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RESEARCH PAPER

Development Optimization Model of a Zero-Defect Single Sampling Plan

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Abstract
One way to control the quality of products is to inspect the lot inputs. The focus of this paper is on a non-linear integer programming model for determining an optimal single sampling plan for inspecting different parts so that the total cost of the quality control is minimized and we try to improve the quality of inputs to the assembly line by applying a rectifying inspection policy. The optimization model includes the cost of the inspection, the cost of non-conforming items entering the assembly line and the cost of rejecting the items. In this research, it is assumed that the inspection is perfect and zero acceptance number policy is employed for inspection. If a non-conforming item is found in the sample, the total lot is rejected. Each part is different in the risk of non-conforming items, the cost of non-conforming items, the size of the lot and the cost of the inspection. In the practical example, it can be seen that the rate of defective items, followed by the cost of defective items and the cost of lot rejection, have been greatly reduced following the proposed methods and minimized the cost of quality control.

Keywords:
Acceptance Sampling;
Non-linear Integer;
Programming;
Incoming Inspection;
Rectifying Inspection

Introduction

The complexity of the products that are being produced today has increased dramatically, and consequently, quality control of the products is more difficult, and the customer demands a durable product, as a result, quality is an important aspect in product development and is a key factor in achieving target markets and gaining competitive advantages. In order to assure the quality of the product, there are various tools; one of these tools is to apply sampling for acceptance, so that after collecting a random sample from the lot, the selected quality characteristic is inspected, then according to the information obtained from this inspection, a decision is made on the acceptance or rejection of the lot. Accepted lots in the production line are used and rejected lots are reworked or returned to the supplier [1].

Over the past two decades, the use of zero-acceptance sampling has been growing. In this method, a lot is accepted if there are not any non-conforming items in the sample; this means that the number of non-conforming items should be equal to zero. Sampling method with zero acceptance number emphasizes the lack of non-conforming items and it is easier to use for manufacturers and consumers. In this method, since the acceptance number is zero, and the lot quality has a reciprocal relationship with sample size, thus its optimization is of great importance.

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Determining the sample size is significant because of its effect on the sampling costs. If the consumer does not inspect, then the cost of the non- conforming items will be incurred and if the inspection is 100%, the inspection cost will increase significantly, so the choice of the optimal sample size should make a balance between these two costs.

To determine the sample size, there are different methods that one of these methods is to design the economic model that determines all related qualitative costs and aims to minimize the total cost of the capability to optimize the sample size.

Literature Review

In order to have a product with high quality, the components in the product and the relationships between the components should fall within the tolerances for the quality characteristics. Therefore, the manufacturer must inspect the components prior to assembling, in order to ensure that the components are assembled in the product. According to the standards, if the faulty items enter the assembly line, the system or product will not function properly, and this will lead to the additional cost for the company. In order to minimize production costs, sampling plans are a method to prevent the usage of non-conforming components into the final product. 100% inspection is expensive and time-consuming and does not guarantee acceptance without inspection of mismatched items in finished product quality; so this research has been done to optimize sampling plans to reduce costs. We will continue to study the research performed in this field.

Most of the activities carried out in the field of sampling have been analytical; Wetherill and Chiu [2] have also specifically referred to this issue. Several articles have been written in the field of sampling that most of them are based on the articles written by Hald [3]. One of the main reasons for the acceptance sampling approach is to consider the cost of the inspection methods. For the first time, Lieberman and Resnickov [4] proposed a statistical sampling approach for the inspection of the incoming lot, but the economic design of acceptance sampling was presented by Bennett et al. [5]. Schmidt et al. [6] also introduced another model for variable sampling plan, in which a step-by-step model was developed to calculate the cost of quality. In the proposed model, as long as a quality characteristic is within the acceptance region, the cost is considered to be zero, and when the quality characteristic is outside of the acceptance region, fixed costs will be incurred. With the introduction of Taguchi's [7] loss function, there was a revolution in the vision towards the quality costs.

There are different approaches to design an acceptance sampling plan. One of the important approaches in this area is to consider the risk of accepting or rejecting the lot and economic factors. In this approach, an optimal sampling plan is obtained by taking into account the cost of rejection or acceptance.

Humzic et al. [8]. Presented an economic model based on a single sampling plan with a zero-acceptance number. In this paper, two models have been investigated. The first model was designed, regardless of labor costs and the second model is based on labor costs for inspection. The proposed model is able to solve and provide an optimal solution for the problem. Qin et al [9] also provided a non-linear integer programming model in 2015 to determine a single sampling plan with a zero-acceptance number for inspection of the items entering to the assembly line. This model can determine the optimal sample size for different parts. In this research, a three-step solution algorithm is proposed that significantly reduces the problem-solving time.

Willemain et al. [10]. Presented an economic model for the 100% inspection problem and single sampling plan with the presence and absence of inspection errors so that they used the Taguchi model to analyze the difference between the real value of the quality characteristic and its target value. The proposed scheme is a new design due to the use of a continuous loss function for comparison study and analyzing the effect of the inspection errors and the greater clarity of the model in treating with the effect of the error and also the possibility of using the model in designing control charts.

Ferrell and Choker [11] proposed an economic method for determining the optimal sampling plan by taking into account consumer and producer loss functions. In their study, they proposed an approach for designing a single acceptance sampling plan for obtaining sample sizes and optimal acceptance numbers. The Taguchi continuous loss function has been used to determine the amount of deviation from the target level in their study. They also considered the inspection error in their model.

Niaki and Fallahnezhad [12] used Bayesian inference and dynamic planning for designing a sampling plan in quality control environments. The proposed model has considered cost and risk function in the objective function to obtain an optimal policy. Niaki and Fallahnezhad initially modeled the problem as a dynamic planning and then minimized the cost / (1-risk) ratio to identify the optimal decision. Ultimately, they designed and defined control thresholds to make decisions easier.

Fallahnezhad and Hosseini Nasab [13] introduced a new control policy for acceptance sampling schemes. Decisions are made on the basis of the number of faulty items in the inspected sample. The purpose of this model is to find a constant control level that minimizes total costs, including the cost of rejection, the cost of the inspection, and the cost of a non-conforming item. Optimization is carried out using negative binomial distribution and Poisson distribution.

Hsu and Hsu [14] provided an economic plan for determining an optimal sampling scheme in a two-stage supply chain based on the cost of quality and internal failure by taking into account consumer and producer risks. They concluded that the proposed design was very sensitive to the product quality of the manufacturer.

Fallahnezhad and Niaki [15] developed an optimization model to determine the optimal values of the control thresholds so that they satisfy the constraints on the probabilities of the type 1 and 2 errors. They used a Markov model to extract the total cost of the acceptance sampling plan, including the cost of acceptance and inspection.

Fallahnezhad and Aslam [16] provided a new sampling plan for decision-making based on the cost-objective function. They used Bayesian inference to update the probability distribution function of non-conforming proportion. In addition, Bayesian inference, along with a recursive induction, has been used to estimate the expected cost of different decisions.

Li and et al. [17] reviewed the military standard MIL-STD-1916. This standard is in the form of a zero acceptance number. This means that if the non- conforming items are not detected then the lot will be accepted and if any non- conforming items are found, the total lot will be rejected. In this research, the author states that if non-conforming items are not found in the sample, it does not mean that the entire lot is in accordance with the specification. Military standards also require a large workforce to inspect the sample with large sizes that are not feasible in the real world. Champernowne's research [18] focuses on the economic outcomes of the problem by using a sampling plan as the quality of a tool in the process. Champernowne has considered three states in his problem:

1. Average lot quality for testing and the change between qualitative lots of average.
2. The cost of inspection and its dependence on the amount of the inspection.
3. Costs associated with making a mistake to accept or making mistakes to reject any lot and dependence of this cost to the cost of quality in each lot.

Champernowne (1953) developed an economic model to determine whether a lot should be accepted or rejected. He focused mainly on the economic aspects of the problem. This means that as long as the results are within the economic range, the lot is accepted even if the non-conforming items are found in the sample. Bernard [19] believes that Champernowne's assumptions are not available in the real world. Bernard proposed that assigning a probability distribution for the number of non-conforming items is required to solve the problem.

Hamaker [20] has provided three different sampling methods: 1- sampling tables, 2- data collection, and 3- design of sampling plans. He also presented an economic and concluded that it would be ineffective to inspect all items in the lot if there is little chance to detect non-conforming items in the lot. He also suggested that a sampling plan should first be selected and monitored and, if necessary, it should be used for new data.

Calvin [21] state similar concerns with regard to non-conforming items. The author believes that many managers only look for ways to reach the non-conforming zero, and they do not pay attention to staying at the non-conforming level of zero. The author believes that there are various statistical methods, including control charts and acceptance sampling schemes that managers can use for their products to stay at a non-conforming level of zero.

If lot is rejected then production may decrease, or may be stopped due to lack of parts. Salameh and Jaber [22] focused on the optimal inventory level of items that may include non-conforming items. They found that the number of items in each order increases if the number of non-conforming item increases. On the other hand, Maddah and Jaber [23] observed that ordering a large number of items with poor quality is not always very profitable, so a reasonable balance between transportation costs and inventory costs is needed in order to achieve more profit.

Taghipour and Benjewik [24] and Taghipour and et al. [25] considered the economic aspect of the sampling plan. They considered two different types of failures in the system: a hard failure that breaks a system and a soft failure that does not break the system but reduces the system's effectiveness. Therefore, if the failures are not soft, the system will not be efficiently implemented and will increase the system cost.

Shi and Zhou [26] provided a brief overview of different methods to improve the quality control for the processes with multiple stages. Some of the important methods are the physical method, the data-driven model and statistical process control. Physical methods require past data about the process. The data-driven model requires sufficient knowledge in mathematics and statistics as well as a database for proper estimation. Data-driven models are more attractive because they do not need past information to use in the process. Statistical process control has a probability of wrong alarm and according to the findings of Shi and Zhou; there is no ability to discriminate between changes in different stages.

Starbird [27] has shown that if a manufacturer uses a zero-acceptance number for the inspection of incoming items, it exacerbates the supplier to deliver the lots with zero non-conforming items. In addition, when manufacturers use 100% inspection or zero acceptance number for inspection, this method is more optimal for the manufacturer to inspect output products.

Fernandez [29] has presented a nonlinear integer programming problem to find an acceptance sampling plan for defective items per unit with producer and consumer risks that minimizes expected surplus costs. An algorithm to determine the optimal number of units for inspection and decision criterion is also presented. In addition, Fernandez [30] has proposed a nonlinear integer programming problem to determine a binomial sampling scheme to investigate large lot with consumer and producer risks.

Lu Cui et al. [31] have presented a new review to redesign sample size as a time-adapted sequence sampling design to determine sample size. The new approach results in optimizing the design over a wide range of design parameters.

Sommer and Steland [32] have proposed a new sampling method as a multi-stage framework in which the accumulation is monitored at several time points and is accepted only if it goes through all stages.

Ahmadi et al. [33] provided a bayesian problem of predicting future observations with an exponential distribution based on an observed sample, taking into account both total cost of the experiment and the mean square error of prediction in order to determine the sample size.

Determining the sample size is important when a zero-acceptance number method is used. The sample size for inspection is determined by different methods, including:

1. Using standard sampling inspection tables such as Dodge and Romig [28].
2. The use of acceptance criteria, such as the acceptance quality level (AQL) and the lot tolerance proportion defective (LTPD), based on producer and consumer risks.
3. Developing an economic model to consider all the costs associated with quality.

Among these methods, the economic model provides a better ability to optimize the sample size [9].

Sample size determination is of great importance due to its cost impact, and by obtaining the optimal sample size we can reduce the quality control costs. The proposed model is able to provide an optimal and economic sample size considering all costs, which minimizes the cost of quality control.

The Problem Formulation

In the sampling plan, when the acceptance number is zero random samples from are selected the lot of each section by the inspectors if the non-conforming items are not found in the sample, the lot is accepted; otherwise, the total lot is rejected and rectifying inspection will be done. Inspections in different sections can be formulated as an optimization problem, which aims to reduce total costs by selecting the optimal sample size for each section. By increasing the sample size in each section, the non- conforming item inputs to the assembly line decreases and reduces the expected cost of non- conforming items. Also increasing the sample size increases the inspection time. Therefore, we must choose the optimal sample size by determining the right strategy.

To determine the sample size in each section, the inspection time must be specified. Also, the values of input variables to the assembly line are different in each segment. This problem can be modeled as a nonlinear integer programming problem as follows:

Index:

$I : \{i \mid i = 1, 2, ..., M\}$

Variables:

n_i : sample size

d_i :The number of non- conforming items in each lot from section i , which varies from zero to N_i.

$d_i : \{0, 1, 2, ..., N_i\}$

Parameters:

N_i :Lot size.

T :Available time of inspection.

C_i :The cost of non- conforming item in the section i if accepted.

C_i' :The cost of a non- conforming item in the section i if rejected.

L :Labor cost per time unit.

r_i :The rate of non- conforming items in the section i .

t_i :Average time of inspection of one item in the section i .

α :Producer risk (probability of rejecting a good lot).

β : Consumer risk (probability of accepting a bad lot).
AQL : Acceptable quality level.
LQL : Limiting quality level.
$b(d_i | N_i, r_i)$: The probability that section i has d_i non- conforming items so that the lot size is N_i and the rate of non- conforming items is r_i .
$h(0|N_i, d_i, n_i)$: The probability of finding a non- conforming item in the section i such that the lot size is N_i and the number of non- conforming items is d_i and the size of the sample size is n_i.

$$p = \min f(n_1, n_2, ..., n_M) = Min\sum_{i=1}^{M}\left\{\left(\frac{C_i\sum_{d=0}^{N_i} b(d_i|N_i, r_i).d_i.h(0|N_i, d_i, n_i)}{\sum_{d=0}^{N_i} b(d_i|N_i, r_i).h(0|N_i, d_i, n_i)} + L.t_i.n_i\right)\times\left(\sum_{d=0}^{N_i} b(d_i|N_i, r_i).h(0|N_i, d_i, n_i)\right) + \left(1-\sum_{d=0}^{N_i} b(d_i|N_i, r_i).h(0|N_i, d_i, n_i)\right)\times(N_i.t_i.L + N_i.r_i.C_i')\right\} \quad (1)$$

$$\sum_{i=1}^{M} t_i n_i \le T \quad (2)$$

$$b(d_i | N_i, r_i) = \binom{N_i}{d_i} r_i^{d_i}(1-r_i)^{N_i-d_i} \quad (3)$$

$$h(0|N_i, d_i, n_i) = \frac{\binom{N_i - d_i}{n_i}}{\binom{N_i}{n_i}} \quad (4)$$

$$\sum_{d=0}^{N_i} b(d_i | N_i, AQL).h(0|N_i, d_i n_i) \ge 1-\alpha \quad (5)$$

$$\sum_{d=0}^{N_i} b(d_i | N_i, LQL).h(0|N_i, d_i n_i) \le \beta \quad (6)$$

$$0 \le n_i \le N_i \quad (7)$$

$$n_i = \text{integer} \quad (8)$$

The goal of the problem is to determine the optimal sample size for each section so that the total cost includes the cost of inspection, the cost of non- conforming items and the cost of rejecting the lot would be minimum. The equation for total cost can be elaborated as follows:

(Cost of non-conforming items conditioned on accepting the lot + inspection cost) (probability of acceptance) + (lot rejection probability) * (lot rejection cost + lot total inspection cost)* (9)

The constraint (2) shows the maximum inspection time for inspecting the M parts. For each lot in each part, there are two decisions (acceptance, rejection) and the probability of having d_i non-conforming items in the lot of part i follows the binomial distribution as shown in Eq. 3.

The number of non-conforming items in the sample will be allowed to be zero for accepting the lot and since the inspection is carried out without replacement of non-conforming items, as a result of the hyper-geometric distribution is employed in Eq. 4. For evaluating the probability of accepting the lot, constraint (5) denotes the producer risk that evaluates the probability of accepting a good lot. The constraint (6) is the consumer risk and shows the probability of accepting a bad lot. The constraint (7) shows the sample size range from zero to N_i, and the constraint (8) indicates that the sample size values are integer. If the sample size is zero, the inspection will not be done and the risk of accepting a lot with d_i non-conforming items will be high. If the size of the sample is N_i, the inspection is done 100% and the acceptance risk for a lot with d_i non-conforming items is zero, but the cost of the inspection will be high. By increasing the sample size, the probability of accepting lot with d_i non-conforming items decreases. Therefore, the larger sample size is expected cost to have a less defective input to the assembly line.

Since we are using zero acceptance number, thus the sample is inspected and if non-conforming items are not found, lot is accepted, hence the acceptance cost includes the inspection cost and cost of non- conforming items entering the assembly line, so that the cost of a defective input to The assembly line is C_i, thus the cost of accepting non-conforming items entering the assembly line is calculated as following:

$$C_i E(d_i \mid Accepting\ the\ Lot) = C_i E(d_i \mid all\ items\ are\ conform\ in\ the\ sample) = \left(\frac{C_i . b(d_i \mid N_i, r_i) . d_i . h(0 \mid N_i, d_i, n_i)}{b(d_i \mid N_i, r_i) . h(0 \mid N_i, d_i, n_i)} \right) \tag{10}$$

Now, in order to calculate the expected total cost for a given part, taking into account all possible values of d_i, the equation can be as following:

$$\left(\frac{C_i \sum_{d_i=0}^{N_i} b(d_i \mid N_i, r_i) . d_i . h(0 \mid N_i, d_i, n_i)}{\sum_{d_i=0}^{N_i} b(d_i \mid N_i, r_i) . h(0 \mid N_i, d_i, n_i)} \right)$$

If a non-conforming item is found in the sample, then the lot is rejected. In this case, the costs will include the cost of inspecting all items in the lot and the cost of replacing or repairing non-conforming items with the conforming items and it will be calculated as following:

$$\left(N_i . t_i . L + N_i . r_i . C_i' \right)$$

Finally, in order to find the expected total cost for Section M, the values are aggregated and calculated as Eq. 1.

It should be noted that the model presented in the reference does not consider the cost of lot rejection, but in the model of this study, in addition to the cost of defective items entering the assembly line and inspection costs, the cost of lot rejection means the cost of replacement and replacement of defective items and rectifying inspection is included in the model.

Results and Analysis

In this study, we plan to obtain optimal sample size values for 20 different parts (M = 20). Inspection time is 8 hours and 10 inspectors are in the process, so the total inspection time is 80 hours (T = 4800 min), and each inspector's wage rate is \$ 0.05 per minute; as a result, the inspection cost is \$ 240.

Input values are: t_i: Average time of inspecting one item. C_i: The cost of non- conforming items entering the assembly line. C_i' : Cost of replacing or repairing one non-conforming item. N_i: Lot size. r_i: Rates of non-conforming items.

Because of the added cost of lot rejection and lot Rectifying Inspection, the entire lot will be inspected if a defective item is found and by considering other base article input values, each lot input rate and inspection time are less than the base article values.

Solution Method

The proposed model is programmed in the MATLAB environment. First the values $d_i = \{0,1,2,...,N_i\}$ are substituted in the objective function for different parts and the value of cost objective. Function is calculated, then the optimal sample size will be determined by determining the minimum value of the objective function. When the sample size is obtained, the percentage of non-conforming items after the inspection is calculated for different parts. Now, with a non-conforming item price, the inspection cost and the cost of rejecting the lot after the inspection can be calculated. For example, the calculations performed for the first part can be as follows:

$$AOQ = \frac{(P_a \times r)(N-n)}{N} \tag{11}$$

In Eq. 11, P_a is the probability of accepting the lot. For N = 168 and n = 28, the AOQ value is 0.0071. The cost of non-conforming items and the lot rejection cost after the inspection can now be obtained as follows:

Cost of non- conforming items after inspection = $(N-n)\times C_i \times AOQ = 140\times 86\times 0.00713 = 85.8$

Cost of rejecting the lot after inspection = $(N-n)\times C_i' \times AOQ = 140\times 77\times 0.00713 = 76.8$

Table 1 shows the optimal sample size values and the cost of different decisions and reduction percentages in the non-conforming proportion. As shown in Table 1, the first part is the input values that are different for each part, the second part is the results and the problem that shows the optimal sample size, and The third section shows the rate of defective items, the cost of defective items, and the lot rejection cost before and after the inspection.

By comparing the model results presented in this article and the base article, the percentage of inspection (N / n) in all sections improved compared to the baseline results. This improvement is also shown in the effectiveness section. The average efficiency of the rate of defective items after inspection in this article is 83.6%, while in the base article it is 46.5%.

As can be seen in Table 2, the quality control costs have dropped significantly after the inspection. Expected total cost is 3326$, which includes 240$ for inspection costs, 1462$ for lot rejection costs and 1624$ for non- conforming items entering the assembly line.

The following graphs can be used to estimate the number of defective items and the cost of rejection and non-conforming items before and after the inspection.

Table 1. problem description

Number of parts, M	20	α	0.05
Time of inspection available, T	4800	β	0.1
Horly rate of inspection wage, L ($/ min)	0.05	AQL	0.001
		LQL	0.2

Input						Decisions		Effectiveness								
Parts i	InspectionTime per item (min) t	NC cost per item ($), C	Cost of rejection ($), C ’	lot size N	NC rate r	Sample size, n	pct to be inspected n/N	NC rate before inspection	NC rate after inspection	pct change in NC rate	NC cost before inspection	NC cost after inspection	pct change in NC Cost	Cost of rejection before inspection	Cost of rejection after inspection	pct change in Cost of rejection
1	1	86	77.4	168	0.08	28	17	0.08	0.007	91	1156	86	93	1040	77	93
2	0.5	129	116.1	105	0.03	43	41	0.03	0.005	84	406	39	90	366	35	90
3	1.1	121	108.9	145	0.07	27	19	0.07	0.009	88	1228	123	90	1105	111	90
4	0.3	182	163.8	165	0.1	26	16	0.1	0.006	94	3003	158	95	2703	142	95
5	2	60	54	129	0.08	22	17	0.08	0.011	86	619	73	88	557	66	88
6	1	61	54.9	148	0.1	23	16	0.1	0.009	91	903	65	93	813	59	93
7	1.8	40	36	160	0.02	47	29	0.02	0.006	72	128	25	80	115	22	80
8	0.3	76	68.4	100	0.07	15	15	0.07	0.021	70	532	135	75	479	121	75
9	2	111	99.9	110	0.01	51	46	0.01	0.003	68	122	3	97	110	3	97
10	1.6	74	66.6	123	0.09	21	17	0.09	0.011	87	819	86	90	737	77	90
11	1.9	182	163.8	134	0.04	36	27	0.04	0.007	83	976	124	87	878	111	87
12	2.3	189	170.1	170	0.05	35	21	0.05	0.007	86	1607	176	89	1446	159	89
13	2	28	25.2	128	0.07	21	16	0.07	0.013	81	251	40	84	226	36	84
14	0.3	69	62.1	158	0.04	40	25	0.04	0.006	85	436	49	89	392	44	89
15	1.7	67	60.3	170	0.05	33	19	0.05	0.008	85	570	71	88	513	64	88
16	0.8	104	93.6	150	0.04	38	25	0.04	0.007	84	624	77	88	562	69	88
17	2	45	40.5	130	0.06	23	18	0.06	0.012	79	351	60	83	316	54	83
18	0.3	129	116.1	169	0.06	33	20	0.06	0.007	89	1308	117	91	1177	105	91
19	0.2	82	73.8	155	0.1	24	15	0.1	0.008	92	1271	83	93	1144	74	93
20	1.7	49	44.1	150	0.03	40	27	0.03	0.007	78	221	36	84	198	32	84

Table 2. total benefits

	Before inspectin $	After inspection $	Change $	Change %
Inspection cost	-	240	240	0
Cost of rejection	14877	1462	13415	90.173
NC cost	16530	1624	14906	90.175
total cost	31407	3326	28561	90.938

Fig. 1 shows the rate of defective items before and after the inspection as the graph is visible, the rate of defective items decreased significantly after the inspection, indicating that the consideration of the lot rejection cost, ie the cost of rejecting and replacing the defective items and rectifying inspection in the model, significantly reduced the inputs to the assembly line. And the quality of the product comes out. Figs. 2 and 3 also show the cost of defective items and the lot rejection cost and show the reduction in costs after inspection.

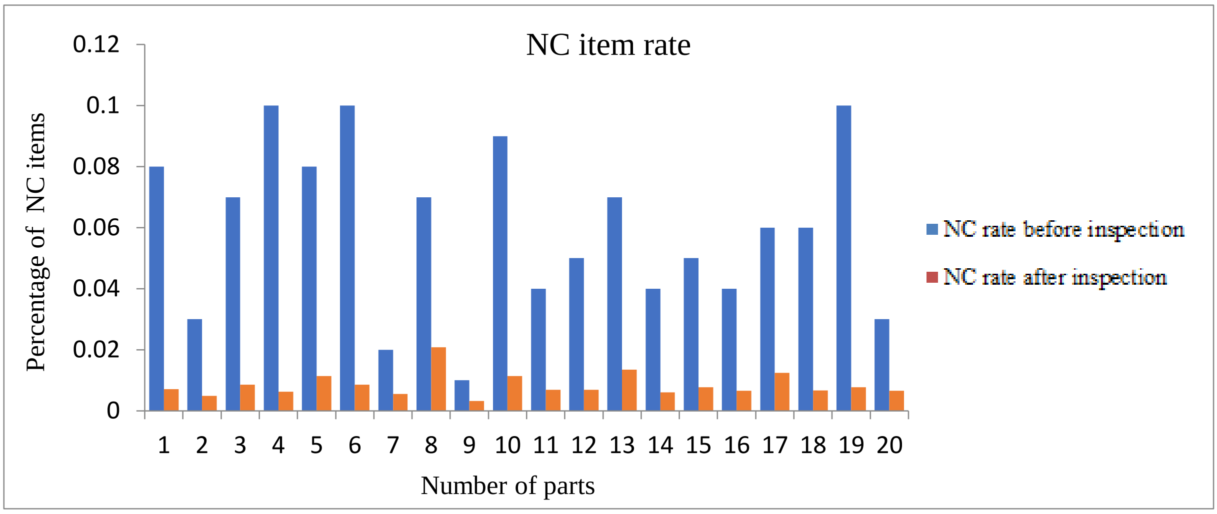

Fig. 1. Non-conforming item rates before and after inspection

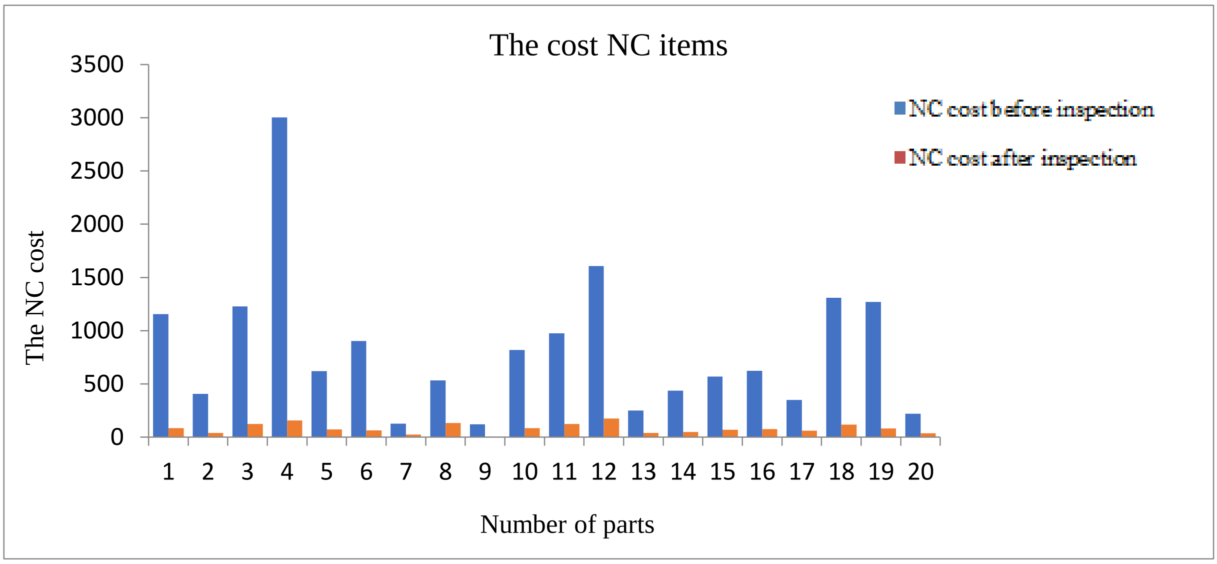

Fig. 2. Cost of non-conforming items before and after inspection

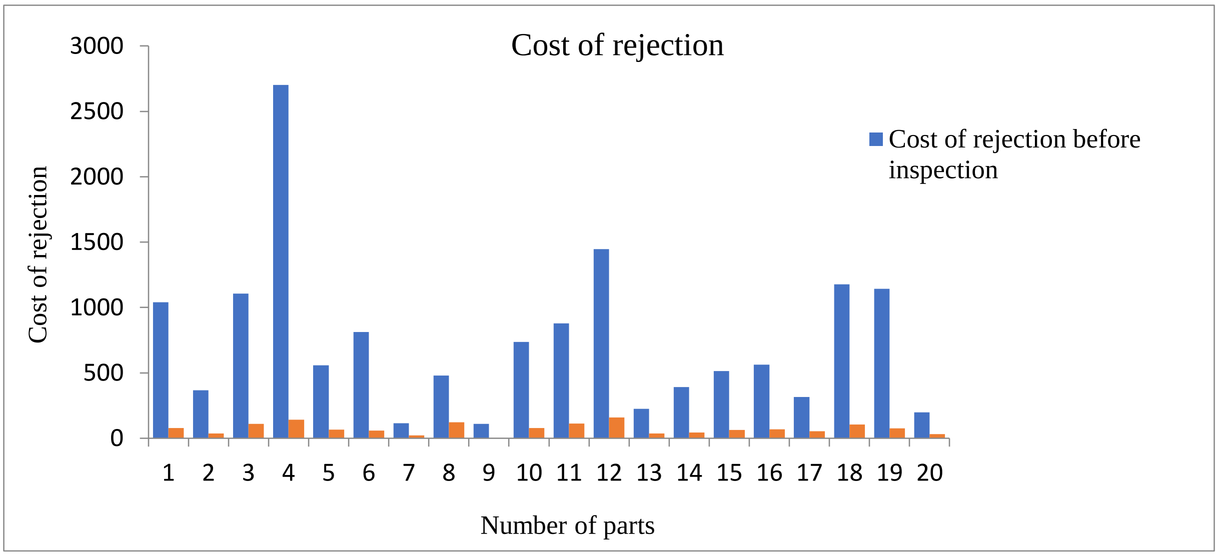

Fig. 3. Cost of rejection before and after inspection

By comparing the model presented in this paper and the model presented by Qin et al. [9] and the results of these models, it can be concluded that the lot rejection cost means the cost of rejecting and replacing defective items with healthy items and rectifying inspection has greatly reduced costs. In general, there are three different states for the objective function of the inspection plan in the system:

First state: The objective function for section *i* is strictly increasing, meaning that the cost of inspecting one is more than the expected cost of non-conforming items, so inspecting the items is not optimal.

Second state: The objective function is strictly decreasing. This means that the cost of inspecting all items is less than the expected cost of non-conforming items. In this case, the 100% inspection is optimal.

Third state: The objective function is convex. This means that there is a specific sample size that will minimize the total cost of sampling plan. In the third case, the model results in a sampling plan with a given sample size as the solution.

Conclusion

The purpose of this research is to find the most economical solution using a mathematical model to determine the optimal sample size in order to control the quality of the products in the input lot. In order to achieve this goal, the model should determine an optimal balance between the cost of the inspection, the cost of non-conforming items entering the assembly line and the cost of the lot rejection.

The model of this research is an integer non-linear programming model for designing a single sampling plan with a zero-acceptance number in order to inspect the items entering the assembly line. The proposed model is able to provide an optimal solution for the problem.

In this research, a practical example with 20 sections is presented to illustrate the application of the model. The input parameters were given to the model and the optimal sample size was determined. In the numerical example, it can be seen that the rate of non-conforming items, the cost of non-conforming items and the cost of rejecting the lot, and the total cost of the quality control, has decreased after implementing the proposed method, and the use of the model by companies and manufacturers reduces costs and thus increases the profits.

The model presented in this study is based on a single sampling plan. For future research, double or other sampling schemes can be used to compare with a single sampling plan. Also, in the proposed model, the acceptance number is zero; in future models, other acceptance numbers can be analyzed. In addition, it is possible to consider the parameters as fuzzy instead of constant and compare the results with the results. The objective of the proposed model is to reduce the cost of quality control, thus a model with the aim of reducing the average number of inspections can be designed.

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