Multi-Objective Two-Sided Robotic Mixed-Model Assembly Line Balancing Problem Considering Energy Consumption and Smoothing Workload

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Abstract

In this paper, a type II robotic mixed-model two-sided assembly line balancing problem is considered. This paper presents a new mixed-integer programming model for type II robotic mixed-model two-sided assembly line balancing to minimize the cycle time, energy consumption, and purchased cost of robots for a given number of workstations. We provided an effective framework for optimizing the multi-objective robotic mixed-model two-sided assembly line balancing problem considering energy consumption and smoothing workload in the make to order environment to help the decision-makers make the right decisions under stochastic demand. An augmented epsilon constraint and Lp-metric methods were applied to solve the problem, and then, with the help of defining two vertical and horizontal criteria, we attempt to help the decision-maker to choose a more efficient solution to make the production line more smooth workload. We demonstrate the efficiency of the proposed method by designing the numerical experiments.

Keywords:
Balancing; Energy Consumption; Robotic Mixed-Model Assembly Line; Smoothing Workload; Two-Sided Assembly Line

Introduction

In 1913, Henry Ford transformed production systems and reduced production costs by introducing a system designed as a moving belt called assembly line. According to the literature, the goal of line balancing is to reduce the expectations due to the unequal amount of production time at work centres and to produce the amount of production time in all processes [1].

The most fundamental problem in this area is the balancing of a high-volume assembly line for a product called simple assembly line balancing (SALB) problem. Salveson [2] formulated the first model for the SALB problem. In these lines, the assembly of each task, such as \( a_i \), is allocated to a workstation, and the processing time it takes, \( P_{a_i} \), is matched to the working time of that station. The station with the longest accumulation time is known as the bottleneck and the associated time with that is called cycle time. Given the current market conditions of consumers and the decline in the product lifecycle, flexible manufacturing systems are needed to produce products with different characteristics in a single line. Assembly lines manufacturing products with different characteristics in a line are called mixed-model assembly lines (MMALs) [3].

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Nowadays, energy is expensive and electricity is one of the essential types of energy in these high-emissivity production lines. Because of the use of electrical energy, carbon dioxide emissions in the world have increased by 20%; therefore, factories need to focus on energy consumption and deal with more friendly environments [4]. Fysikopoulos et al. [5] showed that during the production process in an automobile company, about 9% through 12% of the total cost of production is included in the cost of energy consumption, and by only 20% reduce of this cost, from about 2 to 2.4% of production costs is reduced, and thus reducing this cost will lead to better performance of the industry than competitors. Therefore, in advanced industries, a different category of the assembly line is used that known as robotic assembly lines (RAL). In RAL, instead of using a workforce, a robot is applied to do assigned tasks that increase productivity product quality, flexibility, safety, as well as reducing the need for skilled labour. Another critical benefit of RAL is to work without fatigue and 24-hour weekly assembly lines [6]. In the robotic assembly line balancing (RALB) problem, works and robots should be assigned to the stations, and the robot is responsible for the dedicated works of that station [7]. In this case, robots are generally heterogeneous; that is, the time which is required to complete the work depends on the features and type of robot. Generally, The RALB problem is divided into four categories. The first category of them, RALB-I, with the given time cycle, seeks to decrease the number of stations. RALB-II seeks to decrease the cycle time by knowing the number of stations. In the problem of the E-type robotic assembly lines (RALB-E) balancing, the cycle time and the number of stations are not specified, and in type-F (RALB-F) both are determined [8].

In general, Make to stock (MTS) and Make to order (MTO) are two categories of MMALs in manufacturing environments. In MTS environments, goods are manufactured before the arrival of consumer demand, while in MTO systems, customer orders are initially received and then it started to be produced. The robotic mixed-model assembly line balancing (RMMALB) problem want to increase productivity by increasing flexibility and increasing the speed of response to demand changes for different product models. One of the features considered in this research is the MTO environment. In this manufacturing system, product features are selected by the customer based on available designs and the nature of the product demand is stochastic. The MTO system has higher flexibility than traditional systems and is more common for expensive products such as the automobile industry. Customer satisfaction in this system increases due to satisfying his/her demand. The goal of this system is to maximise performance.

As aforementioned, the MMALs produces various types of products in a single line. Yagmahan [9] investigated an MMALB-I problem to optimization of the number of stations, line productivity and linearity. Akpinar and Bayhan [10] presented the MMALB-I problem to optimization of the number of stations, the task-connection and the task-related communication in the stations. Liao [11] minimized the variance of station time and total cycle time by studying the MMALB problem. They addressed a zoning constraint that positive zoning is the case where two activities must be assigned to a station, and negative zoning prevents the allocation of two incompatible activities to a station. Pereira et al. [12] studied the RALB problem using a cost-based mathematical model. Kucukkoc et al. [13] formulated the lexicographic bottleneck MMALB problem to adjust the workload distribution of the workstations. Samouei and Ashayeri [14] studied the MMALB problem in a semi-automatic environment so that costs and cycle time are improved. They provided a robust solution for managers to achieve reliable results. Yang and Cheng [15] formulated a model for optimization of the two-sided MMALB problem considering the forward and backward setups. In the study of Zhang and Xu [16], a model has been developed to optimization of the U-shaped assembly line balancing (ALB) problem under energy concerns. The formulated model studies the objectives of workload uniformity between stations and the energy aspect. Chutima and Jirachai [17] examined the parallel U-shaped MMALB problem by considering conflicting objectives including task
unrelatedness, various workloads between stations, number of stations and workstations. Their proposed approach to solve the model enables decision-makers to balance decisions. Li et al. [18] formulated the MMALB-I problem to optimization of the number of stations and mated-stations and solved it by local search approaches. Li et al. [19] provide the exact and heuristic approaches to deal with the ALB-II problem. Liu et al. [20] formulated the MMALB problem by optimizing the complexity of the station, the difference in workload within the station, and the efficiency in an uncertain environment.

The first problem of the RALB was reviewed by Rubinnovitz [21]. Aghajani et al. [22] investigated the problem of two-sided RALB-II. In this assembly line, right, left, or both sides of the production line are used, simultaneously. Rabbani et al. [8] formalised a problem of the U-shape MMALB-II, with three objective functions, minimizing cycle time, cost of purchasing a robot, and the cost of setup time for a robot. Çil et al. [23] developed a heuristic method for optimization of the RMMALB problem with the aim of minimizing the total cycle time. Janardhanan et al. [24] studied the RALB-II problem to optimization of the cycle time so that the setup time depends on the sequence of tasks. Li et al. [25] proposed two new mathematical models to optimize cycle time in a U-shaped RALB problem. Li et al. [26] provided the two-sided RALB problem by presenting an integer programming model and considering the start-up times to improve the cycle time. Çil et al. [27] investigated the MMALB problem in a semi-automatic system to minimizing the total cycle times. Li et al. [28] studied the RALB issue to minimizing buying cost of robots and cycle time. Rabbani et al. [29] optimized the aims of minimizing the number of mated-station and the cost of employing various agents in a human-robot four-sided MMALB problem. Sun and Wang [30] focused on the RALB problem and minimized the cycle time by considering the precedence constraint and applied the branch-and-bound approach to optimize the model. Li et al. [31] formulated the problem of RALB to improve the cycle time and the cost of purchasing under different purchase costs.

Moreover, it seems the literature on the energy consumption sector in robots is limited. Nilakantan et al. [32] investigated a simultaneous minimisation of the cycle time and power usage in a SALB. Zhang et al. [33] optimized the objectives of cycle time and power usage in the U-shaped RALB problem. Zhang et al. [34] presented the U-shaped RALB problem to minimizing the energy consumption, noise of robots and cycle time. Sun et al. [35] studied the RALB problem to minimize energy consumption and cycle time. Zhou and Wu [36] formulated the RALB problem to optimize energy usage and productivity. In this study, the efficiency and power consumption rate by robots are considered variable. Zhang et al. [37] developed the U-shaped MMALB and sequencing problem to optimization the objectives of energy consumption and makespan.

In the MTO production system, the nature of the product demand is stochastic. Bukchin et al. [38] investigated an MMALB-II problem and used the Bottleneck criteria for performance evaluation of the assembly line in the MTO environment. Manavizadeh et al. [39] provided a mixed-integer programming assembly line balancing, which uses vertical and horizontal balancing measures to evaluate performance. They simultaneously minimized the number of workstations and time cycles by a multi-objective evolutionary algorithm in an MTO and stochastic environment. Some criteria for vertical and horizontal balancing are mentioned by [40]. We also use the same criteria for our problem. Emde et al. [40] considered vertical balancing measures as criteria for assessing the equal distribution of workloads between workstations, and horizontal balancing measures are also within each workstation. A summary of the previous studies is provided in Table 1.
Table 1. The comparison table of literature review

<table>
<thead>
<tr>
<th>Article</th>
<th>Model</th>
<th>Objective functions</th>
<th>Shape</th>
<th>Solution procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cycle time</td>
<td>Energy consumption</td>
<td>Robot purchasing costs</td>
</tr>
<tr>
<td>[8]</td>
<td>II</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<tr>
<td>[22]</td>
<td>II</td>
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<td>II</td>
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<tr>
<td>[25]</td>
<td>II</td>
<td>*</td>
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<td>*</td>
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<tr>
<td>[27]</td>
<td>II</td>
<td>*</td>
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<td>*</td>
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<td>[28]</td>
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<tr>
<td>[36]</td>
<td>II</td>
<td>*</td>
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<td>*</td>
</tr>
<tr>
<td>This study</td>
<td>II</td>
<td>*</td>
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<td>*</td>
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</tbody>
</table>

We attempt to improve the environmental situation, workload smoothing and reduce production costs considering the robots energy consumption. Nowadays, sustainable development has received a great deal of attention so that a significant part of it is energy efficiency in the manufacturing industry [41]. According to statistics, one of the key users of energy is the manufacturing industry, and most of the energy is mainly consumed by robots used in the production sector [42]. In the automotive industry, the electrical energy used by robots in the production sector averages about 8% of the total energy consumed in the product life cycle [41]. Since robots are used as the main component in almost all automated production processes, decreasing the power consumption of robots has become the key effort in the improvement of green production environments. Reducing the robots’ energy consumption automatically reduces operating costs and CO₂ emissions. In fact, according to the current policy on CO₂ emissions and increase in energy costs, decreasing energy consumption in robots is important so that reducing it leads to improving the efficiency of production systems [42]. Robotic power consumption has become a key goal for many researchers and robot manufacturers, with several researchers defining tools for measuring and analyzing robot energy consumption and optimizing energy consumption to improve the economic aspect and minimize environmental impacts have worked [43].

Moreover, considering that in the production environment MTO, customer demand is randomized, so in order to analyse and assist by the decision-maker with the aim of increasing the workload smoothing, we define the two vertical and horizontal criteria of the balance, so that the decision-maker has a better choice.

Therefore, the contributions of the current study are as follows: First, we proposed a new mixed-integer nonlinear programming (MINLP) model for the two-sided RMMALB-II problem. The proposed model is formulated by considering the energy consumption and smoothing workload so that it leads to sustainable development, improvement of green production and reducing production costs. Second, the proposed model includes objectives of
minimizing cycle time, energy consumption, and the purchased cost of robots for a given number of workstations and has been studied in an MTO environment. According to the best of our knowledge, simultaneously addressing the optimization of the mentioned objective functions in the two-sided RMMALB problem is the first study to be considered. Third, the stochastic nature of the MTO environment is controlled by both horizontal and vertical criteria, not only provide better decision-making for the decision-maker, but also increase the speed of response to changes in demand and flexibility.

The different parts of the research are structured as follows: explanation of the problem and mathematical modelling addressed in Section 2. In Section 3, the procedure solution will be discussed, and in Section 4, the numerical example will be mentioned. Finally, conclusion remarks are explained in the last section.

**Problem description and mathematical modelling**

In this section, we introduce our proposed problem in addition to the definition of decision variables, parameters, sets and mathematical modelling.

**Problem Definition**

In this study, we investigate a balancing problem for an RMMAL. In the problem, only robots are utilized in order to carry out activities, and assembly of several different types of goods are performed in a single line. The type-II assembly line is considered in which we tend to minimise the operating cycle time, assuming the number of stations is known. Due to the supreme energy consumption of these assembly lines and the creation of environmental pollution, the objective function is to reduce energy consumption in these lines.

Furthermore, the production environment is MTO. Given that in this kind of manufacturing environment, demand has a random nature, it must be planned in such a way that changes in demand does not increase the time of idle or overtime in workstations. In order to take into account these conditions, two balance measures are considered, namely, vertical and horizontal. Emde et al. [40] showed that a function that reduces the time difference between the real-time model process and cycle time with Manhattan’s distance is suitable for these conditions and hence this type of target function is considered. The shape of the assembly line is intended to be two-sided (Fig. 1), which is suitable for assembling expensive and heavy goods. Before we go into problem modelling, we define the assumptions as follows:

- All models of the product can be assembled in one production line.
- Each product has a precedence diagram that integrates these diagrams and generates a single precedence graph.
- Robots are capable of doing all works and there is no limit to allocating works and robots to stations.
- The time it takes for each activity to be done by means of the specific type of robot assigned to that workstation is constant.
- Each activity can be assigned to only one station and breaking activities is not allowed.
- The energy consumption rate of each robot is given.
- The assembly line is two-sided.
**Notation**

The following characteristics are applied to the formulation of the problem:

<table>
<thead>
<tr>
<th>Indices:</th>
<th>Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>w, d, e, l</td>
<td>Work</td>
</tr>
<tr>
<td>v, s</td>
<td>Mated station</td>
</tr>
<tr>
<td>n</td>
<td>Model of item</td>
</tr>
<tr>
<td>q, z</td>
<td>The line side; 1 shows left-side and 2 shows the right-side</td>
</tr>
<tr>
<td>(v, q)</td>
<td>v is station of mated station and q is its direction of operation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Set of work in combined precedence chart; w=1,…,W</td>
</tr>
<tr>
<td>V</td>
<td>Set of the mated station; v=1,…,V</td>
</tr>
<tr>
<td>N</td>
<td>Set of item model; n=1,…,N</td>
</tr>
<tr>
<td>R</td>
<td>Set of the types of robot; r=1,…,R</td>
</tr>
<tr>
<td>A_c</td>
<td>Set of completed works at a left-side</td>
</tr>
<tr>
<td>A_h</td>
<td>Set of completed works at a right-side</td>
</tr>
<tr>
<td>A_p</td>
<td>Set of completed works at an either side of station</td>
</tr>
<tr>
<td>P(w)</td>
<td>Set of immediate predecessors of work w</td>
</tr>
<tr>
<td>P_p(w)</td>
<td>Set of all predecessors of work w</td>
</tr>
<tr>
<td>S(w)</td>
<td>Set of immediate successors of work w</td>
</tr>
<tr>
<td>S_s(w)</td>
<td>Set of all successors of work w</td>
</tr>
<tr>
<td>F_0</td>
<td>Set of works that have no immediate predecessors</td>
</tr>
<tr>
<td>t_{w,n,r}</td>
<td>The amount of time required to process work w for model n using robot</td>
</tr>
<tr>
<td>ω</td>
<td>A big amount</td>
</tr>
<tr>
<td>C(w)</td>
<td>Set of works whose directions of operation are opposite to direction of operation of work w</td>
</tr>
<tr>
<td>K(w)</td>
<td>Set of indicating the preferring direction of operation of work w</td>
</tr>
<tr>
<td>S_{w,n,r}</td>
<td>Setup time of robot r for processing work w of model n</td>
</tr>
<tr>
<td>PZ</td>
<td>Set of pairs of compatible works for positive zoning</td>
</tr>
<tr>
<td>NZ</td>
<td>Set of pairs of compatible works for negative zoning</td>
</tr>
<tr>
<td>α_n</td>
<td>The proportion of the demand model n from the overall demand</td>
</tr>
<tr>
<td>P_C_r</td>
<td>The energy consumed by the robot r during operation</td>
</tr>
<tr>
<td>S_P_C_r</td>
<td>The energy consumed by the robot r during standby</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables:</th>
<th></th>
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</table>

**Fig. 1.** Robotic two-sided assembly line
The cycle time of model \( C_n \):

If work \( w \) allocated to the station \( (v,q); 1 \) and otherwise; \( 0 \)

\( X_{wvq} \):

If robot \( r \) allocated to the station \( (v,q); 1 \) and otherwise; \( 0 \)

\( Y_{rvq} \):

Completion time of work \( w \) for model \( n \)

\( Z_{we} \):

If work \( w \) allocated before work \( e \) in the equal station; \( 1 \) and vice versa; \( 0 \)

Mathematical modelling

We formulated a new MINLP model for the RMMALB problem considering energy consumption and the MTO environment. Optimization of the cycle time, energy consumption and purchased cost of the robot are the goals of the problem.

\[
\min \sum_{n \in N} C_n \quad (1)
\]

\[
\min \sum_{r \in R \forall v \in V} \left( \sum_{q \in Q(w) \forall w \in W} \left( \sum_{r \in R \forall n \in N} \sum_{w \in W} (PC_r t_{wnr} X_{wvq} Y_{rvq}) \right) + \left( \sum_{r \in R \forall n \in N} \sum_{w \in W} (SPC_r Y_{rvq} \cdot \left( \sum_{n \in N} C_n - \sum_{r \in R \forall n \in N} \sum_{w \in W} t_{wnr} X_{wvq} Y_{rvq} \right) \right) \right) \quad (2)
\]

\[
\min \sum_{r \in R \forall v \in V} \sum_{q \in Q} C_r Y_{rvq} \quad (3)
\]

Subject to:

\[
\sum_{v \in V} X_{wvq} = 1 \quad \forall w \in W \quad (4)
\]

\[
\sum_{s \in S} s \cdot X_{dsq} - \sum_{v \in V} \sum_{q \in Q(w) \forall w \in W} v \cdot X_{wvq} \leq 0 \quad \forall w \in W - F_0; \forall d \in F_0 \quad (5)
\]

\[
\forall v \in V; \forall n \in N \quad \forall w \in W - F_0; \forall d \in F_0; \forall v \in V; \forall n \in N \quad (6)
\]

\[
t_{wn} - t_{dn} + \omega \left( 1 - \sum_{q \in Q(w) \forall w \in W} X_{dsq} \right) + \omega \left( 1 - \sum_{q \in Q(w) \forall w \in W} X_{wvq} \right) \geq \sum_{r \in R} t_{wnr} Y_{rvq} \quad (7)
\]

\[
t_{en} - t_{wn} + \omega (1 - X_{ewq}) + \omega (1 - X_{wvq}) + \omega (1 - Z_{we}) \geq \sum_{r \in R} (t_{ern} + S_{ern}) Y_{rvq} \quad (8)
\]

\[
t_{wn} - t_{en} + \omega (1 - X_{ewq}) + \omega (1 - X_{wvq}) + \omega (Z_{we}) \geq \sum_{r \in R} (t_{wnr} + S_{wnr}) Y_{rvq} \quad (9)
\]

\[
t_{wn} \geq t_{wn} \quad \forall w \in W; \forall n \in N \quad (10)
\]
Eq. 1 is formulated to minimise the cycle time. Eq. 2 minimises the amount of power consumed by the robots and consists of two parts. In the first component, the amount of energy consumed when the robot is engaged in the operation and in the second component calculates the amount of energy consumption when the robot is ready to work. Eq. 3 minimises the cost of purchasing a robot and using it.

Constraint (4) confirms that each activity is allocated to only one station. Constraint (5) controls the pre-requisite constraint for all activities. When each pair of activities, such as \( w \) and \( d \), is allocated to a workstation, the constraint (5) is activated and checks these predecessors. Constraint (6) guarantees that the completion time should be lower than the cycle time. Constraint (7) establishes a prerequisite for each pair of activities assigned to a station, and the activity \( w \) can only begin when activity \( d \) has been completed. Constraints (8) and (9) for each pair of activities assigned to a station are activated and takes into account the preparation time between activities. Constraint (10) ensures that the activity of the model \( m \) is greater than or equal to the completion time of the same activity. Constraint (11) indicates that no more than one robot is allocated to any station. Constraint (12) confirms that each robot is assigned a maximum of one workstation.

As previously mentioned, the MMALs produces various types of products in a single line, and thus the MTO production environment is considered. In an MTO environment, quick response to customer demand is an important consideration. Emde et al. [40] showed that the solution in this production environment is appropriate to minimise the difference between the real processing time of the model and the working cycle time regarding Manhattan distance. Therefore, in the current study, we rewrite the two indices introduced by them and provide the decision-maker as a benchmark for comparing different solutions. Eqs. 13 and 14 indicate the vertical and horizontal measures, respectively. \( T_v^* \) is average processing time at station \( v \) and \( \bar{T} \) is the average time of station supposing works can be distributed at will, so that the values of \( T_v^* \) and \( \bar{T} \) are obtained from Eqs. 15 and 16. The vertical measure is defined as criteria for measuring the equal distribution of workloads between workstations, and horizontal measure is as a criterion for evaluating the equal distribution of workloads within each workstation.

\[
\sum_{v \in V} \left| T_v^* - \bar{T} \right| = \sum_{v \in V} \left| \sum_{n \in N} \sum_{w \in W} \sum_{q \in Q} \sum_{r \in R} t_{wnr} x_{wvq} y_{rvq} - c_n \right| (13)
\]

\[
\sum_{v \in V} \sum_{n \in N} \alpha_n \left| \sum_{w \in W} \sum_{q \in Q} \sum_{r \in R} t_{wnr} x_{wvq} y_{rvq} - c_n \right| (14)
\]

\[
T_v^* = \sum_{w \in W} \sum_{n \in N} \sum_{q \in Q} \sum_{r \in R} \alpha_n t_{wnr} x_{wvq} y_{rvq} (15)
\]

\[
\bar{T} = \frac{1}{\|V\|} \sum_{v \in V} T_v^* (16)
\]

Augmented Epsilon Constraint for RMMALB

In the Augmented Epsilon Constraint (AEC) method rather than having a distinctive objective function, only one of the goals in the single-objective problem is optimized and the rest of the objectives are considered as constraints. The single-objective model for the \( j^{th} \) objective function is the model (17):
\[
\min f_j(x) \\
\text{s.t. } f_k(x) \leq \varepsilon_k \; \forall k = 1, ..., P \text{ and } k \neq j \\
x \in X
\]  
(17)

\(\varepsilon\) is an epsilon vector containing the elements \(\varepsilon_j\), which is not included in the model. Now, with systematic change, the vector \(\varepsilon\) of various efficient solutions can be produced. The AEC method is able to generate non-dominated solutions that conjugate composition is covered by other non-recessive solutions.

Regardless of the multiple advantages of the AEC method than weighted sum, the special attention to two points increases the efficiency of the algorithm: one, determining the range of variations of each of the objective functions on a set of efficient solutions; and two, creating assurance in the production of efficient solutions in each repetition of the AEC method. Since the optimal solution for each single-objective \(\varepsilon\)-constraints model, with the condition that the limit constraints of \(f_k(x) \leq \varepsilon_k\) are active at that point, an answer is an efficient solution, therefore, to ensure the generation of efficient solutions from each of the one-objective \(\varepsilon\)-constraints models, we use the augmented \(\varepsilon\)-constraints single-objective model (18). Thus, we generate a payoff matrix with a lexicographic approach and then determine the \(\varepsilon\) value by dividing the feasible solution range of each objective obtained from the payoff matrix. Finally, we optimize the selected objective by generating the Pareto frontier.

\[
\min f_j(x) - \delta \sum_{k \neq j} s_k \\
\text{s.t. } f_k(x) + s_k = \varepsilon_k \; \forall k = 1, ..., P \text{ and } k \neq j \\
x \in X; s_k \in R^+
\]  
(18)

The value of \(\delta\) is a small value fluctuated between \(10^{-6}\) and \(10^{-3}\), and usually, the first objective is used as the objective function of the AEC method. The AEC model helps to select, if there are multiple optimal solutions for the primary probabilistic problem, to find the difference between the values of other target goals and the actual answer (\(s_k\) are positive).

**Numerical examples**

In this section, we provide a number of numerical examples to test the performance of the proposed model, and some of their data is provided in Tables 2 and 3. In MMALB, for each product model, there is a prior and farther diagram that indicates the method of balancing or balancing the line and is relatively complex. There are two ways to overcome this problem:

- Combining all precedence and latency charts and creating a new chart
- Matching the process time of tasks

Most researchers used a hybrid primitive approach for their research [39]. In the other approaches, the mean of the average working process-time is used, which is considered for each model of the repeated product. In this research, the first approach has been used to determine the priority and descending graph.

For example, in test problem 2, we have two item models A and B, which combine the precedence diagrams of the preceding and the late-graphs in Fig. 2. In Fig. 2, a number of tasks are related only to model A, a number of to model B and a number of to both types of models, which are shown respectively with (A), (B) and (A, B).
As aforementioned, in the two-sided assembly lines, several activities should be completed left, several right and several on both sides of the lines. The number of tasks in test problems 1, 2, and 3 are equal to 7, 10, and 20, respectively. Table 2, contains the required data for test problem 2. The processing time of the activities was generated as a random number. Table 3, shows the robot’s features used in the numerical examples.

**Fig. 2.** The precedence diagram and model of the 9-work problem for test problem 2

**Table 2.** Data for test problem 2

<table>
<thead>
<tr>
<th>Task</th>
<th>Immediate Precedence(s)</th>
<th>Immediate Successor(s)</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>4</td>
<td>R</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>6</td>
<td>E</td>
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<tr>
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<td>6</td>
<td>R</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>7</td>
<td>L</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>7,8</td>
<td>R</td>
</tr>
<tr>
<td>6</td>
<td>2,3</td>
<td>9</td>
<td>E</td>
</tr>
<tr>
<td>7</td>
<td>4,5</td>
<td>10</td>
<td>E</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>10</td>
<td>E</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>10</td>
<td>L</td>
</tr>
<tr>
<td>10</td>
<td>7,8,9</td>
<td>-</td>
<td>L</td>
</tr>
</tbody>
</table>

**Table 3.** Data for robot

<table>
<thead>
<tr>
<th>Robot</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.5</td>
<td>0.55</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.65</td>
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<td>0.7</td>
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<td>0.75</td>
<td>0.78</td>
<td>0.78</td>
<td>0.8</td>
</tr>
<tr>
<td>Power</td>
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<td>0.1</td>
<td>0.11</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.125</td>
<td>0.125</td>
<td>0.13</td>
<td>0.132</td>
<td>0.135</td>
<td>0.14</td>
<td>0.142</td>
<td>0.142</td>
</tr>
</tbody>
</table>

Test problem 1: 5 robots,
Test problem 2: 8 robots,
Test problem 3: 14 robots
The problem is solved as a single objective, and the pay-off table for various objective functions for test problem 2 is shown in Table 4.

| Table 4. Pay-off table of three objective functions by AEC method for test problem 2 |
|---------------------------------|-----------------|-----------------|-----------------|
| Objective function              | Z₁ (Cycle time) | Z₂ (Energy consumption) | Z₃ (Purchase cost of the robot) |
| min Z₁ (Cycle time)            | 150.1*          | 377.19            | 1220            |
| min Z₂ (energy consumption)    | 162.8           | 304.17*           | 1220            |
| min Z₃ (Purchase cost of the robot) | 157.2          | 510.8             | 780*            |

*Optimal value function of each objective function

The problem is coded with the AEC method in the GAMS 24.8.5 software and efficient solutions are shown in Table 5. As previously stated, the model presented in part 2 is a MINLP that is used to codify nonlinear relationships from the linear transformation to ensure that the model provides global optimal solutions. All of the results are presented in Table 5.

| Table 5. Efficient solution by AEC method |
|---------------------------------|-----------------|-----------------|-----------------|
| NO.                            | Z₁              | Z₂              | Z₃              |
| Test problem 1                |                 |                 |                 |
| 1                              | 129.8           | 238.4           | 970             |
| 2                              | 112.9           | 241.8           | 970             |
| 3                              | 109.2           | 253.1           | 970             |
| 4                              | 121.5           | 250.99          | 920             |
| 5                              | 134.7           | 247.2           | 920             |
| 6                              | 117.12          | 250.83          | 900             |
| 7                              | 120.01          | 244.39          | 900             |
| 8                              | 108.4           | 251             | 900             |
| 9                              | 105.5           | 259.8           | 900             |
| 10                             | 131.9           | 233.67          | 880             |
| 11                             | 110.8           | 280.91          | 880             |
| 12                             | 98.11           | 283.18          | 880             |
| 13                             | 112.96          | 267.2           | 870             |
| 14                             | 103.2           | 274.5           | 870             |
| 15                             | 124.78          | 270.9           | 850             |
| 16                             | 147.2           | 261.12          | 850             |
| 17                             | 138.1           | 283.7           | 830             |
| 18                             | 102.97          | 289.11          | 830             |
| 19                             | 120.6           | 272.45          | 800             |
| 20                             | 123.08          | 270.16          | 800             |
| 21                             | 98.4            | 291.77          | 780             |
| 22                             | 137.19          | 281.2           | 780             |
| 23                             | 128.6           | 298.5           | 780             |
| 24                             | 121.78          | 302.33          | 780             |
| 25                             | 150.8           | 300.4           | 780             |
| 26                             | 127.51          | 311.5           | 670             |
| 27                             | 115.7           | 314.2           | 670             |
| 28                             | 158.5           | 309.8           | 670             |

The results are presented in Table 5.
As we have already said, in multi-objective problems, it is often not possible to optimise all the goals for an answer vector simultaneously but to have some unique optimal solutions. We will deal with Pareto optimal solutions. We will analyse the results of the model here. First, with the help of the AEC method, we produced efficient solutions, and these points are presented in Table 5, and the Pareto front of test problem 2 shown in Fig. 3, which, according to the definition of any of these points, has no advantage over the other.

![Efficient solution of test problem 2 by AEC method](image)

Figs. 4-6 were drawn based on numerical example results to validate the proposed approach to solving the studied problem. These are comparative forms of the Pareto Front results for solving multi-objective problems. Conflict of these objectives with each other shows that the proposed approach works well and is valid.
Fig. 4. Comparison of the results of the objectives $Z_1$ and $Z_2$

Fig. 5. Comparison of the results of the objectives $Z_1$ and $Z_3$
As mentioned, the Pareto Front results are plotted in Figs. 4-6 to examine the conflict between objectives. Fig. 4 shows the conflict between the first and second objectives. Decreasing the Pareto front values of the first objective function (i.e. cycle time) leads to increasing the Pareto front values in the second objective function. In fact, the use of more robots reduces cycle time while increasing energy consumption. This creates a logical conflict between objectives of cycle time and energy consumption, which is well illustrated in Fig. 4. Moreover, Fig. 5 shows the conflict between the first and third objectives. When fewer robots are assigned to the stations, their completion time will increase and leads to an increase in cycle time. Obviously, allocating fewer robots also leads to a downward trend in the objective of the robots' purchase cost (as shown in Fig. 5). According to Fig. 6, increasing the values of the second objective function (i.e. energy consumption) will lead to reducing the values of the third objective function (i.e. cost of purchasing robots). It is clear that robots that are more efficient in terms of energy consumption will need more cost and investment. Therefore, the cost of purchasing robots and their energy consumption are in conflict with each other.

In multi-objective problems, the validation of the proposed model is studied by examining the results of the Pareto front and their logical conflict. Moreover, solving the proposed model using another valid solution method and comparing its results with the main solution approach is another way to checking the validity of the model and its results. In this paper, we examined three numerical examples in different sizes and shown the logical conflict of the problem objectives in Figs. 4, 5, and 6. In addition, the proposed model is solved using the Lp-metric method and the results are compared with the AEC method. So to benchmark the proposed model and compare it with another solution method, the present problem is solved by the Lp-metric method for test problem 2. This method minimizes the total power of the relative deviations of the objective functions from their optimal value. Results of the Lp-metric method are reported in Table 6. Pareto solutions are achieved based on different combinations of the importance of each objective function ($W_i$).
Table 6. Efficient solution by Lp-metric method

<table>
<thead>
<tr>
<th>NO.</th>
<th>$Z_1$</th>
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<th>$Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>151.94</td>
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<td>1220</td>
</tr>
<tr>
<td>2</td>
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<td>40</td>
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<td>780</td>
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</table>

The Pareto front results of the AEC and Lp-metric methods are presented in Fig. 7. This figure visually demonstrates the validity of the results of the proposed approaches. As it is clear, the Pareto front resulting from both methods is close to each other, which indicates the validity of the proposed model and its results based on the solution method.
Then for analysis of this problem, one important analysis is considered. According to the data in Table 7, product type prediction values directly affect the decision-maker's choice. It is shown that the best mode for the assembly line balance is a situation where only one product and with this distance, vertical and horizontal measure increase. In the above-mentioned problem, due to the existence of different precedence relations for products, the numbers have not been reduced uniformly.

In this paper, we review two vertical and horizontal measures, as we said before, that these measures give the decision-maker the possibility to compare the Pareto front in terms of smoothing workload. To simplify our analysis of our results, in this numerical example, we will only review 5 efficient solutions obtained by the AEC method. These efficient solutions are described in Table 7. In Figs. 8 and 9, we calculated and plotted the two horizontal and vertical measures for different values of $\alpha_A$ and $\alpha_B = 1 - \alpha_A$.

---

**Table 7. Data of analysis for horizontal and vertical balance measure**

<table>
<thead>
<tr>
<th>No.</th>
<th>Cycle time</th>
<th>Energy consumption of robot</th>
<th>Purchased cost of robot</th>
<th>Horizontal measure</th>
<th>Vertical measure</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha_A$ $\alpha_B$</td>
<td>$\alpha_A$ $\alpha_B$</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
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<td>162.8</td>
<td>364.17</td>
<td>1220</td>
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<td>0.25 0.75</td>
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<td>0.5   0.5</td>
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<td>780</td>
<td>209.58 213.34</td>
<td>145.12 147.77</td>
</tr>
</tbody>
</table>

---

**Fig. 7. Efficient solution by AEC and Lp-metric methods**
The decision-maker can perform two more horizontal and vertical measures for two points in the same way as shown in Figs. 10 and 11 for a closer examination. We calculated and plotted these two graphs for different values of $\alpha_A$ and $\alpha_B = 1 - \alpha_A$.

These shapes allow the decision-maker to check the line status for different quantities of different products of the smoothing workload status line and can help them predict which response can be more efficient if market conditions change. Of course, it should be noted that this analysis can only be used for choices that the decision-maker wishes to choose, and only helps the decision-maker to choose an option to increase the smoothing workload of the product line. Based on the shapes, there can be no specific relationship between the amount of cycle time and the values of horizontal and vertical measures. If the demand for definitive products and values of $\alpha_A$ and $\alpha_B = 1 - \alpha_A$ are fixed, then two horizontal and vertical measures can be considered as two minimization objectives.
Application and implications of research

Due to the development of manufacturing industries and increasing competition in today's environment, paying attention to various internal and external issues of industries will lead to a significant share of profitability. Governments' attention to environmental issues, the pressure of environmentalists, and the importance of sustainable development have led industries to improve their situation. In today's manufacturing industry, the use of robots or a combination of humans and robots in production lines has expanded due to their benefits. The use of robots play a significant role in energy consumption and efficiency, and since robots are used as the main component in almost all automated production processes, decreasing the power consumption of robots has become the key effort in the improvement of green production environments. Reducing the robots' energy consumption automatically reduces operating costs.
and CO₂ emissions. With this in mind, the level of energy consumption has been optimized in this study. Moreover, the use of robots affects various aspects of production, especially cycle time. Achieving optimal cycle time was another goal of this study, which includes benefits such as desirable lead time and higher productivity levels. Apart from this, the purchase of robots, regardless of their cost, requires a high cost and investment, and it is necessary to consider the purchase cost as other objectives. Thus, this study helps industries to achieve an appropriate level of energy consumption and cycle time in their production lines with the desired investment. Therefore, they will benefit from a wide range of economic and environmental benefits. Moreover, the move of manufacturing industries towards the use of robots that consume less energy will also require robot manufacturers to produce higher-efficiency robots according to the needs of the industry. In this way, environmental issue improvement also occurs indirectly in the external chain. Another issue raised in the research was to pay attention to the random nature of demand by presenting two criteria, horizontal and vertical. The stochastic nature of the MTO environment is controlled by both horizontal and vertical criteria, not only provide better decision-making for the decision-maker, but also increase the speed of response to changes in demand and flexibility.

Conclusions

We investigated the RMMALB problem considering the power usage of robots in the MTO environment. Considering the increasing pollution at the global level and the necessity of reducing energy consumption while paying attention to the competitive environment of production and reducing production costs, in this paper, minimization of cycle time, energy consumption and cost of purchasing a robot is considered that with the approaches of multi-objective mathematical programming, AEC method, modelling and solving a problem, and a numerical example for the problem was investigated. Taking into account the production environment of the manufacturing, MTO, we are trying to increase productivity by increasing the flexibility and increasing the speed of response to changes in demand for different models. In order to match the manufacturing environment with these conditions and convenience the decision-maker to select one of the efficient results, we rewrite two measures whose performance has already been examined. The two vertical and horizontal measures are in accordance with the make to order environment.

This paper studied in the framework of the two-sided line balancing problem and future researchers can use other types of layouts including U-shaped and four-sided as a direction to expand the present study. The objectives of the problem are balanced only by exact methods, so the study of them in a real study in higher dimensions by meta-heuristic methods is also suggested. In addition, examining other objectives within the framework of the current proposed model could be a good suggestion for future research. Also, in this study, only the presence of robots in the production line was investigated. Simultaneous study of the relationship between robots and humans in production lines could be another direction for future research.

References


