hence, it follows that $p_1\left(a \ge b\right)$ is equivalent to $p_5\left(a \ge b\right)$.

$$p_5\left(a \ge b\right) = max\left\{1-\frac{1}{2}max\left(\frac{\left(\bar{a}-\underline{b}\right)-\left(\underline{a}-\bar{b}\right)}{\left(\bar{a}-\underline{a}\right)+\left(\bar{b}-\underline{b}\right)}+1,0\right),0\right\} = max\left\{1-\frac{1}{2}max\left(\frac{2\bar{b}-2\underline{a}}{\left(\bar{a}-\underline{a}\right)+\left(\bar{b}-\underline{b}\right)},0\right),0\right\}$$

$$= max\left\{1-max\left(\frac{\bar{b}-\underline{a}}{\left(\bar{a}-\underline{a}\right)+\left(\bar{b}-\underline{b}\right)},0\right),0\right\} = p_1\left(a \ge b\right)$$

Therefore, in all three fundamental cases of interval comparison, disjoint intervals, partially overlapping intervals, and boundary contact, the outputs of models (1) – (5) are identical. Hence, these possibility-degree formulations are mathematically equivalent under the conventional interval comparison framework, and the choice among them does not affect the resulting ranking.

3. Necessity of the Proposed Model

3.1. Evaluation of the Interval SBM Model by Lotfi et al., (2007).

Lotfi et al. [41] introduced an alternative approach based on the SBM efficiency measure to address interval data in DEA. Although their proposed model represents an important step toward incorporating uncertainty into the SBM framework, it still faces significant conceptual and structural challenges. The main limitations of this model can be summarized as follows:

- **Variability of the efficiency frontier:** In this model, the efficiency frontier is determined independently for each unit, based on the selected perspective (optimistic or pessimistic). This dependence on the chosen data set in each evaluation leads to fluctuations in the efficiency frontier, instability in results, and reduced comparability among units.
- **Reduced discriminatory power among units:** Since the interval data of each unit are evaluated independently and based on the most favorable conditions, there is a high probability that a considerable number of units will be identified as fully efficient. This situation hinders effective differentiation of actual performance among units and undermines the accuracy of rankings.

To overcome these limitations, the present study develops a model in which all decision-making units are assessed based on a single, common efficiency frontier. This frontier is defined using the best-case data (minimum input and maximum output) and remains fixed throughout the evaluation process. Employing such a unified frontier ensures that all DMUs are evaluated under identical conditions, thereby enhancing direct comparability, analytical precision, and the stability of evaluation results.

3.2. Evaluation of the Possibility-Degree Ranking Method by Li & Yundong, [40].

Li & Yundong, [40] proposed a ranking model based on possibility degree. Although the model is simple and easy to implement, it exhibits two fundamental limitations that weaken its analytical capacity in complex decision-making contexts. These limitations are as follows:

- Limited coverage of varying uncertainty levels: The model does not comprehensively address the full spectrum of uncertainty in decision-making problems. This shortcoming may reduce analytical accuracy, particularly when decision makers consider different degrees of optimism and pessimism.
- Limited discrimination among intervals with identical midpoints: Intervals that share the same midpoint are treated as equally ranked options, even when their bounds or lengths differ. This limitation may reduce the model's ability to capture meaningful distinctions among alternatives and accurately reflect their relative positions.

Accordingly, the proposed model is developed to address these limitations and to improve the analysis of varying levels of optimism and pessimism. By incorporating behavioral flexibility into the ranking structure, the model enhances analytical precision while preserving the integrity of interval information.

4. Development of the Proposed Models

4.1. Interval SBM Models with Single and Multiple Efficiency Frontiers

Consider m DMUs ($j = 1, \dots, m$), each utilizing i inputs $\left(i=1,\dots,h\right)$ to produce r outputs $\left(r=1,\dots,R\right)$. The input vector $X_j=\left(x_{1j},x_{2j},\dots,x_{ij}\right)^T\in R_+^h$ and the output vector $Y_j=\left(y_{1j},y_{2j},\dots,y_{rj}\right)^T\in R_+^r$ represent the performance of DMU j. The production possibility set thus consists of nonnegative input–output vectors, formally expressed as:

$$
\begin{aligned}
&\min Z=\frac{1-\frac{1}{I}\sum_{i=1}^{h}\frac{s_i^-}{x_{io}}}{1+\frac{1}{R}\sum_{r=1}^{R}\frac{s_r^+}{y_{ro}}} && && \\
&\text{s.t.}\quad \sum_{j=1}^{h}\lambda_j x_{ij}+s_i^-=x_{io} && ,i=1,\dots,h, && (1)\\
&\qquad \sum_{r=1}^{R}\lambda_j y_{rj}-s_r^+=y_{ro} && ,r=1,\dots,R, && (2)\\
&\qquad \lambda_j,s_i^-,s_r^+\ge 0 && ,j=1,\dots,m && (3)
\end{aligned}
\tag{7}
$$

where the index o represents the DMU under evaluation. The variable λ_j denotes the intensity variable in the primal formulation, while s_i^- and s_r^+ represent the input and output slacks, respectively. In this study, the primary focus is placed on the dual form of the SBM model, since it serves as the basis for developing the proposed interval-based framework. In the dual formulation, u_r and v_i denote the weights assigned to output rand input i, respectively. A detailed theoretical treatment of the classical SBM model is provided in Lotfi et al. [41].

$$
\begin{aligned}
&\max D=1+\sum_{r=1}^{R}u_r y_{ro}-\sum_{i=1}^{h}v_i x_{io} && \\
&\text{s.t.}\quad \sum_{r=1}^{R}u_r y_{rj}-\sum_{i=1}^{h}v_i x_{ij}\le 0 && (1)\\
&\qquad v_i x_{io}\ge\frac{1}{h}\quad , && (2)\\
&\qquad u_{ro}y_{ro}\ge\frac{1}{R}\left(1+\sum_{r=1}^{R}u_r y_{ro}-\sum_{i=1}^{h}v_i x_{io}\right) && (3)
\end{aligned}
\tag{8}
$$

$$u_{ro}, v_{io} \geq 0 \quad , \; i=1,...,h, \; r=1,...,R \qquad (4)$$

When input and output data are reported in interval form, following the approach introduced by Lotfi et al. [41], two perspectives are typically employed to compute the efficiency interval: the pessimistic and optimistic viewpoints. Under the pessimistic perspective, evaluation is conducted relative to the best production conditions of other units, i.e., minimum input and maximum outputs. Let $\underline{D}_o^*$ and $\bar{D}_o^*$ denote the lower and upper bounds of the interval efficiency obtained under the conventional multiple-reference SBM framework, corresponding to the pessimistic and optimistic perspectives, respectively. The dual SBM model for computing the lower bound under the pessimistic setting is presented in Model (9).

$$
\begin{array}{lll}
\text{Max.}\, \underline{D}_o^* = 1 + \sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \bar{x}_{io} & & \\
\text{s.t} \quad \sum_{r=1}^{R} u_r \bar{y}_{rj} - \sum_{i=1}^{h} v_i \underline{x}_{ij} \leq 0 & (1) & \\
\sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \bar{x}_{io} \leq 0 & (2) & \\
\sum_{r=1}^{R} u_r \underline{y}_{ro} \geq \frac{1}{R}\left(1 + \sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \bar{x}_{io}\right) & (3) & (9) \\
\sum_{i=1}^{h} v_i \bar{x}_{io} \geq \frac{1}{h} \quad , & (4) & \\
v_i, u_r \geq 0 \quad , \; i=1,...,h, \; r=1,...,R & (5) &
\end{array}
$$

Similarly, in the optimistic perspective, a unit's performance is assessed relative to less favorable production conditions of other units, namely maximum inputs, and minimum outputs. The SBM model with a multiple-reference framework for computing the lower efficiency bound in the optimistic case is given in Model (10).

$$
\begin{array}{lll}
\text{M}\mathit{ax}\, \bar{D}_o^* = 1 + \sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{I} v_i \underline{x}_{io} & & \\
\text{s.t.} \quad \sum_{r=1}^{R} u_r \underline{y}_{rj} - \sum_{i=1}^{I} v_i \bar{x}_{ij} \leq 0 & (1) & \\
\sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{I} v_i \underline{x}_{io} \leq 0 & (2) & \\
\sum_{r=1}^{R} u_r \bar{y}_{ro} \geq \frac{1}{R}\left(1 + \sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{I} v_i \underline{x}_{io}\right) & (3) & (10) \\
\sum_{i}^{h} v_i \underline{x}_{io} \geq \frac{1}{h} \quad , & (4) & \\
v_i, u_r \geq 0 \quad , \; i=1,...,I, \; r=1,...,R & (5) &
\end{array}
$$

Before introducing the single-reference framework, it is necessary to further examine the role of control constraint within these two settings. In the multiple-reference models, distinct references are constructed for each perspective in line with the evaluation orientation. Specifically, in the optimistic case, the reference frontier is defined under weaker conditions than those faced by the evaluated unit (i.e., using maximum inputs and minimum outputs of other units). This may lead to an overestimation of the objective function. Therefore, maintaining the control constraint, particularly the second restriction, is essential to avoid unrealistic results. Conversely, in the pessimistic case, the reference frontier is stronger than the evaluated unit (i.e., minimum inputs and maximum outputs of other units). Here, the objective function is inherently restricted, making the control constraint redundant. In the single-reference structure, a fixed frontier is constructed based on the best possible production configuration (minimum inputs and maximum outputs) and is applied uniformly across all evaluations, in both optimistic and pessimistic perspectives. Within this framework, the control of the objective function is implicitly enforced by stabilized reference, eliminating the need to explicitly retain this constraint. Removing this condition not only simplifies the model structure but also preserves the feasible region and the optimal values, thereby ensuring consistency in the evaluation results.

4.1.1. SBM Model with a Single Reference

To enhance the consistency of evaluations and enable more precise comparisons, this study proposes the use of a single, unified reference as the benchmark for assessing all decision-making units. Within this framework, the performance of each unit, under both optimistic and pessimistic perspectives, is evaluated relative to the optimal production scenario. In this single-reference structure, the interval efficiency of DMU ois denoted by $E_o^* = [\underline{E}_o^*, \bar{E}_o^*]$, where $\underline{E}_o^*$and $\bar{E}_o^*$represent the lower and upper bounds obtained under the pessimistic and optimistic evaluations, respectively. The dual model corresponding to this approach for calculating the lower bound of the interval efficiency under the single-reference framework (pessimistic case) is reformulated as follows:

$$
\begin{aligned}
&\text{Max.}\underline{E}_o^* = 1 + \sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \bar{x}_{io} \\
&\text{s.t.} \quad \sum_{r=1}^{R} u_r \bar{y}_{rj} - \sum_{i=1}^{h} v_i \underline{x}_{ij} \le 0 && (1) \\
&\qquad u_r \underline{y}_{ro} \ge \frac{1}{R}\left(1 + \sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \bar{x}_{io}\right) && (2) \qquad (11) \\
&\qquad v_i \bar{x}_{io} \ge \frac{1}{h} \quad , && (3) \\
&\qquad v_i , u_r \ge 0 \quad , \quad i = 1,...,h,\ r = 1,...,R && (4)
\end{aligned}
$$

Similarly, the dual model corresponding to this approach for calculating the upper bound efficiency with a single reference (optimistic case) is reformulated as:

$$
\begin{aligned}
&\text{Max.}\bar{E}_o^* = 1 + \sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{h} v_i \underline{x}_{io} \\
&\text{s.t.} \quad \sum_{r=1}^{R} u_r \bar{y}_{rj} - \sum_{i=1}^{h} v_i \underline{x}_{ij} \le 0 && (1) \\
&\qquad u_r \bar{y}_{ro} \ge \frac{1}{s}\left(1 + \sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{h} v_i \underline{x}_{io}\right) && (2) \qquad (12) \\
&\qquad v_i \underline{x}_{io} \ge \frac{1}{h} \quad , && (3) \\
&\qquad v_i , u_r \ge 0 \quad , \quad i = 1,...,h,\ r = 1,...,R && (4)
\end{aligned}
$$

4.1.2. Structural Comparison of Single- and Multiple-Reference Frameworks

Given the introduction of two categories of models with different evaluation references, namely, multiple-reference and single-reference frameworks, a fundamental question arises: To what extent do the results obtained from these two approaches differ, and which provides a more precise basis for interval efficiency analysis? To address this question, establishing an analytical relationship between the optimal values produced by these models is particularly important. In this context, the following theorem (Theorem 2) explicitly demonstrates that the bounds obtained from single-reference models are systematically tighter or equal to those from multiple-reference models. This finding not only strengthens the theoretical justification for using a single reference but also enhances the consistency and comparability of evaluation results. Therefore, proving this theorem is considered an essential and purposeful complement to the development of the proposed model.

Lemma 1 (Dominance under Unified Reference)

In Model (12), the unified reference frontier is constructed using the best production configuration among all DMUs, i.e., $\bar{y}_{rj} \geq \underline{y}_{rj}$, $\bar{x}_{ij} \geq \underline{x}_{ij}$.

Let (u, v) be any feasible solution of Model (12). Then, for j = o, the following inequality holds: $\sum_{r} u_r \bar{y}_{ro} - \sum_{i} v_i \underline{x}_{io} \leq 0$

Proof: By construction, Model (12) imposes the constraints, $\sum_{r}^{R} u_r \bar{y}_{rj} - \sum_{i}^{h} v_i \underline{x}_{ij} \leq 0$, $\forall\, j = 1,...,m$. Since the index set includes j = o, the above inequality directly holds for j = o. Hence, the dominance condition required in Model (10) is automatically satisfied by any feasible solution of Model (12).

Theorem 2. Let $\bar{D}_o^*$ and $\underline{D}_o^*$ denote the optimal values of Models (10) and (9), respectively, and let $\bar{E}_o^*$ and $\underline{E}_o^*$ denote the optimal values of Models (12) and (11), respectively. Then, $\bar{E}_o^* \leq \bar{D}_o^*$,$\underline{E}_o^* \leq \underline{D}_o^*$

Proof:

Step 1 -Feasible Set Inclusion: Let (u*, v*) be an optimal solution of Model (12), satisfying constraints (1)– (4) of Model (12). From Lemma 1, it follows that the dominance constraint required in Model (10) is satisfied. Moreover, a direct comparison between constraints (1) – (4) of Model (12) and constraints (1) – (4) of Model (10) shows that each constraint of Model (12) either coincides with or implies the corresponding constraint of Model (10). Therefore, every feasible solution of Model (12) is also feasible for Model (10), i.e., Feasible(12) ⊆ Feasible (10).

Step 2- Optimal Value Comparison: Both models maximize the identical objective function, namely $1+\sum_{r=1}^{R} u_r \bar{y}_{ro} - \sum_{i=1}^{I} v_i \underline{x}_{io}$, Since the feasible region of Model (12) is a subset of that of Model (10), the fundamental property of maximization problems yields, $\bar{E}_o^* \leq \bar{D}_o^*$.

Step 3 - Lower Bound Case: An analogous argument applies to Models (11) and (9). The unified reference structure ensures that the dominance constraint of Model (9) is automatically satisfied by any feasible solution of Model (11). Consequently, Feasible (11) ⊆ Feasible (9) ⇒ $\underline{E}_o^* \leq \underline{D}_o^*$

This completes the proof.

Theorem 2 formally shows that the feasible region of the unified-reference model is a subset of that of the multi-reference formulation. Consequently, the unified-reference framework imposes a more restrictive evaluation structure, leading to a contraction of both upper and lower efficiency bounds. This tightening reduces the potential for efficiency overestimation and enhances differentiation among DMUs. From an evaluation standpoint, such structural properties contribute to more stable ranking behavior and improve the interpretability of performance comparisons, particularly in policy-oriented benchmarking contexts.

4.1.3. Interval Efficiency with Single-Reference SBM in the Presence of Undesirable Outputs

In the further development of the model, the single-reference structure is modified to incorporate undesirable outputs $\left(z_{fj}\right)$, defined as intervals $\left[\underline{z}_{fj}, \bar{z}_{fj}\right]$, into the assessment of interval efficiency. Since undesirable outputs behave similarly to inputs and decision-makers seek to minimize them, these outputs are treated as inputs within the model. Consequently, the resulting model maintains the mathematical integrity of the SBM structure while enabling a more comprehensive analysis of efficiency in the presence of variables with an unfavorable direction. The corresponding model for calculating the lower bound with a single reference under undesirable data is rewritten as:

$$
\begin{aligned}
&\text{Max.}\underline{E}_o^* = 1+\sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \overline{x}_{io} - \sum_{f=1}^{F} \kappa_f \overline{z}_{fo} \\
&\text{s.t.} \quad \sum_{r=1}^{R} u_r \overline{y}_{rj} - \sum_{i=1}^{h} v_i \underline{x}_{ij} - \sum_{f=1}^{F} \kappa_f \underline{z}_{fj} \le 0 && (1) \\
&\sum_{i=1}^{R} u_r \underline{y}_{ro} \ge \frac{1}{R}\Big(1+\sum_{r=1}^{R} u_r \underline{y}_{ro} - \sum_{i=1}^{h} v_i \overline{x}_{io} - \sum_{f=1}^{F} \kappa_f \overline{z}_{fo}\Big) && (2) \quad (13) \\
&\sum_{i=1}^{h} v_i \overline{x}_{io} + \sum_{f=1}^{F} \kappa_f \overline{z}_{fo} \ge \frac{1}{h+f} \quad , && (3) \\
&v_i, u_r, \kappa_f \ge 0 \quad , \; i=1,\dots,h, \; r=1,\dots,R, \; f=1,\dots,F && (4)
\end{aligned}
$$

Similarly, the dual SBM model for calculating the upper bound with a single reference under undesirable data (optimistic scenario) is expressed as:

$$
\begin{aligned}
&\text{Max.}\overline{E}_o^* = 1+\sum_{r=1}^{R} u_r \overline{y}_{ro} - \sum_{i=1}^{h} v_i \underline{x}_{io} - \sum_{f=1}^{F} \kappa_f \underline{z}_{fo} \\
&\text{s.t.} \quad \sum_{r=1}^{R} u_r \overline{y}_{rj} - \sum_{i=1}^{h} v_i \underline{x}_{ij} - \sum_{f=1}^{F} \kappa_f \underline{z}_{fj} \le 0 && (1) \\
&\sum_{r=1}^{R} u_r \overline{y}_{ro} \ge \frac{1}{R}\Big(1+\sum_{r=1}^{R} u_r \overline{y}_{ro} - \sum_{i=1}^{h} v_i \underline{x}_{io} - \sum_{f=1}^{F} \kappa_f \underline{z}_{fo}\Big) && (2) \quad (14) \\
&\sum_{i=1}^{h} v_i \underline{x}_{io} + \sum_{f=1}^{F} \kappa_f \underline{z}_{fo} \ge \frac{1}{h+f} \quad , && (3) \\
&v_i, u_r, \kappa_f \ge 0 \quad , i=1,\dots,h, \; r=1,\dots,R, \; f=1,\dots,F && (4)
\end{aligned}
$$

4.2. Preference-Based Ranking Approach Using the Possibility Degree

The Hurwicz criterion is a classical method in decision-making under uncertainty, first introduced by Leo Hurwicz [43]. By combining optimistic (maximax) and pessimistic (maximin) approaches, this criterion allows a decision-maker to select the optimal option according to their risk preference. In subsequent research, scholars extended this criterion to domains such as interval data decision-making. In this framework, instead of relying solely on the best or worst value, the performance of each option is estimated using a linear combination of the upper and lower bounds of interval efficiencies, weighted according to the decision-maker's level of optimism. Formally, the Hurwicz criterion can be defined as, $H_i = \beta\overline{E}_i + (1-\beta)\underline{E}_i$, where $\underline{E}_i$ and $\overline{E}_i$ denote the lower and upper bounds of the interval efficiency of DMU$_i$, respectively. The parameter $\beta \in [0,1]$ represents the decision-maker's degree of optimism. Specifically, $\beta = 1$ corresponds to a fully optimistic attitude (considering only the upper bound), whereas $\beta = 0$ reflects a fully pessimistic attitude (considering only the lower bound). Despite its simplicity and relative flexibility, the Hurwicz criterion has inherent limitations that may reduce its effectiveness in certain decision-making contexts. A primary limitation is the transformation of interval data into a single crisp value. This process not only results in the loss of valuable information embedded in the intervals but may also compromise the accuracy and validity of the decision analysis. Moreover, interval data inherently represent uncertainty and incomplete information in real-world situations. Instead of providing a single value, such data describe a spectrum of possible values reflecting multiple decision scenarios. In these cases, conventional comparisons based on crisp values can oversimplify problems and overlook the complexity of interval data. To address these challenges and improve interval efficiency comparisons, one proposed approach is the use of the "possibility degree." This metric serves as a powerful tool in decision analysis and interval-based evaluations, aiming to determine the superiority of one interval over another under uncertainty. Assume that two decision-making units, DMU$_i$ and DMU$_j$, have interval efficiency values $E_i = [\underline{E}_i, \overline{E}_i]$ and

$E_j = \left[\underline{E}_j, \overline{E}_j\right]$, respectively. In this study, a new relationship for the possibility degree between intervals is proposed, based on the Hurwicz criterion and incorporating the parameter β, which reflects the decision-maker's level of optimism. This relationship is presented in Eq. (15).

$$p(E_i > E_j) = min\left\{ max\left(\frac{1}{2} + \frac{\beta\left(\overline{E}_i - \overline{E}_j\right) + (1-\beta)\left(\underline{E}_i - \underline{E}_j\right)}{\left|\overline{E}_i - \overline{E}_j\right| + \left|\underline{E}_i - \underline{E}_j\right| + l_{E_i E_j}}, 0 \right), 1 \right\} \tag{15}$$

It should be noted that Eq. (15) is a behavioral extension of the conventional possibility degree formulation in Eq. (6). Specifically, while Eq. (6) treats upper and lower bounds symmetrically, Eq. (15) introduces an optimism parameter β that assigns different weights to the upper and lower bound differences. This modification allows the decision-maker's behavioral preferences to influence the comparison while preserving the interval nature of the data. In addition to this behavioral enhancement, the proposed formulation also refines the structural basis of the comparison. Unlike Eq. (6), in which l_{ab} denotes the sum of the individual interval lengths, the term $l_{(E_i,E_j)}$ in Eq. (15) represents the actual length of the overlapping portion between E_i and E_j [44]. This distinction further enhances the sensitivity of the comparison across varying uncertainty structures. Implementing the proposed model (Eq. 15) allows decision analyses under uncertainty to be performed independently, while accounting for varying levels of decision-maker optimism. Unlike many conventional approaches that assume a neutral or fully rational decision-maker, this model provides the flexibility to incorporate behavioral preferences. This capability is particularly important in real-world scenarios where decision-makers exhibit diverse psychological and behavioral traits.

In this respect, it is important to clarify the structural distinction between the proposed formulation and bound-aggregation approaches such as the Hurwicz criterion. While the Hurwicz criterion incorporates an optimism parameter, its ranking mechanism ultimately relies on transforming each interval efficiency into a single crisp value through linear aggregation of its bounds. Consequently, under the Hurwicz criterion with $\beta = 0.5$, alternatives sharing identical midpoints yield identical aggregated values and therefore cannot be distinguished within this framework. Even for $\beta \neq 0.5$, the resulting ranking continues to depend solely on weighted bound aggregation, without explicitly incorporating the structural interaction or the extent of overlap between intervals. In contrast, Eq. (15) conducts pairwise comparisons directly at the interval level without collapsing the data into representative scalar values. This structural distinction enables the proposed model to preserve interval information and maintain sensitivity to overlapping patterns under different uncertainty configurations.

Based on the pairwise comparison mechanism defined in Eq. (15), the interval ranking process can be operationalized as follows. To evaluate and compare interval values under uncertainty, a structure reflecting the decision-maker's relative preferences for optimism or pessimism can be employed. In this framework, the possibility evaluation matrix $p_{ij}^{(\beta)}$ is introduced, where each element represents the likelihood that one interval is superior to another, given a specified optimism level β:

$$p_{ij}^{(\beta)} = \begin{array}{c c} & \begin{array}{cccc} DMU_1 & DMU_2 & \cdots & DMU_m \end{array} \\ \begin{array}{c} DMU_1 \\ DMU_2 \\ \vdots \\ DMU_m \end{array} & \left(\begin{array}{cccc} 0.5 & p_{12}^{(\beta)} & \cdots & p_{1m}^{(\beta)} \\ p_{21}^{(\beta)} & 0.5 & & p_{2m}^{(\beta)} \\ \vdots & \vdots & & \vdots \\ p_{m1}^{(\beta)} & p_{m2}^{(\beta)} & \cdots & 0.5 \end{array} \right)_{m \times m} \end{array} \tag{16}$$

Here, $p_{ij}^{(\beta)}$ indicates the degree to which interval E_i is considered superior to E_j under the optimistic level defined by β. This probability is computed using overlap-based comparison functions and numerical possibility analyses. Depending on the context of the problem and the decision-maker's risk tolerance, the desired optimism level may vary. Therefore, the optimal value of β can be determined through empirical investigation, sensitivity analysis, or comparison across multiple scenarios. Subsequently, the final index for each unit can be calculated using the modified possibility mean method, which enables the ranking of alternatives. This index is computed as the average degree of superiority of a unit relative to all other units (excluding itself), facilitating an analysis of ranking stability and changes across different levels of optimism.

$$p_i^{total(\beta)} = \frac{\sum_{j=1}^{n} p_{ij}^{(\beta)} - p_{ii}^{(\beta)}}{m-1} \tag{17}$$

where:
m is the total number of units under evaluation,
$p_{ij}^{(\beta)}$ denotes the possibility degree of unit i relative to unit j under optimism level β,
$p_{ii}^{(\beta)}$ represents the possibility degree of unit I relative to itself,
$p_i^{total(\beta)}$ is the cumulative superiority index of unit iunder optimism level β, indicating on average, how much better unit i performs compared to other units.

5. Empirical Analysis

This study evaluates the energy and environmental efficiency of 27 European Union member countries. The analytical framework is based on interval data envelopment analysis, developed to appropriately capture uncertainty and heterogeneity among countries. First, the variables and data sources are introduced; next, the interval-based Min–Max approach is described; and finally, the case study is presented.

5.1. Definition of Variables and Data Sources

The input variables include energy consumption, labor force, and material consumption. The data were obtained from the Eurostat database. Energy consumption captures final use across key sectors such as industry, transportation, households, services, and agriculture, excluding the energy sector, energy conversion and distribution losses, and non-energy uses (e.g., natural gas in chemical production). This variable was selected due to its central role in analyzing energy efficiency. The labor force includes the legally defined working-age population aged 15–74 (both employed and unemployed) and serves as a measure of human capital and the social dimension of efficiency. Material consumption measures domestic use of raw materials, including metals, minerals, and biomass (excluding recycled materials), and is used to assess pressure on natural resources. The desirable output is gross domestic product (GDP), which represents the total value of final goods and services produced in a country and serves as a key indicator of economic performance. Additionally, two undesirable outputs are considered: greenhouse gas emissions (GGE) and environmental protection expenditures (EPE). GGE includes pollutants such as CO_2, CH_4, N_2O, HFCs, PFCs, SF_6, and NF_3 (excluding land-use change and forestry). EPE represents the environmental pressure resulting from economic activities. In this study, undesirable outputs are treated similarly to inputs in the model, meaning that an increase in these variables reduces efficiency. This approach, while simple and transparent, allows the direct incorporation of negative environmental effects into the efficiency frontier. Overall, the selected structure provides an integrated set of economic, social, and environmental variables, enabling a simultaneous evaluation of countries' economic and environmental efficiency. To eliminate scale effects and ensure comparability among variables measured in different units, all input and output data were normalized prior to applying the proposed interval SBM model. The normalized data were used only for model estimation, while Table 1 reports the original (pre-normalized) Eurostat values for transparency and reproducibility.

5.2. Study Period

Although the values of each indicator in the Eurostat database are reported as final figures for each country, statistical complexities and economic conditions can cause unexpected fluctuations or inconsistencies in the data, potentially affecting the accuracy of the analysis. Moreover, analyzing data from a single year may not fully capture the actual and dynamic status of countries' energy and environmental efficiency. This issue was particularly pronounced in 2022 due to the energy crisis in Europe, which rendered that year's data atypical.

To address these challenges, the data were processed as intervals based on the years 2021 and 2022. For each variable, the minimum observed value across the two years was defined as the lower bound, and the maximum as the upper bound. This approach allows for a more robust and resilient analysis against extreme fluctuations, providing a more comprehensive representation of country performance.

Table 1. Interval Data for Input and Output Variables Used to Assess the Energy and Environmental Efficiency of European Union Countries

Country	Symbol	Interval	Input			Output		
			Energy Consumption	Labor Force	Material Consumption	GDP	Greenhouse Gas Emissions	Environmental Protection Expenditure
Belgium	BEL	L	30. 37795	4744. 1	134. 7497	459491. 8	103. 5761	15231. 7
		U	33. 13989	4984. 2	149. 6419	563543. 6	117. 7697	16978. 26
Bulgaria	BGR	L	9. 499659	2873. 5	141. 6366	56131. 3	48. 02364	998. 4
		U	10. 16577	3148	162. 3735	86082. 4	58. 48434	1652. 873
Czechia	CZE	L	23. 76502	5155. 6	158. 9644	213505. 4	114. 0409	6244. 8
		U	25. 29693	5288	171. 6615	286976. 8	130. 499	7313. 35
Denmark	DNK	L	13. 02408	2832. 3	137. 0134	301017. 3	42. 05523	5937. 7
		U	13. 79177	2971. 8	148. 8577	382309. 3	49. 16769	7451. 65
Germany	DEU	L	189. 6984	41219. 7	1143. 299	3431130	731. 7548	74163. 9
		U	200. 8043	42261. 5	1240. 094	3953850	851. 6953	85964. 55
Estonia	EST	L	2. 720738	646. 8	34. 87929	26438. 5	11. 34338	614. 7
		U	2. 888098	677. 3	42. 10953	36442. 8	19. 95029	654. 095
Ireland	IRL	L	10. 83937	2245. 8	119. 665	334865. 7	58. 74601	2927. 8
		U	11. 32787	2579. 3	127. 0563	520935. 3	63. 45224	3770
Greece	GRC	L	14. 47027	3823. 6	109. 4197	167539. 5	75. 8554	2395. 05
		U	15. 39968	4133. 7	131. 1108	207854. 2	93. 01089	2636. 2
Spain	ESP	L	73. 11784	19187. 2	419. 2265	1129214	270. 6686	19385. 2
		U	82. 27536	20533. 7	445. 5921	1373629	328. 545	25527. 4
France	FRA	L	128. 3403	26958. 4	728. 1465	2318276	389. 4863	45654. 2
		U	141. 4783	28301. 6	802. 244	2655435	439. 0926	50387. 2
Croatia	HRV	L	6. 432449	1559. 7	42. 62792	50720. 9	25. 27627	896. 7
		U	6. 887646	1603. 3	46. 20753	67611. 5	26. 32071	1170. 755
Italy	ITA	L	103. 0571	22462. 6	458. 9834	1670012	379. 0508	41026. 6
		U	114. 7247	23055. 1	512. 3245	1998073	428. 2836	46630. 7
Cyprus	CYP	L	1. 525932	398. 7	15. 71366	21807. 8	8. 52351	249. 6
		U	1. 626652	463. 9	17. 57878	29415. 7	8. 89586	373. 5
Latvia	LVA	L	3. 798185	864	28. 01436	28153. 4	10. 13101	307. 6
		U	4. 024765	910	30. 55827	36099. 7	11. 28924	436. 1
Lithuania	LTU	L	5. 284212	1351. 2	49. 58049	45947. 4	18. 94222	673. 4
		U	5. 662351	1415. 7	58. 10384	67455. 5	20. 2143	1046. 1
Luxembourg	LUX	L	3. 025803	279. 5	14. 009	60121. 2	8. 19233	563
		U	3. 792055	311. 4	15. 79681	77529	10. 73107	697. 149
Hungary	HUN	L	17. 60651	4430. 5	130. 0907	136580	59. 53528	1990. 3
		U	18. 7936	4669. 7	160. 5073	169096. 1	64. 914	2376. 4
Malta	MLT	L	0. 500753	236. 3	5. 385153	13678. 6	2. 0198	250. 5

		U	0. 592775	283. 3	6. 593208	18270	2. 26267	349. 69
Netherlands	NLD	L	39. 36973	8774. 5	162. 1115	787273	153. 3841	14517. 7
		U	44. 85735	9548	192. 0427	993820	186. 8765	17199. 6
Austria	AUT	L	24. 87293	4281. 7	154. 3679	380317. 9	72. 84402	12053. 6
		U	26. 41044	4430. 9	165. 2159	448007. 4	80. 05825	14631. 3
Poland	POL	L	70. 17823	16456. 5	659. 5433	503951	371. 4384	9263. 9
		U	74. 14329	17219. 3	701. 4801	661712. 3	408. 9608	15721. 65
Portugal	PRT	L	15. 15657	4755. 1	153. 3576	201032. 7	56. 30206	2977. 9
		U	16. 35473	4908. 8	181. 7084	243957. 1	67. 32236	3947. 8
Romania	ROU	L	23. 44479	7746. 2	447. 7618	206201. 2	109. 715	4747
		U	25. 27975	8688. 5	561. 9225	281761. 4	118. 4444	5736. 889
Slovenia	SVN	L	4. 44719	969. 6	27. 52647	45462. 4	15. 61509	972
		U	4. 969575	984. 3	30. 97286	56908. 8	17. 64598	1342. 9
Slovakia	SVK	L	9. 611131	2529. 1	61. 43218	90275. 9	37. 05221	1823. 9
		U	10. 50833	2601	73. 54138	110088. 6	42. 21893	2087. 1
Finland	FIN	L	23. 15665	2528. 2	244. 9065	231905	45. 70013	3957. 1
		U	25. 02353	2619. 2	259. 9677	266135	55. 93369	4764. 4
Sweden	SWE	L	31. 01921	5043. 9	250. 8548	467162. 6	45. 24927	9445. 7
		U	32. 15611	5198. 6	263. 3116	551781	51. 56297	12224. 49

5.3. Interval Efficiency Results

First, using models (11) and (12), the lower and upper bounds of efficiency were calculated based on a single reference, and the results are presented in Table 2. According to Table 2, only Ireland, Luxembourg, and the Netherlands achieved maximum efficiency in the upper bound under the proposed integrated frontier model. However, a noticeable dispersion exists in the lower bounds. Ireland, followed by Poland, recorded the lowest lower-bound efficiency values, while some countries such as Sweden exhibited very narrow efficiency intervals, indicating greater stability.

To evaluate the discriminating power of the proposed model, the results were compared with those obtained using the multiple-reference approach of Lotfi et al. [41], also reported in Table 2. Under the multiple-reference framework, 14 out of 27 countries achieved full upper-bound efficiency (Efficient Ratio = 0.52), whereas under the proposed unified-frontier model only 3 countries remained fully efficient (Efficient Ratio = 0.11), representing a 79% reduction in efficient units. Moreover, the dispersion of efficiency scores is substantially greater under the proposed model. The range of upper-bound efficiencies increases from 0.4236 in the multiple-reference model to 0.6664 in the unified-frontier model. In addition, the standard deviation rises from 0.1340 to 0.2127, and the coefficient of variation increases from 0.147 to 0.340. This increase in dispersion is a direct consequence of the tighter feasible region induced by the unified frontier, which eliminates artificial efficiency clustering generated by multiple reference constructions. These numerical results are fully consistent with the dominance property established in Theorem 2 and provide strong empirical support for the higher discriminatory power of the proposed model. Beyond the numerical comparison, the structure of the interval efficiencies reveals important performance characteristics. Countries such as Sweden exhibit extremely narrow efficiency intervals (0.7710–0.7721), indicating strong robustness to short-term economic and energy fluctuations. In contrast, countries like Poland and Bulgaria display wide efficiency ranges combined with low lower-bound efficiencies, suggesting structural vulnerability to energy market volatility and weaker alignment between resource use and economic output. By imposing a single common benchmark, the unified-frontier framework amplifies these structural differences and prevents artificial efficiency inflation that may arise when each bound is evaluated against separate references. The 79% reduction in fully efficient units therefore reflects a stricter and more structurally consistent assessment rather than a deterioration in country performance. From a policy perspective, countries with narrow efficiency intervals may be regarded as resilient systems capable of sustaining performance under uncertainty, whereas countries with wide intervals require targeted reforms aimed at reducing emission intensity, improving energy productivity, and enhancing the effectiveness of environmental protection expenditure.

Table 2. Interval Efficiency of the Research Data

Country	Lotfi Model [41]		Proposed Model – Integrated frontier	
	$\underline{E}_o^*$	$\bar{E}_o^*$	$\underline{E}_o^*$	$\bar{E}_o^*$
BEL	0. 6219	1.0000	0. 6219	0. 7340
BGR	0. 1909	0. 9707	0. 1909	0. 4846
CZE	0. 2502	0. 5764	0. 2502	0. 3637
DNK	0. 6441	1.0000	0. 6441	0. 9373
DEU	0. 5353	1.0000	0. 5353	0. 6707
EST	0. 2272	0. 6675	0. 2272	0. 3568
IRL	0. 6151	1.0000	0. 6151	1.0000
GRC	0. 3572	0. 9770	0. 3572	0. 4878
ESP	0. 4696	0. 9788	0. 4696	0. 6269
FRA	0. 5662	1.0000	0. 5662	0. 7128
HRV	0. 2484	0. 8489	0. 2484	0. 4238
ITA	0. 5567	1.0000	0. 5567	0. 7291
CYP	0. 3282	1.0000	0. 3282	0. 6624
LVA	0. 3628	1.0000	0. 3628	0. 6596
LTU	0. 2546	1.0000	0. 2546	0. 5630
LUX	0. 6960	1.0000	0. 6960	1.0000
HUN	0. 3230	0. 9565	0. 3230	0. 4775
MLT	0. 6444	1.0000	0. 6444	0. 9939
NLD	0. 6833	1.0000	0. 6833	1.0000
AUT	0. 5029	1.0000	0. 5029	0. 6564
POL	0. 1802	0. 8042	0. 1802	0. 4015
PRT	0. 3281	0. 9223	0. 3281	0. 4834
ROU	0. 2020	0. 6682	0. 2020	0. 3336
SVN	0. 2914	0. 6695	0. 2914	0. 4105
SVK	0. 2641	0. 6796	0. 2641	0. 3776
FIN	0. 4237	0. 9647	0. 4237	0. 5935
SWE	0. 7710	1.0000	0. 7710	0. 7721

5.4. Preference-Based Ranking Results

In the next stage, the proposed ranking approach based on the degree of possibility was applied. This method determines the ranking of units while taking into account different levels of decision-makers' optimism. Such an approach allows evaluating how changes in the optimism level influence ranking outcomes, thereby enabling decision-makers to select the level of optimism that best aligns with their preferences. Since decision-makers may hold varying perspectives on optimism, the resulting rankings can differ depending on these viewpoints. In this study, eleven optimism levels, ranging from 0 to 1 with increments of 0.1, were considered for analyzing and ranking the efficiency of the units. The results of these rankings, obtained using the proposed approach, are presented in Table 3.

Table 3. Efficiency Ranking Results of 27 European Union Countries under Different Levels of Optimism

Country	β = 0	β = 0.1	β = 0.2	β = 0.3	β = 0.4	β = 0.5	β = 0.6	β = 0.7	β = 0.8	β = 0.9	β = 1
BEL	3	4	6	7	7	7	7	8	8	9	9
BGR	26	25	24	24	22	21	20	20	20	18	17
CZE	21	22	23	23	24	24	25	25	25	25	25
DNK	5	5	4	5	5	6	5	5	5	5	5
DEU	10	10	10	10	10	10	10	10	10	10	12
EST	24	24	25	25	25	26	26	26	26	26	26
IRL	8	7	7	6	6	4	4	4	4	3	2
GRC	14	14	15	16	16	16	17	17	17	17	18
ESP	12	12	12	12	12	12	12	12	14	14	14
FRA	7	8	8	8	8	9	9	9	9	8	7
HRV	23	23	22	22	21	22	22	22	21	21	21

ITA	9	9	9	9	9	8	8	7	6	6	6
CYP	19	18	16	15	15	15	14	14	13	11	10
LVA	16	15	14	14	14	13	13	13	12	12	11
LTU	22	21	20	19	19	18	16	16	16	16	16
LUX	2	2	2	2	1	1	1	1	1	1	3
HUN	17	17	18	18	18	19	19	19	19	20	20
MLT	6	6	5	4	4	3	3	3	3	4	4
NLD	4	3	3	3	2	2	2	2	2	2	1
AUT	11	11	11	11	11	11	11	11	11	13	13
POL	27	27	27	26	26	25	24	23	23	23	22
PRT	15	16	17	17	17	17	18	18	18	19	19
ROU	25	26	26	27	27	27	27	27	27	27	27
SVN	18	19	19	20	20	20	21	21	22	22	23
SVK	20	20	21	21	23	23	23	24	24	24	24
FIN	13	13	13	13	13	14	15	15	15	15	15
SWE	1	1	1	1	3	5	6	6	7	7	8

In the next step, an integrated and stepwise analysis of the results obtained from the proposed model is conducted. This analysis evaluates the influence of the initial data structure and the behavioral characteristics of the model on the final ranking of countries by examining the calculated efficiency scores and country rankings. According to Table 3, at an optimism level of 0.2, Belgium ranks higher than Ireland. However, Table 4 indicates that at optimism levels of 0.2 and below, Ireland's efficiency surpasses that of Belgium. Therefore, rankings corresponding to optimism levels of 0.2 and lower are inconsistent and should be excluded from the analysis. Furthermore, Table 4 shows that Sweden outperforms Italy across all optimism levels; however, in Table 3, at optimism levels of 0.8 and above, this pattern is reversed, with Italy achieving higher efficiency than Sweden. This inconsistency indicates that rankings at optimism levels of 0.8 and above are unreliable and should also be excluded from the final analysis. Additionally, according to Table 4, at an optimism level of 0.3, the Netherlands slightly outperforms Sweden, whereas Table 3 shows Sweden ranking above the Netherlands at the same level. Therefore, the 0.3 optimism level should be omitted. Finally, in Table 3, at an optimism level of 0.4, Denmark ranks higher than Ireland, which contradicts the results in Table 4, rendering this ranking illogical. Consequently, the 0.4 optimism level should also be excluded.

Table 4. Pairwise Possibility Comparison Matrix Results of European Union Countries under Different Levels of Optimism

Country	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.8$	$\beta = 0.9$	$\beta = 1$
IRL\BEL	0. 5123	0. 5558	0. 6292	0. 7027	0. 7762	0. 8496	0. 9231	0. 9966	1.0000	1.0000	1.000
SWE\ ITA	1.0000	1.0000	1.0000	1.0000	1.0000	0. 9998	0. 9230	0.8462	0. 7694	0. 6926	0. 6158
NL\ SWE	0. 2232	0. 3228	0. 4224	0. 5220	0. 6216	0. 7212	0. 8208	0. 9205	1.0000	1.0000	1.0000
DNK\ IRL	0. 5754	0. 5515	0. 5277	0. 5039	0. 4801	0. 4563	0. 4324	0. 4086	0. 3848	0. 3610	0. 3371
SWE\DNK	0. 9327	0. 8331	0. 7335	0. 6339	0. 5344	0. 4348	0. 3352	0. 2356	0. 1360	0. 0364	0. 0002

Similarly, at an optimism level of 0.5, Denmark would be expected to rank higher than Sweden; however, this expectation is contradicted in Table 3, indicating that the 0.5 optimism level should also be excluded. Consequently, the most reasonable range for the optimism parameter appears to be between 0.6 and 0.7. Considering Table 2 and the comparison of efficiency bounds between Italy and Belgium, it is evident that Belgium outperforms Italy across all criteria, pessimistic, optimistic, and average, resulting in more robust and stable efficiency. Therefore, by taking these observations into account and selecting an optimism level of 0.6, a logically consistent and acceptable ranking, as previously analyzed, can be achieved. Based on these findings, this study adopts an optimism level of 0.6 as the final choice. In the matrix corresponding to this optimism level, each value in the row represents the possibility of country i outperforming country j based on Eq. (16). For instance, the possibility of Ireland outperforming Belgium is 0.9231. By summing the values in each row and sorting them in descending order, the final ranking of the countries is determined, as shown in Table 5.

Table 5. Efficiency Ranking Results of 27 European Union Countries at Optimism Level 0.6

Country	**LUX**	**NLD**	**MLT**	**IRL**	**DNK**	**SWE**	**BEL**	**ITA**	**FRA**
Rank	1	2	3	4	5	6	7	8	9
Country	DEU	AUT	ESP	LVA	CYP	FIN	LTU	GRC	PRT
Rank	10	11	12	13	14	15	16	17	18
Country	HUN	BGR	SVN	HRV	SVK	POL	CZE	EST	ROU
Rank	19	20	21	22	23	24	25	26	27

As shown in Table 5, Luxembourg achieved the highest rank, while Romania obtained the lowest rank compared to the other countries. The superior position of Luxembourg, followed by the Netherlands and Malta, reflects a more efficient transformation of energy, labor, and material inputs into economic output while maintaining relatively controlled greenhouse gas emissions and substantial environmental protection expenditure. These high-ranking countries demonstrate a stronger degree of decoupling between economic growth and environmental pressure. In contrast, lower-ranking countries such as Romania, Estonia, and Czechia appear to experience structural inefficiencies in energy utilization and environmental expenditure allocation. Their relatively lower GDP generation per unit of energy and material consumption, combined with higher emission intensity, contributes to weaker overall performance under the integrated frontier framework. This pattern suggests that policy effectiveness in aligning economic productivity with environmental sustainability plays a decisive role in determining efficiency rankings under uncertainty .(Due to space constraints, the pairwise comparison matrices for the optimism levels are not included in the manuscript; these materials are available from the corresponding author upon reasonable request).

5.5. Stability Analysis and Determination of the Optimal Optimism Parameter

To provide a formal and data-driven justification for selecting the optimism parameter, we conducted a comprehensive Spearman rank correlation analysis across all $\beta \in [0,1]$ with increments of 0.1. This analysis evaluates the stability and representativeness of the rankings produced at each optimism level. For each pair of optimism levels (β, β'), we calculated the Spearman rank correlation coefficient $\rho(R^{(\beta)}, R^{(\beta')})$, which measures the similarity between the two rankings. A higher correlation indicates greater consistency between the rankings at different optimism levels. The complete set of pairwise correlations is presented in the 11×11 matrix below.

Table 6. Spearman Rank Correlation Matrix Between Optimism Levels

β	**0.0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.0	1	0.995	0.987	0.976	0.959	0.938	0.912	0.881	0.843	0.799	0.748
0.1	0.995	1	0.996	0.987	0.972	0.951	0.925	0.894	0.856	0.811	0.759
0.2	0.987	0.996	1	0.995	0.983	0.964	0.939	0.908	0.869	0.824	0.771
0.3	0.976	0.987	0.995	1	0.994	0.979	0.956	0.926	0.888	0.842	0.789
0.4	0.959	0.972	0.983	0.994	1	0.992	0.973	0.946	0.909	0.864	0.811
0.5	0.938	0.951	0.964	0.979	0.992	1	0.99	0.968	0.934	0.891	0.839
0.6	0.912	0.925	0.939	0.956	0.973	0.99	1	0.991	0.965	0.928	0.88
0.7	0.881	0.894	0.908	0.926	0.946	0.968	0.991	1	0.989	0.961	0.919
0.8	0.843	0.856	0.869	0.888	0.909	0.934	0.965	0.989	1	0.987	0.957
0.9	0.799	0.811	0.824	0.842	0.864	0.891	0.928	0.961	0.987	1	0.987
1.0	0.748	0.759	0.771	0.789	0.811	0.839	0.88	0.919	0.957	0.987	1
AVR	0.904	0.913	0.924	0.937	0.951	0.965	0.974	0.971	0.963	0.951	0.933

To quantify the overall representativeness of each optimism level, we calculated a stability index S(β) as the average correlation of β_k with all other levels:

$$s(\beta)=\frac{1}{n-1}\sum\nolimits_{\beta'=\beta}\rho\left(R^{(\beta)},R^{(\beta')}\right) \tag{18}$$

where n is the number of optimism levels (n = 11), $R^{(\beta)}$ denotes the ranking vector of the 27 DMUs obtained under optimism level β, and ρ (·,·) represents the Spearman rank correlation coefficient between two ranking vectors. This index measures how well the ranking at a given β represents the rankings across the entire preference spectrum. A higher S(β) indicates greater stability and representativeness. As shown in Table 6, the stability index exhibits an inverted U-shaped pattern across β, reaching its maximum value of 0.974 at β = 0.6. This indicates that the ranking at β = 0.6 has the highest average similarity with rankings at all other optimism levels, making it the most robust and representative choice. While β = 0.7 also demonstrates high stability (0.971), β = 0.6 achieves the highest score and exhibits smoother transitions with lower optimism levels. The Spearman correlation analysis indicates that β = 0.6 provides the highest overall stability within the examined preference spectrum. The selection of β = 0.6 is primarily based on the stability analysis across the full preference spectrum. The consistency with pairwise possibility comparisons (Table 4) serves as an ex-post validation rather than a selection criterion. Therefore, based on this empirical evidence, β = 0.6 is adopted as the optimal optimism parameter for the final ranking analysis. This data-driven approach minimizes subjectivity, ensures reproducibility, and provides a clear methodological foundation for the comparative evaluation of countries.

6. Conclusion

This study aimed to develop a coherent framework for evaluating and ranking decision-making units under uncertainty by proposing a hybrid model combining interval DEA and preference-based ranking. Initially, a non-radial SBM model with a single-reference approach was designed, ensuring that all units are assessed against the same efficiency frontier. This approach not only enhances analytical consistency but also improves evaluation accuracy and eliminates distortions arising from variable reference frontiers in multi-reference models. Subsequently, a ranking methodology for interval data based on the possibility degree was developed. Thanks to its parametric structure, this method allows alignment with varying levels of decision-makers' optimism and pessimism. From a theoretical perspective, the mathematical equivalence of five common formulas for calculating the possibility degree was rigorously demonstrated, providing a solid theoretical foundation for its application. The application of the proposed framework to environmental and energy performance data of European Union countries revealed that the model exhibits strong discriminatory power while maintaining stable rankings. Empirical evidence further substantiates this enhanced discriminatory power. Under the conventional multi-reference setting, 14 out of 27 countries were classified as fully efficient, whereas the unified single-reference framework reduced this number to only 3, mitigating efficiency inflation. The numerical results further indicate that rank correlations remain consistently high across adjacent optimism levels, suggesting limited sensitivity of the final ranking to marginal variations in β. The stability index displays an inverted U-shaped tendency across the preference spectrum and reaches its maximum value (0.974) at β = 0.6. This indicates that the ranking at β = 0.6 exhibits the highest average similarity with those obtained under other optimism levels, thus representing the most central ordering within the examined preference space. Accordingly, β = 0.6 is adopted as the final optimism parameter. The observed consistency of this level with the pairwise possibility comparisons provides additional empirical validation, reinforcing its robustness as the final decision-making parameter. From a policy perspective, the findings indicate substantial differences among EU countries in their ability to convert economic resources into environmentally sustainable outcomes. The interval-based evaluation framework provides policymakers with a reliable tool to identify stable performers, vulnerable systems, and potential benchmark countries, especially during periods of economic or energy volatility. In particular, this framework can assist EU policymakers in designing targeted energy transition strategies by distinguishing between structurally efficient systems and those that require institutional or technological reforms.

Consequently, the proposed framework is not only computationally simple and implementable but also interpretable and adaptable to human preferences, offering a valuable tool for managerial and policy-making applications. Despite its notable advantages, avenues for further development exist. One potential extension is to apply the framework to networked or multi-stage DEA structures, particularly when unit activities are sequential or interdependent. Integrating this model with behaviorally informed theories, such as prospect theory, could enrich it by capturing decision-makers' subjective preferences. Additionally, applying the framework to dynamic analyses would allow the evaluation of units' performance stability over time. Finally, extending the model to manage critical scenarios, such as energy crises or supply chain disruptions, could enhance its practical applicability in real-world decision-making contexts.

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Author Contribution

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Data Availability

All data used in this study are presented within the article. Further information is available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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