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RESEARCH PAPER

On the Monitoring of AR(1) Auto-Correlated Simple Linear Profiles in Multistage Processes

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Abstract

If the quality of a process is described using a linear functional relationship between the response variable and independent variables, such a relationship is called the profile. Today, with the development of manufacturing technologies, multistage processes have found a special position in manufacturing companies and industries. In the present paper, we consider a multistage process with AR(1) auto-correlated simple linear profile in each stage and address the effect of both auto-correlation and cascade property on the efficiency of common monitoring procedures. To eliminate the effect of auto-correlation, we used a transformation method as a remedial measure at first. Then, an approach based on the **U** statistic is applied to eliminate the cascade effect. Next, a modified T^2 control chart is proposed to monitor the process in the second stage. The performance of the proposed control chart is evaluated in terms of the average run length criterion. The simulation studies show that the proposed control chart perform satisfactorily.

Keywords:
Auto-Correlation;
Cascade Property;
Phase II;
Average Run Length

Introduction

Depending on the regression relationship between the response and independent variables, the profiles are divided into various models such as linear, multiple linear, polynomial and nonlinear models. Generally, simple linear profiles have many applications in the manufacturing industries. In the practical cases, Mestek et al. [1], Kang and Albin [2], Mahmoud and Woodall [3] applied linear profiles to study the stability of calibration tools.

In certain cases that the independence assumption of residuals is violated, some model-fitting results may be questionable. Generally, time series models are applied to display the type of dependent relationship between the residuals in the auto-correlated profiles [4]. Amiri et al. [5] introduced a real case from the automotive industry, which can be modeled by a polynomial profile and the first order autoregressive model (AR(1)) correlation structure in the errors. The idea of eliminating the impact of auto-correlation on the monitoring of auto-correlated profiles caused that Jensen et al. [6], Soleimani et al. [7, 8, 9], Koosha and Amiri [10] and Keramatpour et al. [11] present new methods for Phase I and Phase II monitoring of the auto-correlated profiles. Amiri et al. [12] developed a self-starting control chart to monitor the AR(1) auto-correlated simple linear profiles. Wang and Huang [13] used two charts to monitor the auto-correlated linear profiles modeled by the AR (1) time series.

Due to the development of manufacturing technologies, most products and services today are the results of several process stages and steps. Therefore, the products are produced in two

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or more successive manufacturing stages and the quality characteristics are determined by one or more profile relationships. Thus, several methods have been proposed to monitor multi-stage processes with profile quality characteristics [14].

In multi-stage processes, the cascade property usually exists in different stages of a multistage process, where the quality of a stage influences the performance of the next stage. With this in mind, lack of attention to which will lead to wrong results when the control charts are applied in multi-stage processes. Accordingly, in relation to cascade property, Zhang [15] applied a cause-selecting control (CSC) chart to monitor such processes, which later developed this chart in other research. Asadzadeh et al. [16, 17, 18] studied the monitoring of multi-stage processes using cause-selecting control (CSC) charts. Using Hawkins' studies [19], Hauck et al. [20] presented a method based on the **U** statistic for monitoring multi-stage processes in the presence of cascade property. Esmaili et al. [21] considered a two-stage process with a normal quality characteristic in the first stage and a simple linear regression profile in the second stage and proposed two methods to monitor the quality characteristics in both stages. Eghbali et al. [22] and Khedmati and Niaki [23] presented a single max-EWMA-3 control statistic for monitoring all parameters of linear profiles in each stage of multi-stage processes taking into account the cascade effect in Phase II monitoring of these profiles. Ayad and Sibanda [24] proposed a new multi-stage multivariate control chart based on likelihood score equations to monitor the outcomes of health care procedures. Derakhshani et al. [25] developed four control charts for monitoring Poisson regression profiles in multi-stage processes in Phase II.

When the time interval between the samples is short, the collected observations are auto-correlated. Therefore, the lack of attention to auto-correlation within the profiles of multi-stage processes will result in the poor performance of existing control charts for monitoring these types of processes in terms of ARL and consequently the occurrence of false out-of-control signals for control charts.

Thus, in this paper Phase II monitoring of auto-correlated simple linear profile (SLP) in multi-stage processes is addressed. To that end, the 1^{st} order autoregressive model as a justified and broadly used auto-correlation model in manufacturing processes is applied to model the relationship between stages. Hence, it is assumed that profile error terms can be modeled according to AR(1) model and there is no correlation between SLPs. Also, we suppose that the parameters are known and we monitor the process in Phase II. In this study, first, the cascade effect on the Phase II monitoring of auto-correlated simple linear profiles in a two-stage process has been shown. Then, to monitor the auto-correlated simple linear profile in each stage, a transformation method is applied to eliminate the impact of auto-correlation within simple linear profiles and a remedial measure, which is the U statistic proposed by Hauck et al. [20], is applied to eliminate the effect of cascade property between stages. Next, a control chart for monitoring auto-correlated simple linear profiles is discussed and the performance is evaluated via the average run length (ARL) criterion. The remainder of this study is organized as follows: Assumptions and modeling of the problem are presented in Section 2. Using simulation studies, the impact of auto-correlation and cascade property on the performance of auto-correlated simple linear profiles in a multi-stage process has been presented in Section 3. The transformation method, remedial measure and monitoring method is discussed in Section 4. Extensive simulation studies are conducted to evaluate the performance of the proposed method in reduction of both effects of the auto-correlation and cascade property in Section 5. Managerial Insights are presented in Section 6. Finally, the concluding remarks and future research directions are presented in Section 7.

Assumptions and modeling the problem

The model is an auto-correlated simple linear profile in a two-stage process, taking into account the presence of cascade property between the stages. Suppose that in the s^{th} stage for sample j, the observations (x_{is}, y_{is}) for the s^{th} stage is collected over time, where $i = 1,2,...,n; j = 1,2,...,m; s = 1,2.$ Furthermore, under in-control statistical conditions for the first stage of the process, the relationship between the response variable, and the independent variable, as well as the residuals, are defined as Eq. 1.

$$y_{\mathrm{ij1}} = \beta_{01} + \beta_{11} x_{\mathrm{i1}} + \varepsilon_{\mathrm{ij1}}, \tag{1}$$

$$\varepsilon_{\mathrm{ij1}} = \rho_1 \varepsilon_{\mathrm{(i\text{-}1)j1}} + a_{\mathrm{ij1}}.$$

Moreover, the relationship between the response variable and the independent variable, and the residuals of the second stage of the process are defined as Eq. 2.

$$y_{\mathrm{ij2}} = \phi y_{\mathrm{ij1}} \alpha_{02} + \alpha_{12} x_{\mathrm{i2}} + \varepsilon_{\mathrm{ij2}}, \tag{2}$$

$$\varepsilon_{\mathrm{ij2}} = \rho_2 \varepsilon_{\mathrm{(i\text{-}1)j2}} + a_{\mathrm{ij2}}.$$

In these two models, it has been assumed that j^{th} sample results from the collection of n observations over time, where y_{ij} is the quality characteristic of the j^{th} sample of the first stage of the process, y_{ij} is the quality characteristic of the j^{th} sample of the s^{th} stage, ρ_1 is the auto-correlation coefficient between the residuals of the profile of the first stage of the process, and ρ_s is the auto-correlation coefficient between the residuals of the profile of the s^{th} stage and ϕ is the auto-correlation parameter between the process stages. Moreover, β_{01} and β_{02} denote the intercept and the slope of the simple linear profile in Eq. 1, respectively. The special impact of stage s on the intercept and the slope are denoted by α_{0s} and α_{1s}, respectively. Furthermore, a_{ij1} and a_{ij2} are independent and identically distributed (iid) normal random variables with mean zero and the variance σ^2. It has been also supposed that the values of x_i are constant and identical in the profiles. In the next section, the impact of cascade property and auto-correlation on the sufficiency of control chart for monitoring the auto-correlated simple linear profiles in the two-stage process is investigated.

The impact of cascade property and auto-correlation on the performance of traditional control scheme for monitoring auto-correlated SLPs in multi-stage processes

To investigate the impact of the cascade property between the two stages, the first stage profile is defined as $y_{ij1} = 3 + 2x_i + \varepsilon_{ij1}$, $(\alpha_{01} = 3, \alpha_{11} = 2)$ and the second stage profile is defined as $y_{ij2} = 2 + x_i + \phi\varepsilon_{ij1} + \varepsilon_{ij2}$, where the variable x_i takes the values 2, 4, 6, and 8. Note that by substituting $\varepsilon_{ij1} = y_{ij1} - 3 - 2x_i$ in $y_{ij2} = 2 + x_i + \phi\varepsilon_{ij1} + \varepsilon_{ij2}$, we have $y_{ij2} = (2 - 3\phi) + (1 - 2\phi)x_i + \phi\varepsilon_{ij1} + \varepsilon_{ij2}$, where $\alpha_{02} = 2 - \alpha_{01}$ and $\alpha_{12} = 1 - \alpha_{11}$, which shows the relationship between the intercept and slope of the profiles in the both stages. Auto-correlated error terms are $\varepsilon_{ij1} = \rho\varepsilon_{(i-1)j1} + a_{ij1}$ and $\varepsilon_{ij2} = \rho\varepsilon_{(i-1)j2} + a_{ij2}$ that are modeled by the first-order autoregressive model (AR(1)), where a_{ij1} and a_{ij2} are independently and identically distributed normal random variables with mean zero and variance σ^2. In this section, the performance of T^2 chart [introduced by Kang and Albin [2]] has been compared under the simultaneous impact of the cascade property and auto-correlation within the profiles based on the ARL criterion. For this purpose, a simulation with a 10,000 times repeat has been applied. The upper control limit of the T^2 chart in the second stage is set to $\chi^2_{0.005,2}$ in order to achieve an in-control ARL of 200. To examine the simultaneous impact of the cascade property and auto-correlation parameter within each profile, the simulation has been done for different values of the cascade property $\phi = [0, 0.1, 0.5, 0.9]$, and different values of the auto-correlation parameter within each profile as $\rho = [0, 0.1, 0.5, 0.9]$ and the ARL values of T^2 chart in the 2nd stage under various values of a shift in the intercept and the standard deviation (SD) of both stages are reported in Tables 1-4. Also, the standard error of ARL for different conditions is given below each ARL. Note that, the results of simulation for shifts in the slope are similar to the intercept shifts. Therefore, we ignore reporting the results related to shifts in the slope.

Table 1. The impact of ϕ and ρ on ARL(Standard error) of the traditional T^2 control chart when β_{011} shifts to $\beta_{011} + \lambda\sigma$.

ρ	ϕ	Chart	λ 0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0	0	T^2	200 (1.94)	199.5 (1.90)	200.1 (1.89)	200 (1.92)	200.1 (1.87)	198.9 (1.74)	200.8 (1.64)	200.01 (1.66)	199.7 (1.54)	200 (1.44)	200 (1.33)
	0.1	T^2	200 (1.95)	199.2 (2.01)	193.4 (2)	186.1 (2.06)	180.2 (2)	176.2 (0.55)	172.2 (0.38)	160.4 (0.27)	152.4 (0.20)	142.4 (0.14)	134.8 (0.10)
	0.5	T^2	200 (0.37)	189.8 (0.24)	153.1 (0.14)	111.7 (0.13)	79.4 (0.75)	53.3 (0.57)	37.1 (0.37)	25.6 (0.37)	18.3 (1.81)	13.1 (1.86)	9.7 (1.79)
	0.9	T^2	200 (1.84)	171.2 (1.77)	111.5 (1.69)	64.8 (1.75)	37.3 (1.90)	22.38 (1.95)	13.31 (1.87)	8.52 (1.92)	5.64 (2.03)	4.03 (2)	2.95 (2.06)
0.1	0.1	T^2	200 (1.45)	200 (1.30)	198.35 (1.09)	196.42 (1.11)	192.25 (1.69)	190.35 (1.64)	188.7 (1.53)	184.1 (1.49)	172.8 (1.97)	163.8 (2.05)	155 (2)
	0.5	T^2	200 (1.04)	186.5 (0.84)	149.2 (0.65)	110.9 (0.65)	78.8 (1.45)	56.2 (1.31)	39.2 (1.12)	27.6 (1.10)	19.9 (1.91)	15.2 (1.96)	10.6 (1.94)
	0.9	T^2	200 (0.73)	165.73 (0.54)	109.5 (0.38)	65.9 (0.37)	38.9 (1.20)	23.1 (1)	14.5 (0.78)	9.6 (0.76)	6.4 (1.93)	4.4 (1.91)	3.4 (1.90)
0.5	0.1	T^2	200 (0.51)	199.8 (0.35)	197.25 (0.23)	196.74 (0.21)	192.4 (0.94)	186.8 (0.74)	181.8 (0.56)	174.5 (0.53)	167.1 (1.86)	163.8 (1.92)	155.6 (1.84)
	0.5	T^2	200 (0.37)	190.4 (0.24)	168.8 (0.14)	131.1 (0.13)	100.5 (0.75)	74.5 (0.57)	57.2 (0.37)	42.8 (0.37)	31.1 (1.81)	24.3 (1.86)	18.9 (1.79)
	0.9	T^2	200 (0.26)	177.9 (0.16)	129.9 (0.09)	86.9 (0.08)	53.9 (0.59)	37.2 (0.42)	24.1 (0.26)	16.7 (0.25)	11.5 (1.76)	8.7 (1.77)	6.2 (1.69)
0.9	0.1	T^2	200 (0.19)	189.4 (0.11)	189.2 (0.06)	189.5 (0.05)	184.3 (0.47)	175.3 (0.32)	182.6 (0.20)	175.3 (0.18)	166.2 (1.77)	168.2 (1.73)	160.9 (1.58)
	0.5	T^2	200 (0.13)	194.8 (0.08)	172.9 (0.04)	143.8 (0.03)	121.3 (0.38)	94.8 (0.23)	74.1 (0.14)	58.5 (0.12)	46.3 (1.65)	37.1 (1.63)	29.6 (1.53)
	0.9	T^2	200 (0.10)	183.1 (0.06)	143.9 (0.03)	104.3 (0.02)	74.2 (0.29)	51.8 (0.18)	36.5 (0.11)	25.7 (0.09)	19.3 (1.64)	14.9 (1.58)	10.9 (1.42)

Impact of cascade property and auto-correlation on ARL performance of T^2 chart in the second stage at various shifts in the intercept of the first stage from β_{01} to $\beta_{02} + \lambda\sigma$ is evaluated through 10,000 simulation runs and the results are summarized in Table 1. Based on the outcomes, by increasing the values of the auto-correlation parameters, the in-control ARL of the T^2 control method in the second stage significantly decreases. In addition, when there is no correlation between two stages $(\phi = 0)$, the shifts in the intercept of the first stage do not affect the ARL of the T^2 chart in stage 2.

Table 2. The impact of ϕ and ρ on ARL(Standard error) of the traditional T^2 control chart when β_{011} shifts to $\beta_{011} + \lambda\sigma$ and β_{02} shifts to $\beta_{02} + \lambda\sigma$

ρ	ϕ	Chart	λ: 0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0	0	T^2	200 (2.03)	137.4 (2.05)	63.6 (2.02)	27.6 (2.01)	13.2 (1.94)	6.9 (1.82)	4.0 (1.84)	2.6 (1.71)	1.9 (1.68)	1.4 (1.59)	1.2 (1.54)
	0.1	T^2	200 (2.45)	127.1 (1.55)	53.7 (1.05)	22.1 (0.66)	9.7 (0.46)	5.3 (0.32)	3.1 (0.23)	2.0 (0.17)	1.5 (0.12)	1.3 (0.09)	1.1 (0.05)
	0.5	T^2	200 (2.44)	107.6 (1.54)	37.0 (0.99)	13.2 (0.55)	5.6 (0.36)	2.9 (0.24)	1.8 (0.16)	1.3 (0.12)	1.1 (0.08)	1.0 (0.06)	1.0 (0.03)
	0.9	T^2	200 (2.39)	104.2 (1.41)	32.4 (0.87)	11.3 (0.44)	4.7 (0.27)	2.5 (0.17)	1.6 (0.11)	1.3 (0.08)	1.1 (0.06)	1.0 (0.04)	1.0 (0.01)
0.1	0.1	T^2	200 (2.47)	147.3 (1.52)	62.4 (0.89)	26.6 (0.44)	12.1 (0.27)	6.4 (0.16)	3.7 (0.11)	2.4 (0.07)	1.8 (0.05)	1.4 (0.04)	1.2 (0.01)
	0.5	T^2	200 (2.49)	109.9 (1.96)	40.3 (1.53)	14.6 (1.08)	6.4 (0.83)	3.4 (0.66)	2.1 (0.51)	1.5 (0.41)	1.2 (0.32)	1.1 (0.25)	1.1 (0.14)
	0.9	T^2	200 (2.53)	102.3 (2)	34.9 (1.53)	12.7 (0.98)	5.5 (0.74)	2.9 (0.55)	1.8 (0.41)	1.4 (0.31)	1.1 (0.24)	1.0 (0.18)	1.0 (0.09)
0.5	0.1	T^2	200 (2.43)	148.2 (1.91)	75.7 (1.39)	37.4 (0.85)	19.7 (0.60)	10.9 (0.44)	6.4 (0.31)	4.3 (0.23)	2.9 (0.17)	2.1 (0.14)	1.7 (0.05)
	0.5	T^2	200 (2.50)	129.8 (1.92)	56.1 (1.41)	24.5 (0.86)	11.6 (0.60)	6.3 (0.43)	3.7 (0.31)	2.5 (0.22)	1.8 (0.17)	1.4 (0.13)	1.2 (0.05)
	0.9	T^2	200 (2.41)	126.5 (2.39)	50.9 (2.31)	20.9 (2.27)	10.2 (2.17)	5.3 (2.12)	3.2 (2.06)	2.2 (1.99)	1.6 (1.90)	1.3 (1.84)	1.2 (0.81)
0.9	0.1	T^2	200 (2.47)	151.9 (2.43)	93.6 (2.36)	52.2 (2.31)	29.8 (2.24)	17.7 (2.13)	11.2 (2.07)	7.5 (1.95)	5.1 (1.92)	3.5 (1.77)	2.7 (0.77)
	0.5	T^2	200 (2.42)	147.5 (2.42)	73.9 (2.41)	37.4 (2.24)	18.5 (2.18)	10.8 (2.14)	6.5 (2.03)	4.2 (1.96)	2.9 (1.85)	2.2 (1.76)	1.7 (0.71)
	0.9	T^2	200 (2.34)	136.3 (2.36)	67.1 (2.34)	33.4 (2.21)	16.8 (2.13)	9.4 (2.06)	5.5 (1.95)	3.7 (1.85)	2.6 (1.81)	1.9 (1.70)	1.5 (0.70)

The ARLs of the T^2 control chart in stage 2 under simultaneous shifts in the intercept of both stages are presented in Table 2. The outcomes show that when the values of ϕ and ρ parameters increase, the in-control Average Run Length efficiency of the control chart decreases. Moreover, obtained results for shifts in the error standard deviation are the same.

The impact of cascade property and auto-correlation on ARLs of the control method for various levels of the shift sizes in the error SD in the first stage is shown in Table 3 in terms of ARL. The results indicate that the in-control performance of the traditional T^2 control chart deteriorates.

Table 3. The impact of ϕ and ρ on ARL(Standard error) of the traditional T^2 chart when σ_{01} shifts to $\gamma\sigma_{01}$

ρ	ϕ	Chart	γ: 1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0	0	T^2	200 (1.94)	200.1 (2)	201.0 (2.02)	200.5 (2.10)	200.6 (2.03)	201.1 (2.08)	200.5 (2.02)	200.0 (2.05)	200.8 (1.97)	200.0 (2.03)	200.0 (2.03)
	0.1	T^2	200 (1.97)	189.5 (1.99)	187.5 (1.96)	181.1 (2.14)	177.1 (1.98)	172.1 (2.09)	162.1 (2.03)	155.1 (2.04)	149.1 (1.96)	135.6 (2.04)	135.6 (2.04)
	0.5	T^2	200 (2.01)	136.5 (2.01)	89.15 (1.90)	57.90 (2.03)	40.11 (1.95)	28.69 (2.11)	19.65 (2.06)	15.31 (2.09)	11.84 (1.93)	7.58 (2.07)	7.58 (2.07)
	0.9	T^2	200 (2.05)	84.45 (2.05)	41.06 (1.95)	22.87 (2.06)	14.12 (2.02)	9.68 (2.02)	7.13 (2)	5.55 (2.10)	4.30 (2.01)	3.26 (2.04)	3.26 (2.04)

0.1	0.1	T^2	200 (1.94)	217.8 (2)	208.5 (2.02)	203.9 (2.10)	193.3 (2.03)	192.5 (2.08)	180.9 (2.02)	174.7 (2.05)	165.2 (1.97)	150.8 (2.03)	150.8 (2.03)
	0.5	T^2	200 (1.98)	131.7 (1.98)	85.74 (1.98)	57.01 (2.10)	37.84 (2.02)	27.20 (2.05)	20.64 (2.01)	15.37 (2.07)	11.80 (1.97)	7.79 (2.07)	7.79 (2.07)
	0.9	T^2	200 (1.97)	83.25 (1.99)	39.45 (1.96)	22.41 (2.14)	14.18 (1.98)	9.60 (2.09)	6.88 (2.03)	5.50 (2.04)	4.43 (1.96)	3.24 (2.04)	3.24 (2.04)
0.5	0.1	T^2	200 (1.94)	198.7 (2.04)	195.6 (1.91)	188.4 (2.04)	183.3 (2.02)	176.2 (2.05)	167.3 (1.99)	159.7 (2.03)	154.5 (1.97)	136.2 (2.01)	136.2 (2.01)
	0.5	T^2	200 (1.99)	138.7 (2.04)	90.05 (1.93)	60.03 (2.07)	41 (1.99)	28.55 (2.04)	21.56 (2.07)	16.13 (2.08)	12.64 (1.94)	8.39 (2.06)	8.39 (2.06)
	0.9	T^2	200 (1.96)	87.69 (2.05)	43.18 (2)	23.72 (2.07)	15 (1.98)	10.03 (2.08)	7.37 (2.03)	5.63 (2.08)	4.54 (2)	3.32 (2.05)	3.32 (2.05)
0.9	0.1	T^2	200 (1.96)	198.7 (2.04)	195.6 (1.92)	188.3 (2.09)	183.2 (2.03)	176.2 (2.06)	167.3 (2)	159.7 (1.99)	154.5 (2.03)	134.8 (2.03)	134.8 (2.03)
	0.5	T^2	200 (2)	135.9 (2.05)	89.98 (1.98)	61.99 (2.11)	41.70 (2.02)	30.41 (2.03)	21.88 (2.05)	16.67 (2.07)	13.26 (1.91)	8.27 (2)	8.27 (2)
	0.9	T^2	200 (2)	89.25 (1.99)	43.76 (1.96)	24.01 (2.07)	15.42 (1.98)	10.65 (2.06)	7.71 (2.03)	5.94 (2.01)	4.79 (1.94)	3.35 (2.01)	3.35 (2.01)

Table 4. The impact of ϕ and ρ on ARL(Standard error) of the traditional T^2 chart when σ_{01} shifts to $\gamma\sigma_{01}$ and σ_{02} shifts to $\gamma\sigma_{02}$

ρ	ϕ	Chart	γ = 1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
0	0	T^2	200 (0.003)	40.0 (0.002)	15.2 (0.002)	7.8 (0.002)	5.1 (0.003)	3.8 (0.003)	3.0 (0.002)	2.5 (0.002)	2.2 (0.002)	1.9 (0.002)	1.8 (0.003)
	0.1	T^2	200 (0)	39.2 (1.971)	14.9 (0.032)	7.8 (0.012)	5.1 (0.007)	3.8 (0.005)	2.9 (0.004)	2.5 (0.003)	2.2 (0.003)	2.0 (0.003)	1.8 (0.002)
	0.5	T^2	200 (1.995)	40.8 (2.019)	15.2 (2.016)	7.8 (2.064)	5.3 (1.971)	3.8 (2.031)	3.1 (2.013)	2.6 (2.098)	2.2 (2)	2.0 (1.999)	1.8 (2.009)
	0.9	T^2	200 (0.035)	40.6 (0.033)	15.3 (0.032)	8.0 (0.033)	5.1 (0.036)	3.8 (0.034)	3.0 (0.032)	2.5 (0.032)	2.2 (0.035)	1.9 (0.034)	1.8 (0.032)
0.1	0.1	T^2	200 (0.013)	43.0 (0.013)	15.9 (0.012)	8.4 (0.012)	5.5 (0.013)	3.9 (0.013)	3.1 (0.012)	2.5 (0.012)	2.2 (0.013)	2.0 (0.012)	1.8 (0.012)
	0.5	T^2	200 (0.008)	39.7 (0.008)	14.8 (0.007)	8.1 (0.008)	5.1 (0.008)	3.8 (0.008)	3.0 (0.007)	2.5 (0.007)	2.2 (0.008)	2.0 (0.008)	1.8 (0.007)
	0.9	T^2	200 (0.006)	38.8 (0.005)	14.8 (0.005)	7.9 (0.005)	5.1 (0.006)	3.7 (0.006)	3.0 (0.005)	2.5 (0.006)	2.2 (0.006)	1.9 (0.006)	1.8 (0.005)
0.5	0.1	T^2	200 (0.004)	42.3 (0.004)	15.8 (0.004)	8.4 (0.004)	5.3 (0.005)	3.9 (0.004)	3.1 (0.004)	2.6 (0.004)	2.2 (0.004)	2.0 (0.004)	1.8 (0.004)
	0.5	T^2	200 (0.004)	43.2 (0.004)	16.1 (0.004)	8.6 (0.004)	5.5 (0.004)	3.9 (0.004)	3.1 (0.004)	2.6 (0.004)	2.2 (0.004)	2.0 (0.004)	1.9 (0.004)
	0.9	T^2	200 (0.003)	41.6 (0.003)	15.9 (0.003)	8.3 (0.003)	5.4 (0.003)	3.9 (0.003)	3.1 (0.003)	2.6 (0.003)	2.2 (0.003)	2.0 (0.003)	1.8 (0.003)
0.9	0.1	T^2	200 (0.003)	41.7 (0.003)	16.0 (0.003)	8.4 (0.003)	5.6 (0.003)	4.0 (0.003)	3.1 (0.003)	2.6 (0.003)	2.3 (0.003)	2.0 (0.003)	1.9 (0.003)
	0.5	T^2	200 (0.003)	43.4 (0.003)	16.4 (0.002)	8.6 (0.002)	5.7 (0.003)	4.0 (0.002)	3.2 (0.002)	2.7 (0.002)	2.3 (0.002)	2.1 (0.002)	1.9 (0.002)
	0.9	T^2	200 (0.002)	41.7 (0.002)	16.5 (0.002)	8.6 (0.002)	5.6 (0.002)	4.1 (0.002)	3.2 (0.002)	2.7 (0.002)	2.3 (0.002)	2.1 (0.002)	1.9 (0.002)

In Table 4, the impact of cascade property and auto-correlation on the ARL of the traditional control method in the 2nd stage under simultaneous shifts in the error SD of stages 1 and 2 is summarized. Similar to the previous case, it should be noted that the performance of the control chart deteriorates dramatically. The impact of the cascade property and auto-correlation were the same when concurrent shifts occur in the error terms SD of the 1st and 2nd stages were examined.

The proposed method

In this segment, a transformation scheme is applied for the first-order autoregressive simple linear profiles to eliminate within-profile auto-correlation. Then, a remedial measure, which was proposed by Hauck et al. [20], is extended to remove the cascade effect involved in multistage processes. Finally, a modified control chart is extended to monitor the process in the second stage. The framework of the proposed method is shown in Fig. 1.

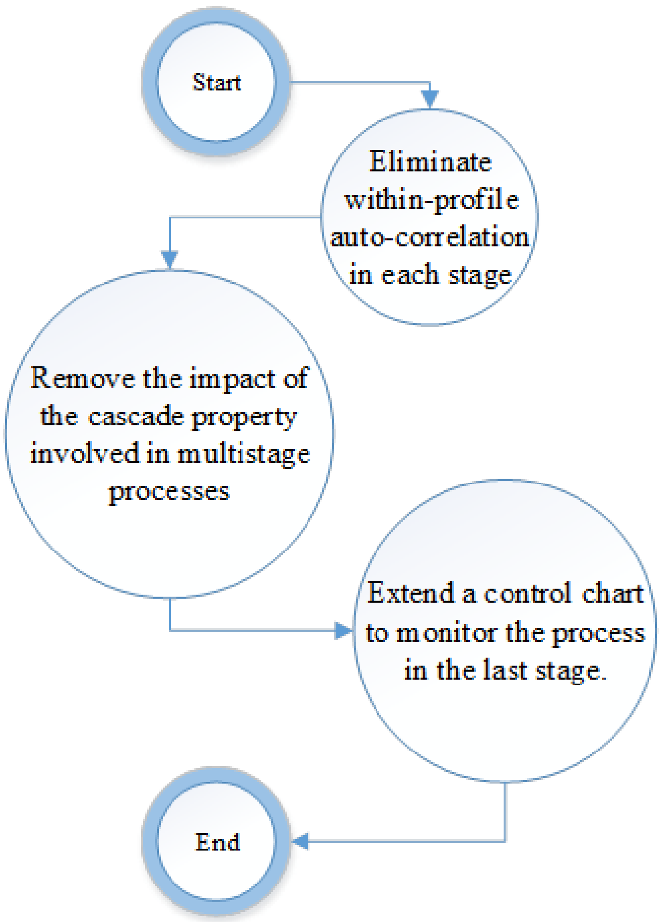

Fig. 1. Framework of the proposed method

The transformation method

The transformation of the auto-correlated observations is proposed as a first step to eliminate the within profile auto-correlation. For this purpose, each observation in each stage is converted via the following transformation scheme:

$$Y'_{ijs} = Y_{ijs} - \rho_s Y_{(i-1)js}. \tag{3}$$

If observations Y_{ijs} and $Y_{(i-1)js}$ in Eq. 3 are replaced by their values from the regression model in Eq. 1, an SLP model with independent error terms is obtained as follows:

$$Y'_{ijs} = \beta_{01s}(1-\rho) + \beta_{11s}(x_{i1s} - \rho x_{(i-1)1s}) + (\varepsilon_{ij1s} - \rho\varepsilon_{(i-1)j1s}), \tag{4}$$

leading to:

$$Y'_{ijs} = \beta'_{01s} + \beta'_{11s} x'_i + a_{ijs}, \tag{5}$$

where $Y'_{ijs} = Y_{ijs} - \rho_s Y_{(i-1)js}$, $x'_{is} = x_{is} - \rho_s x_{(i-1)s}$, $\beta'_{01s} = \beta_{01s}(1-\rho)$, $\beta'_{11s} = \beta_{11s}$ and a_{ijs} 's are independent random variables with mean zero and variance σ^2. Since we consider Phase II monitoring of simple linear profiles, the parameters β_{01s}, β_{11s} and ρ are assumed to be known. As a second step of the proposed method, a remedial measure is used to eliminate the cascade property between two stages.

Remedial measure

The $\mathbf{U}$ statistic, which was introduced by Hauck et al. [20], is used to reduce the impact of the correlation between stages. Based on the remedial measure proposed by Hauck et al. [20], for the j^{th} profile in the first stage of a multi-stage process, we have:

$$\mathbf{U}_{j1} = [\hat{\beta}'_{011}, \hat{\beta}'_{111}] \tag{6}$$

And for the s^{th} stage we have:

$$\mathbf{U}_{js} = [\hat{\beta}'_{01s}, \hat{\beta}'_{11s}] - \mathbf{\Sigma}_{s,s-1}\mathbf{\Sigma}^{-1}_{s-1,s-1}[\hat{\beta}'_{01(s-1)}, \hat{\beta}'_{11(s-1)}];\; s = 2,3,...,S \tag{7}$$

where $[\hat{\beta}'_{011}, \hat{\beta}'_{111}]$, $[\hat{\beta}'_{01(s-1)}, \hat{\beta}'_{11(s-1)}]$ and $[\hat{\beta}'_{01s}, \hat{\beta}'_{11s}]$ denote the intercept and slope vector estimators of the transformed model in stage 1, stage $(s-1)$ and stage s, sequentially. $\mathbf{\Sigma}_{s,s-1}$ is the covariance matrix of $[\hat{\beta}'_{01s}, \hat{\beta}'_{11s}]$ and $[\hat{\beta}'_{01(s-1)}, \hat{\beta}'_{11(s-1)}]$, and $\mathbf{\Sigma}_{s-1,s-1}$ is the covariance matrix of $[\hat{\beta}'_{01(s-1)}, \hat{\beta}'_{11(s-1)}]$. The above-mentioned covariance matrices are obtained by using Eqs. 8, 9, and 10, respectively:

$$\mathbf{\Sigma}_{1,1} = \begin{pmatrix} \sigma^2(\frac{1}{n-1} + \frac{\overline{\mathbf{x}'}^2}{S_{x'x'}}) & -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} \\ -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} & \frac{\sigma^2}{S_{x'x'}} \end{pmatrix}, \tag{8}$$

$$\mathbf{\Sigma}_{s,s-1} = (\phi^{2s-3} + \sum_{r=2}^{s-1} \phi^{2[(s-1)-r]+1}). \begin{pmatrix} \sigma^2(\frac{1}{n-1} + \frac{\overline{\mathbf{x}'}^2}{S_{x'x'}}) & -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} \\ -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} & \frac{\sigma^2}{S_{x'x'}} \end{pmatrix}, \tag{9}$$

$$\mathbf{\Sigma}_{s,s} = (\frac{1-\phi^{2s}}{1-\phi^2}). \begin{pmatrix} \sigma^2(\frac{1}{n-1} + \frac{\overline{\mathbf{x}'}^2}{S_{x'x'}}) & -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} \\ -\sigma^2 \frac{\overline{\mathbf{x}'}}{S_{x'x'}} & \frac{\sigma^2}{S_{x'x'}} \end{pmatrix}, \tag{10}$$

where $\mathbf{x}'$ is the vector of transformed observations, $\overline{\mathbf{x}'}$ is the mean of the transformed observations and $\mathbf{S}_{\mathbf{x}'\mathbf{x}'}$ is the sum of the squares of the difference between $\mathbf{x}'$ and $\overline{\mathbf{x}'}$.

The vector of average values and the covariance matrix of **U** in the 1st stage are presented in Eq. 11 and Eq. 12, sequentially.

$$\mu(\mathbf{U}_{j1}) = [\beta'_{011}, \beta'_{111}] \tag{11}$$

$$\mathbf{\Sigma}_{\mathbf{U}_{j1}} = \mathbf{\Sigma}_{\beta'_{011},\beta'_{111}} = \mathbf{\Sigma}_{1,1} \tag{12}$$

For stage *s*, the vector of average values and the covariance matrix of **U** are presented in Eq. 13 and Eq. 14, respectively.

$$\mu(\mathbf{U}_{js}) = [\beta'_{01s}, \beta'_{11s}] - \mathbf{\Sigma}_{s,s-1}\mathbf{\Sigma}^{-1}_{s-1,s-1}\mu(\mathbf{U}_{j(s-1)}) \tag{13}$$

$$\mathbf{\Sigma}_{\mathbf{U}_{js}} = \mathbf{\Sigma}_{s,s} - \mathbf{\Sigma}_{s,s-1}\mathbf{\Sigma}^{-1}_{s-1,s-1}\mathbf{\Sigma}_{s-1} \tag{14}$$

The modified T^2 method

This control scheme is based on the T^2 method, which was suggested by Kang and Albin [2]. In this control chart, the intercept and slope parameters of the 1st stage and s^{th} stages in the traditional model are changed by the transformed ones in Eq. 5 and the **U** statistic of the j^{th} profile is attained by using Eq. 6 for the first stage and Eq. 7 for the s^{th} stage. Then, the improved T^2 statistic for each stage is as Eq. 15:

$$\mathrm{T}^2_{\mathbf{U}_{js}} = (\mathbf{U}_{js} - \mu(\mathbf{U}_s))\mathbf{\Sigma}^{-1}_{\mathbf{U}_s}(\mathbf{U}_{js} - \mu(\mathbf{U}_s))^T, \tag{15}$$

Where $\mathbf{U}_{js}$ is the remedial measure of the intercept and slope vector estimators of the transformed model in stage *s*, $\mathbf{\Sigma}^{-1}_{\mathbf{U}_s}$ is the covariance matrix of $\mathbf{U}_j$ in the s^{th} stage and $\mu(\mathbf{U}_s)$ is the mean vector of $\mathbf{U}_j$ in the s^{th} stage.

Performance evaluation

In this section, similar to the example of Section 3, an example is considered to evaluate the suggested method for monitoring the 2nd stage of a two-stage process with AR(1) auto-correlated simple linear profile in each stage. The simulation has been done for different values of $\phi = [0, 0.1, 0.9]$ and different values of $\rho = [0, 0.1, 0.9]$. To achieve an in-control ARL of approximately 200 for T^2 chart in the second stage, the upper control limit is set to $\chi^2_{0.005,2}$. The ARL performance of the proposed control chart under different shifts in the regression parameters of the first stage and the second stage are summarized in Tables 5–9.

Tables 5 and 6 contain the ARL values of the proposed chart in the 2nd stage under various values of shifts in the intercept of profiles in both stages, sequentially. Based on the outcomes, by applying the modified method the performance of the second stage chart are unaffected by the shifts in the intercept of stage 1. Table 7 contains the ARL values of the proposed control chart in the 2nd stage under concurrent shifts in the intercepts of profiles in both stages. The results show that the control chart performs satisfactorily.

Table 5. ARL performance of the proposed control chart for monitoring the second stage when β_{011} shifts to $\beta_{011}+\lambda\sigma$

ρ	ϕ	Chart	λ										
			0	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
0.9	0.9	Modified T^2	199.9	200.56	198.9	199.2	200.0	200.1	199.7	197.9	199.9	200.8	199.8
0.1		Modified T^2	200.0	200.02	200.1	199.7	197.9	200.0	198.9	199.0	197.9	200.0	197.9
0.9	0.1	Modified T^2	200.1	200.03	199.9	199.9	199.9	200.0	200.6	198.9	199.2	200.0	199.9
0.1		Modified T^2	199.9	200.01	199.9	199.7	197.9	199.9	200.8	199.8	199.0	199.9	199.7

Table 6. ARL performance of the proposed chart for monitoring the second stage when β_{012} shifts to $\beta_{012}+\lambda\sigma$

ρ	ϕ	Chart	λ										
			0	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
0.9	0.9	Modified T^2	200.95	158.61	94.35	45.727	25.23	14.78	8.344	5.24	3.719	2.50	1.987
0.1		Modified T^2	200.01	199.42	198.5	196.89	195.6	193.2	190.2	189.7	188.0	186.3	184.6
0.9	0.1	Modified T^2	200.05	187.42	134.6	105.24	68.69	46.48	32.14	21.07	14.16	10.48	7.468
0.1		Modified T^2	200.05	200.1	199.4	197.87	195.2	193.9	192.2	191.1	190.9	190.9	190.9

Table 7. ARL performance of the proposed chart for monitoring the second stage when β_{011} shifts to $\beta_{011}+\lambda\sigma$ and β_{012} shifts to $\beta_{012}+\lambda\sigma$

ρ	ϕ	Chart	λ										
			0	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
0.1	0.1	Modified T^2	200.0	165.55	97.929	48.275	25.34	13.914	8.739	5.154	3.557	2.455	1.98
0.9		Modified T^2	200.0	199.85	198.25	197.65	196.9	196.02	190.5	188.9	185.7	183.8	182.0
0.1	0.9	Modified T^2	201.3	197.57	152.04	111.07	69.33	45.236	33.19	21.51	15.82	10.52	7.5
0.9		Modified T^2	200.0	199.42	198.52	196.89	195.6	193.20	190.2	189.7	188.0	186.3	184.7

Table 8 contains the ARL values of the control chart in the second stage under simultaneous shifts in the error standard deviations of profiles in both stages. According to the results, the performance of the proposed control chart under both strong and weak auto-correlation coefficients is reasonable. The results of Table 9 show the ARL values of the proposed control chart in the second stage at various values of shifts in the error SD term in the 2^{nd} stage. These outcomes show that the modified control method performs suitable for all shifts and both auto-correlation coefficients. In addition, according to the results (not shown here) for the changes in error standard deviations of stage 1, the monitoring method is passable and is not affected by the correlation between the stages.

Table 8. ARL performance of the proposed chart for monitoring the second stage when σ_{01} shifts to $\gamma\sigma_{01}$ and σ_{02} shifts to $\gamma\sigma_{02}$

ρ	ϕ	**Chart**	γ										
			1	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
0.1	0.1	Modified T^2	200	39.553	14.707	7.399	5.124	3.769	2.963	2.503	2.22	1.953	1.804
0.9	0.1	Modified T^2	201.02	50.355	19.41	10.255	6.497	4.655	3.486	2.996	2.63	2.254	1.983
0.1	0.9	Modified T^2	200.07	41.78	15.34	8.22	5.39	3.822	3.039	2.583	2.294	1.898	1.893
0.9	0.9	Modified T^2	199.95	45.898	18.337	10.01	6.713	4.76	3.635	2.99	2.51	2.166	1.981

Table 9. ARL performance of the proposed chart for monitoring the second stage when σ_{02} shifts to $\gamma\sigma_{02}$

ρ	ϕ	**Chart**	γ										
			1	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
0.1	0.1	Modified T^2	199.93	40.946	15.224	8.27	5.386	3.866	3.033	2.512	2.189	1.985	1.827
0.9	0.1	Modified T^2	200.07	50.32	19.087	10.712	6.712	4.72	3.878	3.035	2.618	2.381	2.17
0.1	0.9	Modified T^2	199.56	92.1	48.161	27.552	17.049	11.57	8.893	6.417	5.094	4.241	3.652
0.9	0.9	Modified T^2	199.98	95.65	53.899	31.799	20.955	13.99	10.29	8.15	6.25	5.31	4.38

Managerial insights

1. If the quality characteristic is an auto-correlated simple linear profile, the auto-correlation effect first should be removed for monitoring. Because auto-correlation has a negative effect on the statistical performance of control charts.
2. The cascade property exists in different stages of a multistage process, where the quality of a stage influences the quality of the next stages. Due to the cascade property, when a control chart in each stage signals and shows an out-of-control situation, it is not clear whether the problem is from that stage or has been transferred from the previous stages. Therefore, in order to correctly interpret the control chart signal and eliminate the assignable causes, first, the cascade effect must be removed, then the relevant control chart should be developed.
3. Since simple linear profile monitoring requires monitoring the *Y*-intercept and the slope of the regression line, a multivariate control chart should be used after eliminating both effects of auto-correlation and cascade property. Because the *Y*-intercept and the slope estimators are correlated and must be monitored simultaneously. Using Hotelling's T^2 control chart due to its quadratic structure and using a variance-covariance matrix, can be a suitable choice for monitoring error of SD as well as the Y-intercept and the slope of a simple linear profile.

Conclusion and future research

In this study, a control method was introduced for Phase II monitoring of auto-correlated simple linear profile in a multi-stage process. The results showed that both auto-correlation and cascade property affect the efficiency of the traditional control chart, which is offered for monitoring SLPs in a multi-stage process. To eliminate this impact, a transformation method on the Y-values at the first step of the proposed method was applied. At the second step, the cascade effect of multi-stage processes was eliminated by using **U** statistic. Then, a control chart namely modified T^2 was developed for monitoring auto-correlated simple linear profiles in a multi-stage process. The results of simulation studies indicate that the proposed control chart is applicable for monitoring the auto-correlated simple linear profiles in multi-stage processes. The performance of the modified control method was sufficiently good, under different shifts. In this paper, a remedial measure to eliminate the cascade effect was considered, but other approaches such as using the state-space models can be applied for modeling and addressing this impact for future research.

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