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RESEARCH PAPER

The Dual-Channel Green Supply Chain Performance Under Government Monitoring: A Game Theoretic Approach

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Abstract

Nowadays, consumers tend to use more green and healthy products. Consumer awareness of environmental issues increases today. Declining natural resources, rising disease rates, and rising global temperatures due to industry-wide pollution have raised public concerns. The supply chain, as an important issue and involved with these problems, has always attracted the attention of researchers. A two-channel supply chain is considered in this study. The supply chain consists of one green producer and two retailers along with a Third-party Logistics Company (TPL). The government is also considered as the leader of the structure. The results show that the strategy adopted by the government has a major effect on other members' decisions of the supply chain and can therefore increase/decrease pollution. On the other hand, cooperating with a TPL company also reduces environmental pollution, despite rising costs. The obtained results emphasize that by growing the value of subsidies allocated to the producer, the government can reduce the amount of pollution in the SC. Moreover, increasing the amount of subsidy will lead to an increase in the degree of product greenness. Also, increasing the degree of the greenness of the product does not always increase the profitability of SC members.

Keywords:
Game Theory;
Pricing;
Government Monitoring;
Green Supply Chain;
Outsourcing activities

Introduction

The resources available in nature are very limited. In recent years, consumer needs have been expanding day by day. The variety of consumer needs has led producers to consume too much of their resources in order to maintain their presence in the market, also increase their production to meet the needs. This reduces both natural resources and causes pollution. In recent years, it has been proven that excessive greenhouse gas emissions such as CO2 are the cause of global warming. Rising global temperatures are a reason for the spread of all kinds of diseases and disasters such as floods. Pollution does not enter the atmosphere only by producers. The transportation sector also has an important role in the pollution rate. The use of old and non-standard means of transportation causes excessive greenhouse gas emissions. Therefore, these two important factors should be studied together in research. Governments and managers are aware of the devastating effects of greenhouse gas emissions as well as the needs of consumers. Therefore, in recent years, corporate executives and managers have been looking for

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appropriate solutions to address this issue. Governments in different countries have developed specific laws and regulations to address these problems, and they have strict control over these issues, forcing producers and consumers to comply with environmental regulations. Examples of these rules are: limiting the consumption of goods, determining the maximum amount allowed for the use of resources by producers, and determining the maximum amount of emission of gases produced by the producer [1-5]. Most of the activity of manufacturing and non-manufacturing companies has caused a great deal of environmental pollution, which has endangered the life of the creatures on earth. On the one hand, the natural concern of human beings for their survival and on the other hand the pressures of various governments and organizations have forced different companies and industries to consider the environmental impacts of their business [6-9]. To this end, managers and planners of various industries are increasingly aware of the flaws in their respective Supply Chains (SC). Increasing economic growth requires continuous improvement of logistics processes [10]. At the beginning of the 21st century, the lack of SC greenness has become an important issue for many businesses and the main challenge for their production and logistics management [11]. In addition to concerns about production processes that were more relevant to producers in the supply chain, the issue of sustainable activities at the logistics stage also attracted the attention of many managers and decision-makers. Therefore, the discussion of sustainable activities for TPL companies has become a challenge [12]. International companies not only seek to improve the service quality and improve their effectiveness but also seek to reduce costs with the use of TPL companies [13]. To be more effective in sustainability, many TPLs are beginning to change their operations and strategies [14]. Hence, TPLs predict that environmental sustainability will soon be the criterion for their selection [15]. Previous studies have shown well that government fiscal policies have a good impact on SC activities and encourage them to pursue sustainable activities [16]. Competitive advantage can also be defined as the set of factors or capabilities that always enable companies to perform better than their competitors[17].

Given the particular importance of logistics activities in the SC, the role of TPL in the SC cannot be ignored. This study examines the effects of TPL companies as well as government financial interventions on how an SC operates and how it affects other members' profit of the SC. By extending previous research and developing a new model, this study assists SC decision-makers in choosing equilibrium strategies, enhancing member profits, and trying to reduce pollution in an SC.

To improve the quality of their products, companies work with suppliers, maintain SC continuity, and reduce company risk and SC management. This partnership aims to expand aspects of sustainability, such as environmental, social and economic sustainability. Given the importance of the SC in business, the focus of academic research on the environment and other sustainability issues in the SC has begun about two decades ago, and studies in the sustainable SC are growing rapidly [18-21].

Literature review

Green supply chain

Fleischmann, Bloemhof-Ruwaard [22], in their study, showed that the production of green products by the producer increases the demand for that product. This makes producers move towards greener products and enjoy the competitive advantages created by the target market. Swami and Shah [23] examined an SC involving a producer and a retailer. In this study, the effect of product greenness on demand increasing is also taken into account. Using game theory and considering Stackelberg competition among members, they came to the conclusion that the optimal rate of greenness by the producer and retailer was equal to the cost rate of green.

Rezaee, Dehghanian [24] designed a GSC in a carbon transact environment. They concluded that there is a positive nonlinear relationship between the budget availability and carbon price for the reconfiguration of SCs and the greening of SCs. Liu, Liu [25] studied the pricing problem in a dual-channel SC for two different products. In this study, game theory was used to determine the best strategy. The results showed that the manufacturer can sell products that are popular with consumers in the direct channel to earn more profit and avoid selling products in indirect channels.

Supply chain and third part logistic company

There has been relatively little research on sustainability strategies in the area of SC and TPL companies. For example, environmental sustainability has rarely been the subject of research related to TPLs in the literature [26, 27]. Lieb and Lieb [28] noted that environmental sustainability activities in the field of TPL companies got little attention. Regarding TPL cooperation in SC, solving this problem according to game theory, some studies have been done in the field of Closed-Looped SC or reverse logistic. In most of these problems, TPL is considered as a product collector or recycler. Maiti and Giri [29], for example, considered a Closed-Loop SC in which the producer sells its product with an appropriate quality to the customer through retail. A TPL company collects products used by customers and transfers them to the producer. The producer then reproduces the product and resells it to customers with the same original quality. In this study, different scenarios such as a decentralized game, decentralized, Nash and Stackelberg games were performed and finally, the best scenario for the optimal situation was introduced. Huang and Wang [30] considered a Closed-Loop SC consisting of one producer, one retailer, and one TPL. Producer, as channel leader, tends to collaborate with retailers and TPL by issuing technology licenses. The ability to reproduce products by SC members and their sustainability were analyzed in this problem. The problem was solved by playing Stackelberg in order to obtain equilibrium strategies for the benefit of all members of the SC. Yan, Xiong [31] examined two models for the sale of reproduced products. In the first model, it is supposed that the producer sells the products to customers through a retailer. For reproducing it is the retailer who is responsible for collecting the products, and through the Internet channel, the re-produced products are sold to customers. In the second model, it is assumed that products are sold to customers through a retailer. But to collect products for reproduction and resale use TPL. Huang, Yang [32] considered a Closed-Loop SC in which there is competition between the retailer and TPL for collecting recyclable products. They analyzed the competition between retailer and TPL with game theory and determined the price of each member. Xiao-hua and Zhen-ning [33] studied the interaction between the selection of recycler and the pricing decisions of a product in a forward channel when competing among retailers was allowed. Jiang, Wang [34] considered decision-making and harmony in an SC involving one producer and two retailers. This third-party logistics provider chain also provides logistics services due to the low cost of services. There is competition between retailers for selling products and working with TPL. This model was examined with the game theory approach.

According to the previous studies and the aim of the current study, the government has key roles in the economics world and the efficiency of members of the supply chain. Hence, most companies are influenced by government laws. Therefore, this study tries to consider this important factor and answer the questions related to this factor. In this research, we try to present answers for the following questions:

1. What is the equilibrium level for the tariff allocated by the government to the producer?

2. What is the pattern of the players' profit functions regarding the tariffs set by the government?
3. How does the tariff set by the government affect the amount of released pollution?

Table 1 demonstrates the specifications of the current study and previous studies.

Table 1. Specifications of the reviewed papers

Author	Green Supply Chain	Third part logistic company	Delivery Time	pricing	Government intervention	Game Theory
Hua, Wang [35]			✓	✓		✓
Sheu and Chen [36]	✓			✓	✓	
Ghosh and Shah [37]	✓			✓		✓
Cao and Zhang [38]	✓			✓		✓
Maiti and Giri [29]		✓		✓		✓
Esmaeili, RASTI [39]				✓		✓
Huang, Wang [40]	✓	✓	✓	✓		
Hafezalkotob and Mahmoudi [41]				✓	✓	✓
Jafari, Hejazi [42]				✓		✓
Huang, Fan [43]		✓				✓
Taleizadeh and Sadeghi [44]				✓		✓
Current Study	✓	✓	✓	✓	✓	✓

This study is prepared in 7 sections. First, in Sections 1 and 2, a summary of the studies conducted in the literature and research gaps were examined. Then, Section 3 will provide general conditions about the current problem and the functions of supply chain members will be defined. In Section 4, the considering model for the government will be presented. And finally, in Sections 5, 6, and 7 a numerical example to investigate the feasibility of the model will be proposed and results and managerial insights will be proposed.

Problem statement

This study considers an SC with a green manufacturer, two retailers and a logistics service provider. The logistics service company transports the product from the manufacturer to the first retailer. In this regard, as stated before, the government monitors the SC by allocating tariffs (tax or subsidy) to the producer. The investigated problem is presented in Fig. 1.

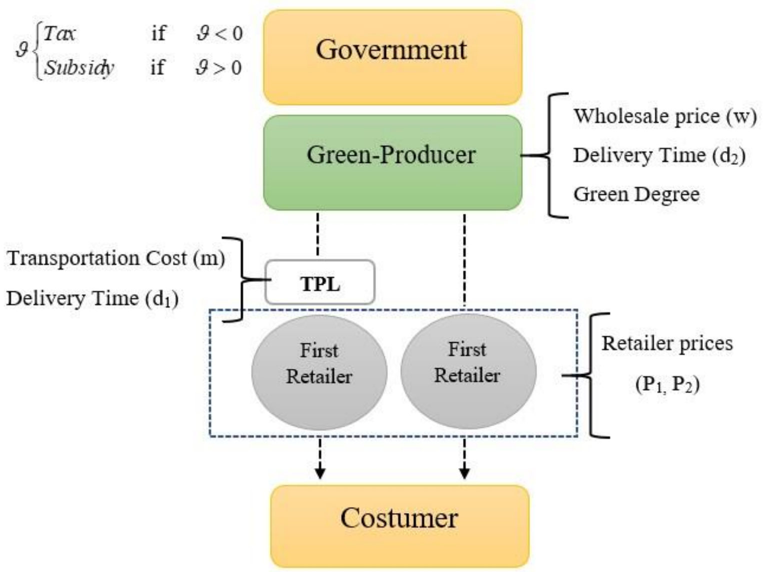

Fig. 1. The General schematic of the problem

The indices, parameters, and variables used in problem modeling are as follows:

Indexes

i	Channel of transportation
r	index of retailers

Parameters

σ_i	the market base in channel *i*
β	the impact of the greenness degree of the product on the demand of producer *i*
γ	The product demand sensitivity coefficient of the product price in other channels
λ_i	Product demand sensitivity coefficient to the degree of the greenness of the product
k	Product demand sensitivity coefficient to delivery time
q	The amount spent by TPL to transport each unit of goods
t	Cost paid by manufacturer for transportation each unit in the second retailer channel
c	Cost of production for each unit of a product by the manufacturer
ξ	The amount of contamination for transportation each unit of product produced in the normal state
L_m	The minimum amount of profit set by the manufacturer to enter the game
L_{TPL}	The minimum amount of profit set by the TPL Company to enter the game
L_j	The minimum amount of profit set by the *j* retailer to enter the game
U_G	The maximum amount of pollution released by the entire SC members

Decision variables

p_j	Product sales price by retailer *j*
α	Degree of the greenness of the product
w_i	Wholesale price through *i* channel
m	Costs received by TPL from the retailer for transportation each unit
d_1	Product delivery time in the first channel
d_2	Product delivery time in the second channel
ϑ	The amount of tariffs allocated by the government to the manufacturer

Dependent Variables

π_j	The profit of retailer *j*
π_m	The amount of manufacturer profit
π_{TPL}	The profit of TPL
y_i	The amount of product demand through channel *i*

Assumptions

The following assumptions are taken to bring modeling closer to the real world and to make the problem possible.

(I) In this study, it is assumed that retailers' prices are higher than the manufacturer's wholesale prices. The wholesale price is also higher than the total cost of the product. These assumptions have been used in previous studies [45, 46].

(II) Basic market demand is large enough and exceeds other model parameters [18, 47].

(III) Changing the price in one channel is more effective than changing the price in other channels on demand in that channel. This hypothesis has been used in previous studies by Chen, Fang [48].

(IV) The impact of the price change coefficient on demand is higher than the degree of the greenness of the product [45, 49].

Problem formulation

In this section, different game structures are analyzed, and a game-theoretic approach is proposed to study each game.

Decentralized Model (DCM)

The profit and demand functions of each SC member are written as Eqs. 1-6.

$$y_1 = \sigma_1 - \beta p_1 + \gamma p_2 + \lambda\alpha + k(d_2 - d_1) \tag{1}$$

$$y_2 = \sigma_2 - \beta p_2 + \gamma p_1 + \lambda\alpha + k(d_1 - d_2) \tag{2}$$

$$\pi_1 = (p_1 - w - rm)y_1 \tag{3}$$

$$\pi_2 = (p_2 - w)y_2 \tag{4}$$

$$\pi_{TPL} = (m-q)y_1 - (g - ud_1)^2 \tag{5}$$

$$\pi_m = (w + \vartheta - c - (1-r)m)y_1 + (w + \vartheta - c - t)y_2 - \delta\frac{\alpha^2}{2} - (g - ud_2)^2 \tag{6}$$

In the problem, it is assumed that there is a Stackelberg Game between members of the SC. The government plays the role of the leader in the SC. The government also intervenes in the SC by allocating tariffs to the producer. Other members of the SC follow the government. In a decentralized model, SC members make their own decisions and seek to maximize their profits. In this article, it is also assumed that *r* percent of the cost that TPL has demanded provided by the retailer and the rest by the manufacturer. Delays in delivering the goods to the customer will also be costly. According to previous studies Desiraju and Moorthy [50] this cost is equal to $(g - ud)^2$. Where *d* is the time of delivery of the goods to the customer, *g* is the constant investment to reduce the delivery time, and *u* is the coefficient of the sensitivity of the delivery time on the cost function. To solve the problem, retailers who compete, first determine the equilibrium values of their decision variables. TPL then determines the equilibrium value of its decision variables with respect to the equilibrium values obtained for retailers. The manufacturer, by incorporating the equilibrium values obtained in preceding steps in its profit function, and to maximize profit, determines the equilibrium value of the wholesale price, the degree of the greenness of the product produced, and the time of delivery of the products.

Lemma 1. The equilibrium retail price for the first and second retailers is obtained by Eqs. 7 and 8.

$$p_1 = \frac{\begin{pmatrix} 2mr\beta^2 + (2\beta+\gamma)(w\beta + \alpha\lambda) + k(-2\beta+\gamma)d_1 \\ +k(2\beta-\gamma)d_2 + 2\beta\sigma_1 + \gamma\sigma_2 \end{pmatrix}}{4\beta^2 - \gamma^2} \tag{7}$$

$$p_2 = \frac{\begin{pmatrix} mr\beta\gamma + w\beta(2\beta+\gamma) + \alpha(2\beta+\gamma)\lambda + k(2\beta-\gamma)d_1 \\ +k(-2\beta+\gamma)d_2 + \gamma\sigma_1 + 2\beta\sigma_2 \end{pmatrix}}{4\beta^2 - \gamma^2} \tag{8}$$

Proof. To obtain equilibrium values of first and second retailers profit functions compares to their first-order decision variable, we obtain the first-order derivative. Therefore, equilibrium values are obtained by solving the equation system (9).

$$\begin{cases} \dfrac{d\pi_1}{dp_1} = 0 \rightarrow mr\beta + w\beta + \alpha\lambda - kd_1 + kd_2 - 2\beta p_1 + \gamma p_2 + \sigma_1 = 0 \\ \dfrac{d\pi_2}{dp_2} = 0 \rightarrow w\beta + \alpha\lambda + kd_1 - kd_2 + \gamma p_1 - 2\beta p_2 + \sigma_2 = 0 \end{cases} \tag{9}$$

The second-order derivation for retailers' profits functions are as follows:

$$\left\{ \dfrac{\partial^2 \pi_1}{\partial p_1{}^2} = \dfrac{\partial^2 \pi_2}{\partial p_2{}^2} = -2\beta \leq 0 \right. \tag{10}$$

Since b is a positive parameter so that the Eq. 10 is negative. And this shows that the retailers' profit functions are concave with respect to their decision variable.

Lemma 2. The equilibrium value for TPL decision variables obtains from Eqs. 11 and 12.

$$m = \frac{(w\beta + \alpha\lambda) A_2 A_7 - (qA_4 + \alpha A_5 + A_6) A_9 + A_3 \left(A_{12} + A_{11}\left(A_{10} - 2u^2 d_2\right)\right)}{A_8} \tag{11}$$

$$d_1 = \frac{(w\beta + \alpha\lambda) A_{13} + A_{14} + \left(-(\beta+\gamma) A_1 + kA_{11}\right)(\alpha A_5 + A_6 + A_3 d_2)}{A_8} \tag{12}$$

Proof. By replacing the equilibrium expressions obtained in the previous step to the retailers in the TPL demand and profit functions and obtain the first-order derivative of the expression, Eq. 13 is obtained. Solving Eq. 13, the equilibrium value will be obtained for TPL.

$$\begin{cases} \dfrac{d\pi_{TPL}}{dm} = 0 \rightarrow \dfrac{\begin{pmatrix} (-\beta+\gamma)(w\beta+\alpha\lambda) A_2 + (-2m+q) A_4 \\ +\alpha A_5 + A_6 + A_3(-d_1 + d_2) \end{pmatrix}}{4\beta^2 - \gamma^2} = 0 \\ \dfrac{d\pi_{TPL}}{dd_1} = 0 \rightarrow -km + kq + 2gu + \dfrac{(m-q)(\beta+\gamma) A_1}{4\beta^2 - \gamma^2} - 2u^2 d_1 = 0 \end{cases} \tag{13}$$

The Hessian matrix obtained from the profit function of the TPL is defined as follows:

$$H(\pi_{TPL}) = \begin{pmatrix} \dfrac{\partial^2 \pi_{TPL}}{\partial m^2} & \dfrac{\partial^2 \pi_{TPL}}{\partial m \partial d_1} \\ \dfrac{\partial^2 \pi_{TPL}}{\partial d_1 \partial m} & \dfrac{\partial^2 \pi_{TPL}}{\partial d_1{}^2} \end{pmatrix} = \begin{pmatrix} \dfrac{2r\beta\left(-2\beta^2 + \gamma^2\right)}{4\beta^2 - \gamma^2} & -\dfrac{k\beta}{2\beta+\gamma} \\ -\dfrac{k\beta}{2\beta+\gamma} & -2u^2 \end{pmatrix} \tag{14}$$

If the following two requirements are met, the Hessian matrix is negative definite:

$$\frac{2r\beta\left(-2\beta^2+\gamma^2\right)}{4\beta^2-\gamma^2}<0$$

$$\frac{\beta\left(k^2\beta\left(-2\beta+\gamma\right)+4ru^2\left(2\beta+\gamma\right)\left(2\beta^2-\gamma^2\right)\right)}{\left(2\beta-\gamma\right)\left(2\beta+\gamma\right)^2}>0$$

The following equation must be established in order to meet the two requirements.

$$r\geq\frac{k^2}{12u^2\gamma} \tag{15}$$

Finally, to get the producer values of decision variables, we replace the expressions obtained in the previous step on the profit and demand function of the producer.

Lemma 3. The equilibrium values of the degree of the greenness of the product, the wholesale price, and the delivery time of the product will be as Eqs. 16 to 18.

$$\alpha=\frac{\begin{pmatrix}\left(-\left(\vartheta A_{22}+A_{23}\right)A_{25}+\left(\vartheta A_{20}+A_{21}\right)A_{26}\right)\left(-2A_{17}A_{19}+A_{25}A_{26}\right)\\-\left(-A_{19}A_{25}+2A_{15}A_{26}\right)\left(-2A_{17}\left(\vartheta A_{22}+A_{23}\right)+A_{26}\left(\vartheta A_{24}+A_{27}\right)\right)\end{pmatrix}}{2A_{26}\left(A_{17}A_{19}^2+A_{25}\left(A_{16}A_{25}-A_{19}A_{26}\right)+A_{15}\left(-4A_{16}A_{17}+A_{26}^2\right)\right)} \tag{16}$$

$$w=\frac{\begin{pmatrix}2A_{17}A_{19}\left(\vartheta A_{22}+A_{23}\right)+A_{26}\left(-\left(\vartheta A_{22}+A_{23}\right)A_{25}+\left(\vartheta A_{20}+A_{21}\right)A_{26}-A_{19}\left(\vartheta A_{24}+A_{27}\right)\right)\\+2A_{16}\left(-2A_{17}\left(\vartheta A_{20}+A_{21}\right)+A_{25}\left(\vartheta A_{24}+A_{27}\right)\right)\end{pmatrix}}{A_{15}\left(8A_{16}A_{17}-2A_{26}^2\right)-2\left(A_{17}A_{19}^2+A_{25}\left(A_{16}A_{25}-A_{19}A_{26}\right)\right)} \tag{17}$$

$$d_2=\frac{\begin{pmatrix}2A_{16}\left(\vartheta A_{20}+A_{21}\right)A_{25}+A_{19}\left(-\left(\vartheta A_{22}+A_{23}\right)A_{25}-\left(\vartheta A_{20}+A_{21}\right)A_{26}\right)\\+A_{19}^2\left(\vartheta A_{24}+A_{27}\right)+2A_{15}\left(\left(\vartheta A_{22}+A_{23}\right)A_{26}-2A_{16}\left(\vartheta A_{24}+A_{27}\right)\right)\end{pmatrix}}{A_{15}\left(8A_{16}A_{17}-2A_{26}^2\right)-2\left(A_{17}A_{19}^2+A_{25}\left(A_{16}A_{25}-A_{19}A_{26}\right)\right)} \tag{18}$$

Proof. To find equilibrium solutions for decision variables of producer, the equation system derived from the first derivative must be solved.

$$\begin{cases}\dfrac{d\pi_m}{d\alpha}=0\rightarrow 2\alpha A_{16}+wA_{19}+\vartheta A_{22}+A_{23}+A_{26}d_2=0\\ \dfrac{d\pi_m}{dw}=0\rightarrow 2wA_{15}+\alpha A_{19}+\vartheta A_{20}+A_{21}+A_{25}d_2=0\\ \dfrac{d\pi_m}{dd_2}=0\rightarrow \vartheta A_{24}+wA_{25}+\alpha A_{26}+A_{27}+2A_{17}d_2=0\end{cases} \tag{19}$$

Attention. The variable change values are given in the Appendix.

The Hessian matrix obtained from the profit function of the producer is defined as follows:

$$H(\pi_m)=\begin{pmatrix}\frac{\partial^2\pi_m}{\partial\alpha^2} & \frac{\partial^2\pi_m}{\partial\alpha\partial w} & \frac{\partial^2\pi_m}{\partial\alpha\partial d_2}\\ \frac{\partial^2\pi_m}{\partial w\partial\alpha} & \frac{\partial^2\pi_m}{\partial w^2} & \frac{\partial^2\pi_m}{\partial w\partial d_2}\\ \frac{\partial^2\pi_m}{\partial d_2\partial\alpha} & \frac{\partial^2\pi_m}{\partial d_2\partial w} & \frac{\partial^2\pi_m}{\partial d_2^{\,2}}\end{pmatrix}=\begin{pmatrix}-\delta & M_1 & M_2\\ M_1 & M_3 & M_4\\ M_2 & M_4 & M_5\end{pmatrix} \tag{20}$$

Which that:

$$M_1=\frac{2\beta\begin{pmatrix}k^4\beta^2(-2\beta+\gamma)^2-k^2ru^2\beta(2\beta-\gamma)(2\beta+\gamma)(14\beta^2+\beta\gamma-7\gamma^2)\\ +4ru^4(2\beta+\gamma)^2(2\beta^2-\gamma^2)\begin{pmatrix}(2+4r)\beta^2\\ +(-1+2r)\beta\gamma-(1+2r)\gamma^2\end{pmatrix}\end{pmatrix}\lambda}{(2\beta-\gamma)\left(k^2\beta(-2\beta+\gamma)+4ru^2(2\beta+\gamma)(2\beta^2-\gamma^2)\right)^2}$$

$$M_2=\frac{8k(-1+r)ru^4\beta(2\beta+\gamma)^2(2\beta^2-\gamma^2)\lambda}{\left(k^2\beta(2\beta-\gamma)-4ru^2(2\beta+\gamma)(2\beta^2-\gamma^2)\right)^2}$$

$$M_3=\frac{4\beta(\beta-\gamma)\begin{pmatrix}-k^4\beta^2(-2\beta+\gamma)^2+k^2ru^2\beta(2\beta-\gamma)(2\beta+\gamma)(14\beta^2+\beta\gamma-7\gamma^2)\\ -2ru^4(2\beta+\gamma)^2(2\beta^2-\gamma^2)\begin{pmatrix}2(1+5r)\beta^2\\ +(-1+3r)\beta\gamma-(1+5r)\gamma^2\end{pmatrix}\end{pmatrix}}{(2\beta-\gamma)\left(k^2\beta(-2\beta+\gamma)+4ru^2(2\beta+\gamma)(2\beta^2-\gamma^2)\right)^2}$$

$$M_4=\frac{2kru^2\beta(\beta-\gamma)(2\beta+\gamma)\begin{pmatrix}k^2\beta(2\beta-\gamma)\\ -4(-1+2r)u^2(2\beta+\gamma)(2\beta^2-\gamma^2)\end{pmatrix}}{\left(k^2\beta(2\beta-\gamma)-4ru^2(2\beta+\gamma)(2\beta^2-\gamma^2)\right)^2}$$

$$M_5=\frac{\begin{pmatrix}-2k^4u^2\beta^2(-2\beta+\gamma)^2-32r^2u^6(2\beta+\gamma)^2(-2\beta^2+\gamma^2)^2\\ +8k^2r(1+r)u^4\beta(8\beta^4-6\beta^2\gamma^2+\gamma^4)\end{pmatrix}}{\left(k^2\beta(2\beta-\gamma)-4ru^2(2\beta+\gamma)(2\beta^2-\gamma^2)\right)^2}$$

If the following requirements are met, the Hessian matrix is negative definite:

$$-\delta<0$$
$$-\delta M_3-M_1^{\,2}>0$$
$$-M_2^2M_3+2M_1M_2M_4+\delta M_4^2-\left(M_1^2+\delta M_3\right)M_5<0$$

The following equation must be established in order to meet the two requirements.

$$\delta<\frac{-16ru^4\gamma\lambda^2+16r^2u^4\gamma\lambda^2}{k^4-16k^2ru^2\gamma+64r^2u^4\gamma^2} \tag{21}$$

Centralized Model (CM)

In the centralized model, it is assumed that the same level players work together to achieve maximum revenue. In the model under consideration, there are two players at the retail level. As a result, cooperation is only at this level. Like DCM in CM, there is a Stackelberg competition between members of the SC. The order of the game is that retailers first determine the retail price, and then TPL determines the equilibrium values of its decision variables according to the obtained values. Finally, the manufacturer determines the equilibrium price for the wholesale price, the degree of the greenness of the product, and the time of delivery. According to the above description, the profit function of retailers is written as follows:

$$\pi_R = \pi_1 + \pi_2 \tag{22}$$

Lemma 4. Equilibrium retail prices for retailers 1 and 2 are obtained by Eqs. 23 and 24.

$$p_1 = \frac{\begin{pmatrix} (\beta+\gamma)((mr+w)(\beta-\gamma)+\alpha\lambda)+k(-\beta+\gamma)d_1 \\ +k(\beta-\gamma)d_2+\beta\sigma_1+\gamma\sigma_2 \end{pmatrix}}{2(\beta-\gamma)(\beta+\gamma)} \tag{23}$$

$$p_2 = \frac{\begin{pmatrix} (\beta+\gamma)(w\beta-w\gamma+\alpha\lambda)+k(\beta-\gamma)d_1 \\ +k(-\beta+\gamma)d_2+\gamma\sigma_1+\beta\sigma_2 \end{pmatrix}}{2(\beta-\gamma)(\beta+\gamma)} \tag{24}$$

Proof. The profit function of retailers is equal to Eq. 22. Therefore, to obtain the equilibrium values of the decision variables of retailers, the first-order derivative equation is derived. So, the equilibrium values obtained through solving Eq. 25.

$$\begin{cases} \dfrac{d\pi_1}{dp_1} = 0 \rightarrow mr\beta + w\beta - w\gamma + \alpha\lambda - kd_1 + kd_2 - 2\beta p_1 + 2\gamma p_2 + \sigma_1 = 0 \\ \dfrac{d\pi_2}{dp_2} = 0 \rightarrow w\beta - mr\gamma - w\gamma + \alpha\lambda + kd_1 - kd_2 + 2\gamma p_1 - 2\beta p_2 + \sigma_2 = 0 \end{cases} \tag{25}$$

The second-order derivation for retailers' profits functions are as follows:

$$\left\{ \frac{\partial^2 \pi_1}{\partial {p_1}^2} = \frac{\partial^2 \pi_2}{\partial {p_2}^2} = -2\beta \le 0 \right. \tag{26}$$

Since b is a positive parameter so that the Eq. 26 is negative. And this shows that the retailers' profit functions are concave with respect to their decision variable.

Lemma 5. The equilibrium value for TPL decision variables is derived from Eqs. 27 and 28.

$$m = \frac{k^2 q + 4gku - 4u^2(qr\beta + w(-\beta+\gamma)+\alpha\lambda) - 4u^2(kd_2+\sigma_1)}{k^2 - 8ru^2\beta} \tag{27}$$

$$d_1 = \frac{-8gru\beta + k(-(qr+w)\beta + w\gamma + \alpha\lambda) + k(kd_2+\sigma_1)}{k^2 - 8ru^2\beta} \tag{28}$$

Proof. By replacing the equilibrium expressions obtained in the previous step for the retailers in the TPL demand and profit functions and obtaining the first-order derivative of the obtained expression, equation system (29) is obtained. Solving Eq. 29 results in the values of m and d1.

$$\begin{cases} \dfrac{d\pi_{TPL}}{dm} = 0 \rightarrow \dfrac{1}{2}\left(-2mr\beta + qr\beta - w\beta + w\gamma + \alpha\lambda - kd_1 + kd_2 + \sigma_1\right) = 0 \\ \dfrac{d\pi_{TPL}}{dd_1} = 0 \rightarrow \dfrac{1}{2}k\left(-m+q\right) + 2gu - 2u^2 d_1 = 0 \end{cases} \tag{29}$$

The Hessian matrix obtained from the profit function of the TPL is defined as follows:

$$H(\pi_{TPL}) = \begin{pmatrix} \dfrac{\partial^2 \pi_{TPL}}{\partial m^2} & \dfrac{\partial^2 \pi_{TPL}}{\partial m \partial d_1} \\ \dfrac{\partial^2 \pi_{TPL}}{\partial d_1 \partial m} & \dfrac{\partial^2 \pi_{TPL}}{\partial {d_1}^2} \end{pmatrix} = \begin{pmatrix} -r\beta & -\dfrac{k}{2} \\ -\dfrac{k}{2} & -2u^2 \end{pmatrix} \tag{30}$$

If the following two requirements are met, the Hessian matrix is negative definite:

$$-r\beta < 0$$

$$-\frac{k^2}{4} + 2ru^2\beta > 0$$

The following equation must be established in order to meet the two requirements.

$$r > \frac{k^2}{8u^2\gamma} \tag{31}$$

Lemma 6. The equilibrium values of the degree of the greenness of the product, the wholesale price, and the delivery time of the product will be obtained from Eq. 31 to Eq. 34.

$$\alpha = \frac{\begin{pmatrix} 2Z_5Z_8Z_{11} - \vartheta Z_3Z_8Z_{12} - 2Z_5Z_6Z_{14} + \vartheta Z_3Z_7Z_{14} + 2Z_3Z_9Z_{14} \\ +\vartheta Z_2\left(Z_8Z_{11} - Z_6Z_{14}\right) - 2Z_3Z_8Z_{15} \\ +Z_4\left(-2\vartheta Z_7Z_{11} - 4Z_9Z_{11} + 2Z_6\left(\vartheta Z_{12} + 2Z_{15}\right)\right) \end{pmatrix}}{Z_4\left(4Z_{10}Z_{11} - 2Z_6Z_{13}\right) + Z_1\left(-Z_8Z_{11} + Z_6Z_{14}\right) + Z_3\left(Z_8Z_{13} - 2Z_{10}Z_{14}\right)} \tag{32}$$

$$w = \frac{\begin{pmatrix} -\left(\vartheta Z_2 + 2Z_5\right)\left(Z_8Z_{13} - 2Z_{10}Z_{14}\right) + Z_1\left(-\left(\vartheta Z_7 + 2Z_9\right)Z_{14} + Z_8\left(\vartheta Z_{12} + 2Z_{15}\right)\right) \\ +Z_4\left(2\left(\vartheta Z_7 + 2Z_9\right)Z_{13} - 4Z_{10}\left(\vartheta Z_{12} + 2Z_{15}\right)\right) \end{pmatrix}}{Z_4\left(4Z_{10}Z_{11} - 2Z_6Z_{13}\right) + Z_1\left(-Z_8Z_{11} + Z_6Z_{14}\right) + Z_3\left(Z_8Z_{13} - 2Z_{10}Z_{14}\right)} \tag{33}$$

$$d_2 = \frac{\begin{pmatrix} -4Z_5Z_{10}Z_{11} + 2\vartheta Z_3Z_{10}Z_{12} + 2Z_5Z_6Z_{13} - \vartheta Z_3Z_7Z_{13} - 2Z_3Z_9Z_{13} \\ +\vartheta Z_2\left(-2Z_{10}Z_{11} + Z_6Z_{13}\right) + 4Z_3Z_{10}Z_{15} \\ +Z_1\left(\vartheta Z_7Z_{11} + 2Z_9Z_{11} - Z_6\left(\vartheta Z_{12} + 2Z_{15}\right)\right) \end{pmatrix}}{Z_4\left(4Z_{10}Z_{11} - 2Z_6Z_{13}\right) + Z_1\left(-Z_8Z_{11} + Z_6Z_{14}\right) + Z_3\left(Z_8Z_{13} - 2Z_{10}Z_{14}\right)} \tag{34}$$

Proof. By solving the equation system (35), the optimal value of the producer’s decision variables is obtained.

$$\begin{cases} \dfrac{d\pi_m}{d\alpha} = 0 \rightarrow \dfrac{1}{2}\left(\alpha Z_1 + \vartheta Z_2 + wZ_3 + 2d_2 Z_4\right) + Z_5 = 0 \\ \dfrac{d\pi_m}{dw} = 0 \rightarrow \dfrac{1}{2}\left(wZ_{11} + \vartheta Z_{12} + \alpha Z_{13} + d_2 Z_{14}\right) + Z_{15} = 0 \\ \dfrac{d\pi_m}{dd_2} = 0 \rightarrow \dfrac{1}{2}\left(wZ_6 + \vartheta Z_7 + d_2 Z_8 + 2Z_9 + 2\alpha Z_{10}\right) = 0 \end{cases} \tag{35}$$

Attention. The variable change values are given in the appendix.

The Hessian matrix obtained from the profit function of the producer is defined as follows:

$$H(\pi_m) = \begin{pmatrix} \dfrac{\partial^2 \pi_m}{\partial \alpha^2} & \dfrac{\partial^2 \pi_m}{\partial \alpha \partial w} & \dfrac{\partial^2 \pi_m}{\partial \alpha \partial d_2} \\ \dfrac{\partial^2 \pi_m}{\partial w \partial \alpha} & \dfrac{\partial^2 \pi_m}{\partial w^2} & \dfrac{\partial^2 \pi_m}{\partial w \partial d_2} \\ \dfrac{\partial^2 \pi_m}{\partial d_2 \partial \alpha} & \dfrac{\partial^2 \pi_m}{\partial d_2 \partial w} & \dfrac{\partial^2 \pi_m}{\partial {d_2}^2} \end{pmatrix} =$$

$$\begin{pmatrix} -\delta & \dfrac{\begin{pmatrix} k^4 - 2k^2 r u^2 (7\beta + \gamma) \\ +16 r u^4 \beta \begin{pmatrix} \beta + 2r\beta \\ +(-1+2r)\gamma \end{pmatrix} \end{pmatrix} \lambda}{\left(k^2 - 8ru^2\beta\right)^2} & \dfrac{16k(-1+r) r u^4 \beta \lambda}{\left(k^2 - 8ru^2\beta\right)^2} \\ \dfrac{\begin{pmatrix} k^4 - 2k^2 r u^2 (7\beta + \gamma) \\ +16 r u^4 \beta \begin{pmatrix} \beta + 2r\beta \\ +(-1+2r)\gamma \end{pmatrix} \end{pmatrix} \lambda}{\left(k^2 - 8ru^2\beta\right)^2} & -\dfrac{2(\beta - \gamma)\begin{pmatrix} k^4 - 2k^2 r u^2 (7\beta + \gamma) \\ +8 r u^4 \beta \begin{pmatrix} \beta + 5r\beta \\ +(-1+3r)\gamma \end{pmatrix} \end{pmatrix}}{\left(k^2 - 8ru^2\beta\right)^2} & \dfrac{2kru^2\left(k^2 + 8(1-2r)u^2\beta\right)(\beta - \gamma)}{\left(k^2 - 8ru^2\beta\right)^2} \\ \dfrac{16k(-1+r) r u^4 \beta \lambda}{\left(k^2 - 8ru^2\beta\right)^2} & \dfrac{2kru^2\left(k^2 + 8(1-2r)u^2\beta\right)(\beta - \gamma)}{\left(k^2 - 8ru^2\beta\right)^2} & \dfrac{1}{2}\left(-4u^2 + \dfrac{32k^2(-1+r) r u^4 \beta}{\left(k^2 - 8ru^2\beta\right)^2}\right) \end{pmatrix} \tag{36}$$

If the following two requirements are met, the Hessian matrix is negative definite:

$$-\delta+\frac{16(-1+r)ru^4\beta\lambda^2}{\left(k^2-8ru^2\beta\right)^2}<0$$

$$\frac{2(\beta-\gamma)\left(k^4-2k^2ru^2(7\beta+\gamma)+8ru^4\beta\begin{pmatrix}\beta+5r\beta\\+(-1+3r)\gamma\end{pmatrix}\right)\delta-\left(k^2-2ru^2(3\beta+\gamma)\right)^2\lambda^2}{\left(k^2-8ru^2\beta\right)^2}>0$$

$$\frac{2u^2\begin{pmatrix}-2(\beta-\gamma)\begin{pmatrix}k^4+k^2ru^2\left(-3(2+3r)\beta+(-2+r)\gamma\right)\\+8ru^4\beta\left(\beta+5r\beta+(-1+3r)\gamma\right)\end{pmatrix}\delta\\+\left(k^4+4r^2u^4(3\beta+\gamma)^2-4k^2ru^2(\beta+2r\beta+\gamma)\right)\lambda^2\end{pmatrix}}{\left(k^2-8ru^2\beta\right)^2}<0$$

The following equation must be established in order to meet the two requirements.

$$\begin{cases}\delta<\dfrac{16(-1+r)ru^4\gamma\lambda^2}{\left(k^2-8ru^2\gamma\right)^2}\\[2ex]\beta>\dfrac{k^2\delta+u^2\left((-1+r)\lambda^2+r\delta\sqrt{\dfrac{(-1+r)\lambda^2\left(2k^2\delta+(-1+r)u^2\lambda^2\right)}{r^2u^2\delta^2}}\right)}{8ru^2\delta}\end{cases}\tag{37}$$

Government's model

As it is said before, the government has an impact on the SC through the allocation of tariffs (subsidies or taxes) to the producer. In this study, we assume that the government's goal is to increase its profits in the SC. So, the government model is:

$$\max\quad \pi_G=(-\varphi)D_M\tag{38}$$

$$\pi_M\geq L_m\tag{39}$$

$$\pi_{TPL}\geq L_{TPL}\tag{40}$$

$$\pi_j\geq L_j\qquad\qquad \forall j\tag{41}$$

$$\sum_{l=1}^{m}\sum_{i=1}^{n}CP_lD_m+\sigma\xi D_1+\xi D_2\leq U_G\tag{42}$$

$$L_M, L_{3pl}, L_j\geq 0,\ U_G\geq 0,\ \varphi\rightarrow\text{free variables}\tag{43}$$

Eq. 38 is the objective function of the problem that the government seeks to maximize its profit in the SC. Eqs. 39 to 41 represent the minimum amount of satisfying profit for SC members to enter the game and to continue their activity in SC. In addition to exceeding its profits, the government also seeks to balance the amount of pollution released in the cycle. Constraint (42) indicates that the government has imposed a high restriction on the amount of contaminants released in the SC so that players do not violate this amount.

Numerical examples and parametric sensitivity analysis

In this section, we provide a numerical example to understand more and to show how the government influences the SC. It also analyzes how some parameters affect SC members' profits. In Table 2, the values assigned to the problem parameters are shown [49, 51-54].

Table 2. The values assigned to the parameters

σ_1	σ_2	β	γ	λ	c	δ
2000	1800	2	1	0.8	40	1.5
k	r	q	g	u	t	
0.7	0.6	80	400	0.5	110	

By replacing the parameter values as well as solving the centralized and decentralized in the government model, the equilibrium values of decision variables and profit functions for SC members will be obtained as in Table 3.

Table 3. Equilibrium values obtained for decision variables and dependent variables

	CM	DCM
p_1	1928.02	1923.28
p_2	1808.81	1795.63
m	294.787	386.984
d_1	649.649	628.089
d_2	638.681	609.431
α	229.723	294.88
w	1564.61	1605.13
ϑ	-868.653	-870.308
π_1	24039.1	14776.7
π_2	73712.7	72576.1
π_{TPL}	22028.7	45385.7
π_m	188056	241335
π_G	374155.535	481193.546

As can be seen from Table 3, in the decentralized model, TPL requires more cost to carry each unit, and the price of selling the product by retailers is more in decentralized mode than the centralized model. Also, the delivery time to the customer in the decentralized mode is less than the centralized model. According to Table 3, retailers in the centralized model gain more profits due to their cooperation, but TPL and producer profit are less than the decentralized model.

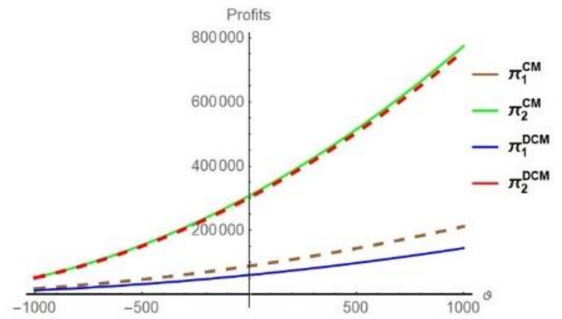

Fig. 3. The impact of government tariff on retailers' profit

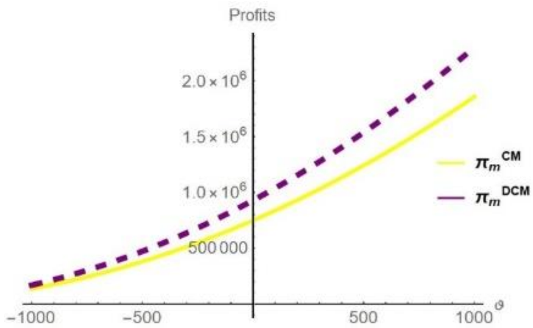

Fig. 2. The impact of government tariff on producers' profit

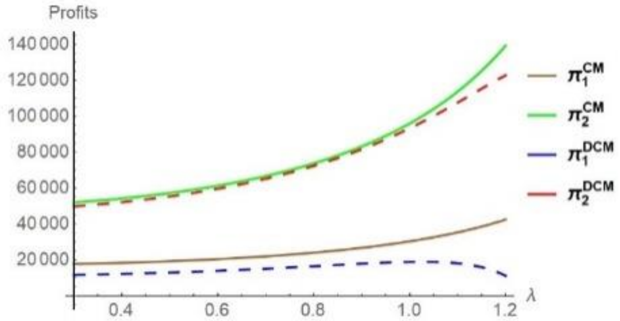

Fig. 5. The impact of λ on retailers' profit

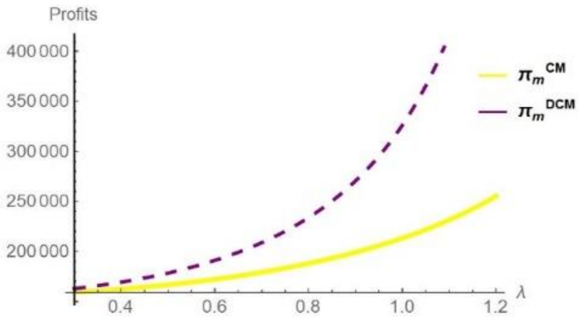

Fig. 4. The impact of λ on producers' profit

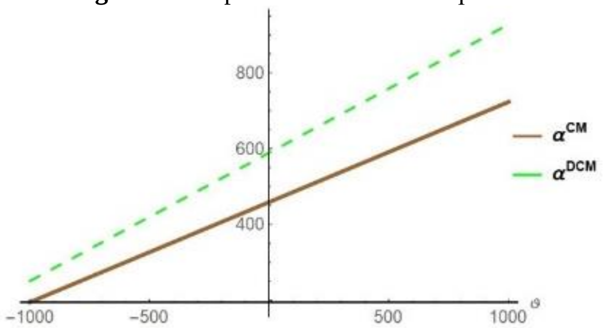

Fig. 7. The impact of government tariff on the product level of greenness

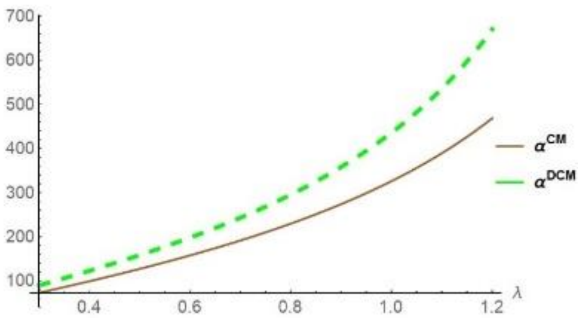

Fig. 6. The impact of λ on the product level of greenness

It can be seen from Figs. 2 and 3 that the SC members' profits increase with the increase in government-allocated tariffs, both in the centralized and decentralized model. Fig. 3 shows that in the centralized model the first retailer profit is more sensitive to the tariff changes than the decentralized one. As Fig. 7 shows, there is a direct relationship between the tariff and the degree of the greenness of the product, and it can be concluded that the increase in tariff allocated to the producer by the government, in addition to increasing the members' profits, also will increase the degree of the product greenness. As a result, the amount of contamination released in the SC will be decreased. Consequently, if the government desires to reduce pollution in the SC, it must encourage members of the chain to do environmental-friendly activities by increasing subsidies. Fig. 4 illustrates how the producer's profit function reacts to λ changes. As can be seen, the increase in λ value will lead to an increase in producer profit, both in a centralized and decentralized model. But in the decentralized model, the producer's sensitivity to λ changes is much more intense. Fig. 5 also shows the effect of λ on retailers' profits. As can be seen in Fig. 5, retailers' profits also increase with increasing λ, but from a point onwards, the amount of first retailer profit in a decentralized model starts to decline. This is due to the decrease in demand through this channel in the decentralized model. As the producer seeks to maximize its profits and, on the other hand, seeks to maintain its presence in the target market, it must, therefore, adopt a strategy that maximizes the profits of all members of the SC. So, a value of 0.8 for λ is not a good strategy in this model. In contrast, as the λ increases, the degree of the greenness of the product increases. And there are reasons for the reduction in pollution.

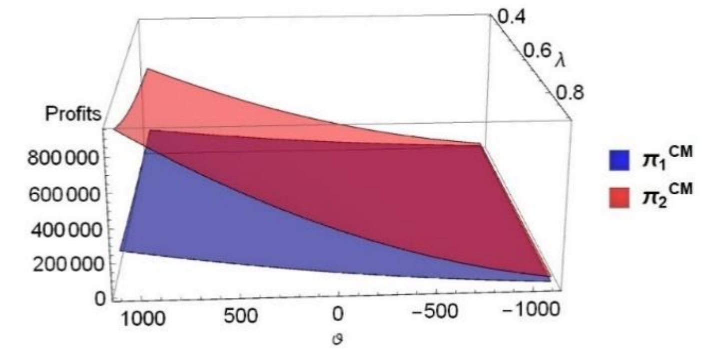

Fig. 8. The impact of government tariff and green product coefficient on retailer profits (Centralized model)

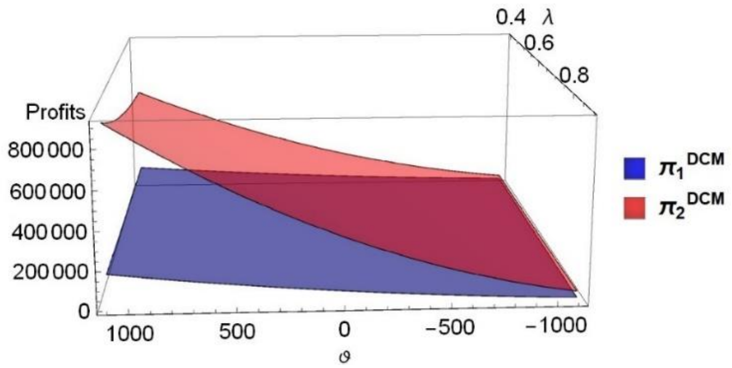

Fig. 9. The impact of government tariff and green product coefficient on retailer profits (Decentralized model)

Figs. 8 and 9 show the simultaneous impact of λ and government tariff (υ) on SC members' profits. These figures clearly show that the profits of producers and retailers in the decentralized model outweigh the centralized model. Figs. 8 and 9 illustrate that the second retailer profit is more sensitive to both λ and tariff changes in both centralized and decentralized modes. Increasing the values of the λ parameters and the government tariff in a positive direction (allocating subside to the producer) has increased the producer profit.

Managerial insights

- The results showed that in the case of cooperating with 3PL companies the SC will perform more sustainable performance. According to this finding, governments can encourage producers to cooperate with 3PLs by making supportive financial and non-financial policies.
- Tariffs are one of the most effective tools to control the SCs' members' activities. Governments can monitor the activities of SCs and affect their sustainability by financial intervention, including taxation and subsidization.
- If the government tend to increase its profits, it must implement a strategy so that other members of the supply chain do not move in unison.

Conclusion and suggestions for future research

This paper considers an SC with a green producer, a TPL company and two retailers. TPL provides services between the producer and the first retailer. It is also assumed that TPL will receive x percent of transportation costs from the retailer and the rest from the producer. The results are as follow. (I) By increasing the amount of subsidies allocated to the producer, the government can reduce the amount of pollution in the SC. (II) Increasing the amount of subsidy will lead to an increase in the degree of product greenness. (III) Increasing the degree of the greenness of the product does not always increase the profitability of SC members. This study shows that considering government and a TPL company, simultaneously, can present a suitable idea for managers to addressing their own company. Considering retailers as the leader of the game or monitoring their activities by the government would be another interesting subject for future research. In addition, the current study investigated the proposed issue at the certain condition. For future studies, people can consider uncertain situations to study. Also, investigators can consider other kinds of game structure (e.g. Evolutionary game theory) to simulate the problem.

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Appendix

$A_1 = k(2\beta - \gamma)$

$A_2 = 2\beta + \gamma$

$A_3 = \left(k\left(4\beta^2 - \gamma^2\right) - (\beta + \gamma)A_1\right)$ $\quad A_4 = r\beta\left(2\beta^2 - \gamma^2\right)$ $\quad A_5 = \left(4\beta^2 - \gamma^2\right)\lambda$

$$A_6 = 2\beta^2\sigma_1 + \beta\gamma\sigma_2 \qquad A_7 = 2u^2(\beta-\gamma)(2\beta-\gamma)(2\beta+\gamma) \qquad A_8 = \left(4k\beta^2 - k\gamma^2 - (\beta+\gamma)A_1\right)A_3 + 4u^2\left(-4\beta^2+\gamma^2\right)A_4$$

$$A_9 = 2u^2\left(4\beta^2-\gamma^2\right) \qquad A_{10} = kq + 2gu \qquad A_{11} = \left(4\beta^2-\gamma^2\right) \qquad A_{12} = -q(\beta+\gamma)A_1$$

$$A_{13} = -(\beta-\gamma)\left(4k\beta^2 - k\gamma^2 - (\beta+\gamma)A_1\right)A_2 \qquad A_{14} = \left(-(kq+4gu)\left(4\beta^2-\gamma^2\right) + q(\beta+\gamma)A_1\right)A_4$$

$$A_{15} = \frac{\beta A_2\begin{pmatrix}(-\beta+\gamma)A_8\left(r\beta(2\beta+\gamma)A_7 + 2A_8\right) - (-1+r)\beta A_2A_7\left(r\beta\left(2\beta^2-\gamma^2\right)A_7 + (\beta-\gamma)A_8\right)\\ -(-1+r)\beta\left(4k\beta^2-k\gamma^2-(\beta+\gamma)A_1\right)A_7A_{13}\end{pmatrix}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{16} = -\frac{\delta}{2} - \frac{(-1+r)(\beta-\gamma)\lambda A_2\left(\lambda A_2A_7 - A_5A_9\right)}{\left(4\beta^2-\gamma^2\right)A_8} + \frac{(-1+r)\left(\lambda A_2A_7 - A_5A_9\right)\begin{pmatrix}k(\beta+\gamma)A_1A_5\\ +\lambda A_8 - k\left(kA_5A_{11}+\lambda A_{13}\right)\end{pmatrix}}{A_8^2}$$
$$-\frac{(-1+r)\left(\lambda A_2A_7 - A_5A_9\right)\begin{pmatrix}(\beta+\gamma)^2A_1^2A_5 + r\beta\left(2\beta^2-\gamma^2\right)\left(\lambda A_2A_7 - A_5A_9\right)\\ -(\beta+\gamma)A_1\left(kA_5A_{11}+\lambda A_{13}\right)\end{pmatrix}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{17} = -\frac{u^2\begin{pmatrix}\left(4\beta^2-\gamma^2\right)A_8^2 + 2(-1+r)\left(k\left(4\beta^2-\gamma^2\right)-(\beta+\gamma)A_1\right)A_3\left((\beta+\gamma)A_1A_3 + A_8\right)A_{11}\\ +2(-1+r)\left(2ru^2\beta\left(2\beta^2-\gamma^2\right) + k^2\left(-4\beta^2+\gamma^2\right) + k(\beta+\gamma)A_1\right)A_3^2A_{11}^2\end{pmatrix}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{18} = \frac{\beta\left(r(\beta-\gamma)\left(\left(qA_4 + A_6\right)A_9 - A_3\left(A_{10}A_{11} + A_{12}\right)\right) + A_8\left(\sigma_1+\sigma_2\right)\right)}{(2\beta-\gamma)A_8}$$

$$A_{14} = \left(-(kq+4gu)\left(4\beta^2-\gamma^2\right) + q(\beta+\gamma)A_1\right)A_4$$

$$A_{15} = \frac{\beta A_2\begin{pmatrix}(-\beta+\gamma)A_8\left(r\beta(2\beta+\gamma)A_7 + 2A_8\right) - (-1+r)\beta A_2A_7\left(r\beta\left(2\beta^2-\gamma^2\right)A_7 + (\beta-\gamma)A_8\right)\\ -(-1+r)\beta\left(4k\beta^2-k\gamma^2-(\beta+\gamma)A_1\right)A_7A_{13}\end{pmatrix}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{16} = -\frac{\delta}{2} - \frac{(-1+r)(\beta-\gamma)\lambda A_2\left(\lambda A_2A_7 - A_5A_9\right)}{\left(4\beta^2-\gamma^2\right)A_8} + \frac{(-1+r)\left(\lambda A_2A_7 - A_5A_9\right)\left(k(\beta+\gamma)A_1A_5 + \lambda A_8 - k\left(kA_5A_{11}+\lambda A_{13}\right)\right)}{A_8^2}$$
$$-\frac{(-1+r)\left(\lambda A_2A_7 - A_5A_9\right)\left((\beta+\gamma)^2A_1^2A_5 + r\beta\left(2\beta^2-\gamma^2\right)\left(\lambda A_2A_7 - A_5A_9\right) - (\beta+\gamma)A_1\left(kA_5A_{11}+\lambda A_{13}\right)\right)}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{17} = -\frac{u^2\begin{pmatrix}\left(4\beta^2-\gamma^2\right)A_8^2 + 2(-1+r)\left(k\left(4\beta^2-\gamma^2\right)-(\beta+\gamma)A_1\right)A_3\left((\beta+\gamma)A_1A_3 + A_8\right)A_{11}\\ +2(-1+r)\left(2ru^2\beta\left(2\beta^2-\gamma^2\right) + k^2\left(-4\beta^2+\gamma^2\right) + k(\beta+\gamma)A_1\right)A_3^2A_{11}^2\end{pmatrix}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$A_{18} = \frac{\beta\left(r(\beta-\gamma)\left(\left(qA_4 + A_6\right)A_9 - A_3\left(A_{10}A_{11} + A_{12}\right)\right) + A_8\left(\sigma_1+\sigma_2\right)\right)}{(2\beta-\gamma)A_8}$$

$$A_{19}=2\lambda\left(1+\frac{(-\beta+\gamma)A_2}{4\beta^2-\gamma^2}\right)+\frac{\beta(\beta-\gamma)\big(-\lambda A_2\big(r(2\beta+\gamma)+2(-1+r)A_2\big)A_7+\big(r(2\beta+\gamma)+(-1+r)A_2\big)A_5A_9\big)}{(4\beta^2-\gamma^2)A_8}$$

$$+\frac{(-1+r)\beta\begin{pmatrix}A_2A_7\big(-(\beta+\gamma)^2A_1^2A_5+2r\beta(2\beta^2-\gamma^2)(-\lambda A_2A_7+A_5A_9)+k(\beta+\gamma)A_1A_5A_{11}\big)\\+(\beta+\gamma)A_1(2\lambda A_2A_7-A_5A_9)A_{13}\end{pmatrix}}{(4\beta^2-\gamma^2)A_8^2}$$

$$+\frac{(-1+r)\beta\big(A_2A_7\big(\lambda A_8+kA_5\big((\beta+\gamma)A_1-kA_{11}\big)\big)+k(-2\lambda A_2A_7+A_5A_9)A_{13}\big)}{A_8^2}$$

$$A_{20}=-\frac{\beta(\beta-\gamma)A_2\big(r\beta(2\beta+\gamma)A_7+2A_8\big)}{(4\beta^2-\gamma^2)A_8}$$

$$A_{21}=\frac{1}{(4\beta^2-\gamma^2)A_8^2}(-1+r)\beta(-(\beta+\gamma)^2A_1^2A_2A_6A_7+A_2A_7\begin{pmatrix}2qr\beta(2\beta^2-\gamma^2)A_4A_9+A_6\big(r(4\beta^3-2\beta\gamma^2)A_9+k^2(-4\beta^2+\gamma^2)A_{11}\big)\\-2r\beta(2\beta^2-\gamma^2)A_3(A_{10}A_{11}+A_{12})\end{pmatrix}$$

$$+(\beta+\gamma)A_1\big(\big((-qA_4-A_6)A_9+A_3(A_{10}A_{11}+A_{12})\big)A_{13}+A_2A_7\big(kA_6(4\beta^2-\gamma^2+A_{11})+A_{14}\big)\big))+\sigma_1$$

$$+\frac{k(-1+r)\beta\big(\big((qA_4+A_6)A_9-A_3(A_{10}A_{11}+A_{12})\big)A_{13}-A_2A_7A_{14}\big)+\beta A_8\big(-ktA_{13}+(-1+r)A_2A_7\sigma_1\big)}{A_8^2}$$

$$+\sigma_2+\frac{(\beta-\gamma)\big((2c+t)\beta A_2-(2\beta+\gamma)(\sigma_1+\sigma_2)\big)}{4\beta^2-\gamma^2}+\frac{1}{(4\beta^2-\gamma^2)A_8}\beta(r(\beta-\gamma)(2\beta+\gamma)\begin{pmatrix}(qA_4+A_6)A_9\\-A_3(A_{10}A_{11}+A_{12})\end{pmatrix}$$

$$+t(\beta+\gamma)A_1A_{13}+A_2\begin{pmatrix}(-1+r)(\beta-\gamma)\big((qA_4+A_6)A_9-A_3(A_{10}A_{11}+A_{12})\big)\\+A_7\big(r\beta\big(-t\beta\gamma+c(\beta-\gamma)(2\beta+\gamma)\big)-(-1+r)\big((2\beta^2-\gamma^2)\sigma_1-\beta\gamma\sigma_2\big)\big)\end{pmatrix})$$

$$A_{22}=\frac{-(\beta-\gamma)\lambda A_2\big(r\beta(2\beta+\gamma)A_7+2A_8\big)+(2\beta+\gamma)\big((4\beta\lambda-2\gamma\lambda)A_8+r\beta(\beta-\gamma)A_5A_9\big)}{(4\beta^2-\gamma^2)A_8}$$

$$A_{23}=(2c+t)\lambda\left(-1+\frac{(\beta-\gamma)A_2}{4\beta^2-\gamma^2}\right)+\frac{1}{(4\beta^2-\gamma^2)A_8^2}(-1+r)(2r\beta(2\beta^2-\gamma^2)(\lambda A_2A_7-A_5A_9)\big((qA_4+A_6)A_9-A_3(A_{10}A_{11}+A_{12})\big)$$

$$-(\beta+\gamma)^2A_1^2\big(\lambda A_2A_6A_7+A_5\big((-qA_4-2A_6)A_9+A_3(A_{10}A_{11}+A_{12})\big)\big)+(\beta+\gamma)A_1(-2kA_5A_6A_9A_{11}+kA_3A_5A_{10}A_{11}^2$$

$$+kA_3A_5A_{11}A_{12}-\lambda A_6A_9A_{13}+\lambda A_3A_{10}A_{11}A_{13}+\lambda A_3A_{12}A_{13}-qA_4A_9(kA_5A_{11}+\lambda A_{13})-A_5A_9A_{14}+\lambda A_2A_7(kA_6A_{11}+A_{14})))$$

$$-\frac{1}{A_8^2}(q(-1+r)\lambda A_4A_8A_9+(-1+r)\lambda A_6A_8A_9-(-1+r)\lambda A_3A_8A_{10}A_{11}+k(-1+r)\lambda A_2A_6A_7\big((-\beta-\gamma)A_1+kA_{11}\big)$$

$$+ktA_5A_8\big((-\beta-\gamma)A_1+kA_{11}\big)-kq(-1+r)A_4A_5A_9\big((-\beta-\gamma)A_1+kA_{11}\big)-2k(-1+r)A_5A_6A_9\big((-\beta-\gamma)A_1+kA_{11}\big)$$

$$+k(-1+r)A_3A_5A_{10}A_{11}\big((-\beta-\gamma)A_1+kA_{11}\big)-(-1+r)\lambda A_3A_8A_{12}+k(-1+r)A_3A_5\big((-\beta-\gamma)A_1+kA_{11}\big)A_{12}+kt\lambda A_8A_{13}$$

$$-kq(-1+r)\lambda A_4A_9A_{13}-k(-1+r)\lambda A_6A_9A_{13}+k(-1+r)\lambda A_3A_{10}A_{11}A_{13}+k(-1+r)\lambda A_3A_{12}A_{13}+k(-1+r)\lambda A_2A_7A_{14}$$

$$-k(-1+r)A_5A_9A_{14}-(-1+r)\lambda A_2A_7A_8\sigma_1+(-1+r)A_5A_8A_9\sigma_1)+\frac{1}{(4\beta^2-\gamma^2)A_8}(-t(\beta+\gamma)^2A_1^2A_5+t(\beta+\gamma)A_1(kA_5A_{11}+\lambda A_{13})$$

$$+A_5A_9\big(r\beta\big(-2c\beta^2+(c+t)\beta\gamma+c\gamma^2\big)+(-1+r)\big((2\beta^2-\gamma^2)\sigma_1-\beta\gamma\sigma_2\big)\big)$$

$$+\lambda A_2\begin{pmatrix}(-1+r)(\beta-\gamma)\big((qA_4+A_6)A_9-A_3(A_{10}A_{11}+A_{12})\big)\\+A_7\big(r\beta\big(-t\beta\gamma+c(\beta-\gamma)(2\beta+\gamma)\big)-(-1+r)\big((2\beta^2-\gamma^2)\sigma_1-\beta\gamma\sigma_2\big)\big)\end{pmatrix})$$

$$A_{24}=\frac{2ru^2\beta(\beta-\gamma)A_3A_{11}}{(2\beta-\gamma)A_8}$$

$$A_{25}=\frac{1}{(4\beta^2-\gamma^2)A_8^2}\beta(-(-1+r)(\beta+\gamma)^2A_1^2A_2A_3A_7+(-1+r)A_2(k(4\beta^2-\gamma^2)A_7A_8+A_3((4ru^2\beta(2\beta^2-\gamma^2)$$

$$+k^2(-4\beta^2+\gamma^2))A_7+2u^2(\beta-\gamma)A_8)A_{11})+2u^2(2\beta+\gamma)A_3A_{11}\big(r(\beta-\gamma)A_8+k(-1+r)(2\beta-\gamma)A_{13}\big)$$

$$+(-1+r)(\beta+\gamma)A_1\big(A_2A_7\big(-A_8+kA_3(4\beta^2-\gamma^2+A_{11})\big)-2u^2A_3A_{11}A_{13}\big))$$

$$A_{26}=\frac{1}{A_8^2}\left(-1+r\right)(-kA_5A_8A_9-2u^2\lambda A_3A_8A_{11}+k^2A_3A_5A_9A_{11}+2k^2u^2A_3A_5A_{11}^2+k\lambda A_2A_7\left(A_8-kA_3A_{11}\right)$$

$$+k\left(\beta+\gamma\right)A_1A_3\left(\lambda A_2A_7-A_5\left(A_9+2u^2A_{11}\right)\right)+2ku^2\lambda A_3A_{11}A_{13})$$

$$-\frac{\left(-1+r\right)\left(\left(\beta+\gamma\right)A_1\left(\lambda A_2A_7-A_5A_9\right)+2u^2\left(-\beta+\gamma\right)\lambda A_2A_3A_{11}\right)}{\left(4\beta^2-\gamma^2\right)A_8}$$

$$+\frac{1}{\left(4\beta^2-\gamma^2\right)A_8^2}\left(-1+r\right)A_3(4ru^2\beta\left(2\beta^2-\gamma^2\right)\left(\lambda A_2A_7-A_5A_9\right)A_{11}-\left(\beta+\gamma\right)^2A_1^2\left(\lambda A_2A_7-A_5\left(A_9+2u^2A_{11}\right)\right)$$

$$-\left(\beta+\gamma\right)A_1A_{11}\left(-k\lambda A_2A_7+kA_5\left(A_9+2u^2A_{11}\right)+2u^2\lambda A_{13}\right))$$

$$A_{27}=kt+2gu-\frac{t\left(\beta+\gamma\right)A_1}{4\beta^2-\gamma^2}+\frac{1}{A_8}(-k\left(-1+r\right)\left(qA_4+A_6\right)A_9+kA_3(t\left(\beta+\gamma\right)A_1-\left(kt-\left(-1+r\right)A_{10}\right)A_{11}$$

$$+\left(-1+r\right)A_{12})-2\left(-1+r\right)u^2A_3A_{11}\sigma_1)-\frac{1}{A_8^2}k\left(-1+r\right)A_3\left(\left(\left(\beta+\gamma\right)A_1-kA_{11}\right)\begin{pmatrix}qA_4A_9+A_6\left(A_9+2u^2A_{11}\right)\\-A_3\left(A_{10}A_{11}+A_{12}\right)\end{pmatrix}-2u^2A_{11}A_{14}\right)$$

$$+\frac{1}{\left(4\beta^2-\gamma^2\right)A_8}(-t\left(\beta+\gamma\right)^2A_1^2A_3+\left(\beta+\gamma\right)A_1\begin{pmatrix}q\left(-1+r\right)A_4A_9+\left(-1+r\right)A_6A_9\\+A_3\left(\left(kt-\left(-1+r\right)A_{10}\right)A_{11}-\left(-1+r\right)A_{12}\right)\end{pmatrix}$$

$$+2u^2A_3A_{11}\left(r\beta\left(-2c\beta^2+\left(c+t\right)\beta\gamma+c\gamma^2\right)+\left(-1+r\right)\left(\left(2\beta^2-\gamma^2\right)\sigma_1-\beta\gamma\sigma_2\right)\right))$$

$$-\frac{1}{\left(4\beta^2-\gamma^2\right)A_8^2}\left(-1+r\right)A_3(4ru^2\beta\left(2\beta^2-\gamma^2\right)A_{11}\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right)$$

$$-\left(\beta+\gamma\right)^2A_1^2\left(qA_4A_9+A_6\left(A_9+2u^2A_{11}\right)-A_3\left(A_{10}A_{11}+A_{12}\right)\right)$$

$$+\left(\beta+\gamma\right)A_1A_{11}\left(kqA_4A_9+kA_6\left(A_9+2u^2A_{11}\right)-kA_3\left(A_{10}A_{11}+A_{12}\right)+2u^2A_{14}\right))$$

$$A_{28}=\frac{1}{\left(4\beta^2-\gamma^2\right)A_8}(-t\left(\beta+\gamma\right)^2A_1^2A_6+g^2\left(-4\beta^2+\gamma^2\right)A_8+kt\left(\beta+\gamma\right)A_1A_6A_{11}$$

$$-r\beta\left(-t\beta\gamma+c\left(\beta-\gamma\right)\left(2\beta+\gamma\right)\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right))$$

$$-\frac{1}{\left(4\beta^2-\gamma^2\right)A_8^2}\left(-1+r\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right)(-\left(\beta+\gamma\right)^2A_1^2A_6+k\left(\beta+\gamma\right)A_1A_6A_{11}$$

$$+r\beta\left(2\beta^2-\gamma^2\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right))$$

$$+\frac{kA_6\left(\left(\beta+\gamma\right)A_1-kA_{11}\right)\left(tA_8-\left(-1+r\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right)\right)}{A_8^2}$$

$$-\frac{\left(k\left(4\beta^2-\gamma^2\right)-\left(\beta+\gamma\right)A_1\right)\left(tA_8-\left(-1+r\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right)\right)A_{14}}{\left(4\beta^2-\gamma^2\right)A_8^2}$$

$$-\frac{1}{\left(4\beta^2-\gamma^2\right)A_8}\beta(\left(-1+r\right)\left(\left(qA_4+A_6\right)A_9-A_3\left(A_{10}A_{11}+A_{12}\right)\right)\left(2\beta\sigma_1+\gamma\sigma_2\right)$$

$$+A_8\left(\left(2c\beta+\left(c+t\right)\gamma\right)\sigma_1+\left(2\left(c+t\right)\beta+c\gamma\right)\sigma_2\right))$$

$$Z_1=-2\delta+\frac{32\left(-1+r\right)ru^4\beta\lambda^2}{\left(k^2-8ru^2\beta\right)^2}\qquad Z_2=\frac{2\left(k^2-2ru^2\left(3\beta+\gamma\right)\right)\lambda}{k^2-8ru^2\beta}\qquad Z_3=\frac{2\left(k^4-2k^2ru^2\left(7\beta+\gamma\right)+16ru^4\beta\left(\beta+2r\beta+\left(-1+2r\right)\gamma\right)\right)\lambda}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_4=\frac{16k\left(-1+r\right)ru^4\beta\lambda}{\left(k^2-8ru^2\beta\right)^2}\qquad Z_5=-\frac{\lambda\begin{pmatrix}k^4t+16gk\left(-1+r\right)ru^3\beta+16r^2tu^4\beta\left(2\beta+\gamma\right)+c\left(k^2-8ru^2\beta\right)\left(k^2-2ru^2\left(3\beta+\gamma\right)\right)\\+2k^2ru^2\left(q\left(-1+r\right)\beta-t\left(6\beta+\gamma\right)\right)-16\left(-1+r\right)ru^4\beta\sigma_1\end{pmatrix}}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_6=\frac{4kru^2\left(k^2+8(1-2r)u^2\beta\right)(\beta-\gamma)}{\left(k^2-8ru^2\beta\right)^2} \qquad Z_7=\frac{4kru^2(\beta-\gamma)}{k^2-8ru^2\beta}$$

$$Z_8=-4u^2+\frac{32k^2(-1+r)ru^4\beta}{\left(k^2-8ru^2\beta\right)^2} \qquad Z_9=\frac{2u\begin{pmatrix} g\left(k^4-8k^2r(1+r)u^2\beta+64r^2u^4\beta^2\right) \\ +kru\begin{pmatrix}\beta\left(-k^2\left(c+q(-1+r)+2t\right)+8r(c+2t)u^2\beta\right) \\ +(c+t)\left(k^2-8ru^2\beta\right)\gamma\end{pmatrix}+8k(-1+r)ru^3\beta\sigma_1\end{pmatrix}}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_{10}=\frac{16k(-1+r)ru^4\beta\lambda}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_{11}=-\frac{4(\beta-\gamma)\left(k^4-2k^2ru^2(7\beta+\gamma)+8ru^4\beta\left(\beta+5r\beta+(-1+3r)\gamma\right)\right)}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_{12}=-\frac{2(\beta-\gamma)\left(k^2-2ru^2(3\beta+\gamma)\right)}{k^2-8ru^2\beta}$$

$$Z_{13}=\frac{2\left(k^4-2k^2ru^2(7\beta+\gamma)+16ru^4\beta\left(\beta+2r\beta+(-1+2r)\gamma\right)\right)\lambda}{\left(k^2-8ru^2\beta\right)^2}$$

$$Z_{14}=\frac{4kru^2\left(k^2+8(1-2r)u^2\beta\right)(\beta-\gamma)}{\left(k^2-8ru^2\beta\right)^2}$$

$$\begin{aligned}Z_{15}=&\frac{1}{2\left(k^2-8ru^2\beta\right)^2}((\beta-\gamma)(k^4(-qr+2t)-4gk^3ru+32gkr(-1+2r)u^3\beta+32r^2u^4\beta\left(-qr\beta+t(2\beta+\gamma)\right)\\&+2c\left(k^2-8ru^2\beta\right)\left(k^2-2ru^2(3\beta+\gamma)\right)+4k^2ru^2\left(q(-1+4r)\beta-t(6\beta+\gamma)\right))\\&+\left(k^4-4k^2ru^2(3\beta+\gamma)+32ru^4\beta\left(\beta+(-1+2r)\gamma\right)\right)\sigma_1+\left(k^2-8ru^2\beta\right)^2\sigma_2)\end{aligned}$$