RESEARCH PAPER



# Proposing a New Method for Transportation Planning Considering Traffic Congestion

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# Abstract

Traffic congestion is one of the issues in transportation planning which imposes environmental consequences and costs. Therefore, decision-makers and policymakers should focus on appropriate transportation planning models. One of the approaches to relieve traffic congestion is imposing tolls on the users. In the present paper, attempts are made to present three transportation planning and traffic congestion management models. The first model assumes that the transportation network and the traffic flows within it are determined. Decision-maker seeks to adjust the transportation network flows so that traffic congestion can be prevented. In the second model, unlike the first one, attempts are made to design urban transportation networks via the development of routes. The third model is a mixture of the first and second models. All models proposed here are bi-objective which were addressed under uncertain conditions and disturbances. According to the results, a decision-making model was extended to rank routes. In the end, a numerical example is considered for analyzing and evaluating the proposed models. The results of the numerical example showed that the first model is the most inefficient and the third model is the most efficient. Since the proposed model can be implemented in road networks in addition to urban transportation networks, the application of the proposed models is demonstrated based on a real-world case study. The case study results showed that the efficiency of the road network depends on the time interval.

## Introduction and literature review

Traffic congestion is a major problem in urban/national roads that endangers people's quality of life. To solve this problem, a dynamic toll system has been designed based on big data. The functionality of this system is to shift the traffic of urban/national roads to a network in which toll is charged. A network with a toll system is an underused toll network, and planning based on this network decreases the cost and time of freight transport. However, the proposed model should be customized based on stakeholders' viewpoints [1]. Charging toll can be an effective solution to decrease the risk of hazardous material shipment where it can lead to pay-off between route selection and toll pricing policy [2]. Demand management can be investigated under different goals, including economic, safety, pollution, and efficiency by the simulation approach. He [3] implemented an optimal simulation model in Maryland to determine the tolls in which this model led to a decrease in the travel time in peak periods. With regard to the toll rates, Tsai and Li [4] proposed a cordon toll for automobile and motorcycle transport systems

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with several origins and one destination. Harks et al. [5] proposed a model in which, after determining the tolls by the policy makers, the users choose their paths to decrease their costs.

One of the main sources of greenhouse gas emission in the transportation sector is road construction; hence, considering the environment-friendly approach, it is required to calculate the emission of greenhouse gases due to the expansion of roads. The road class and length are used to calculate the emission amount [6]. The other factors affecting air pollution include traffic volume and road density which significantly affect the concentration of Co2, No2, and PM10 [7]. Furthermore, some models were developed to decrease human exposure to air pollution [8]. For example, Sobrino et al. [9] proposed a model to estimate the carbon footprint based on the traffic flow. Wang et al. [10] investigated and optimized the tolls for heavy trucks with regard to environmental effects in China freeways. For this purpose, a case study was performed in Heilongjiang. Patil [11] proposed a mathematical model for traffic assignment based on minimizing emission and fuel consumption. However, no system has been exclusively developed for decreasing the emission. Lv et al. [12] developed an inexact programming model to increase road network efficiency and decreasing the emission. In general, the transportation network requires improvement of traffic conditions under different conditions, such as reducing the travel time at the time of disasters [13]. On the other hand, many factors, such as weather conditions, affect the traffic, which should be investigated according to each system's conditions [14]. Resilience mainly consists of two parts in which the first part refers to the system recovery time, and the second part refers to the system adaptation without losing the critical functions. The concept of resilience is closely related to the subject of urban development in the framework of capacity development [15,16]. One of the ways to quantify resilience is to investigate the problem under different disruption conditions [17]. Toll pricing is an appropriate solution to relieve traffic congestion. Shirazi et al. [18] developed a model for efficient application of the network and minimizing the tolls in which, a path-generation algorithm was used to solve this model [19]. The congestion cost can be estimated by piecewise linear functions in which, Stefanello et al. [20] developed a model based on piecewise linear functions and solved it by a random-key genetic algorithm. Xu et al. [21] proposed a model for joint road toll pricing in which, in this model, capacity development was considered in the form of lane increase. Cheng et al. [22] applied a two-level model for dynamic pricing in which, the first level optimizes the whole system, and the second level is based on dynamic user equilibrium. When there are two classes of vehicles, including autonomous and human-driven vehicles in the transportation network, some lanes should be added to the autonomous vehicle class to improve safety and promote its usage. However, it will decrease the traffic efficiency, especially when the traffic flow of the autonomous vehicle class is low. Hence, Liu and Song [23] developed an optimal model for obtaining the tolls under the worst scenario such that, instead of the lane assignment, to decrease using human-driven vehicles by tolling. Odeck and Welde [24] proposed an econometric framework for efficiency assessment in Norway. The results showed high accuracy in the forecasting of traffic volume. Considering multi-criteria decision making (MCDM) as a useful assessment tool, Laurent et al. [25] developed a model based on MCDM and the three criteria related to cost, travel time, and carbon emission amount. Alasad and Motawa [26] applied Monte Carlo simulation for assessing different states of economical and demographical events [26]. One of the approaches used for assessing the sustainability of the cities is data envelopment analysis (DEA). In this approach, the sustainability of the cities is assessed based on the indicators including R&D, cultural interaction, livability, environment and accessibility. Wang [27] showed that traffic congestion has a negative effect on the economy and human well-being.

Based on a thorough review of the literature, some research gaps are identified which have not been addressed in the literature. In the following, the research gaps and innovations of our paper are described.

- The basis of the toll payment in the models is based on the demand and destination points. Unlike the general assumptions of transportation models that consider destinations and origins as specific points, destinations and origins are not known in many cases. In this paper, a data-oriented transportation network is developed which considers the toll based on the traffic flow, regardless of the origins and destinations. Such a view does not require each vehicle to travel from a specific origin to a specific destination. Therefore, the models presented in this paper are flexible compared to the research literature.
- The demands do not depend only on the free travel time and paths capacity. A number of factors affect the demand generation where some are not quantifiable. Therefore, in this paper, the issue of transportation was investigated based on data analysis. In this regard, all the factors affecting the demand are considered in the framework of demand elasticity because data are the result of interaction among all these factors and does not include the assumptions considered in the traditional models. Since the models presented in this paper use data to optimize traffic flow, drivers' behaviors are implicitly considered.
- Tolls have long been considered for vehicles. In many cases, however, identifying less congested paths and offering them as low-subside routes to companies that use heavy and semi-heavy trucks can provide opportunities for the conclusion of such contracts between municipalities and companies. Therefore, a method to determine tolls via a new approach is proposed.
- The other effective issue is the dynamicity of demand. In different periods, the demand shows different patterns and hence, investigation of the demand should be done based on time. For example, the peak period should be divided into different periods based on which, the results are analyzed. Therefore, in this article, the proposed models are multiperiod to consider the dynamics of demand.
- In some cases, the dynamicity of demand occurs under uncertainty; i.e., the demand in different periods is uncertain. Simultaneous consideration of these two issues can make the results closer to reality. Therefore, in our paper, in addition to the demand dynamics, the uncertainty in the data of the proposed models is considered by fuzzy programming.
- Analysis of resilience inhibits disruption condition as far as possible. Investigating adaptability and capacity increase of paths can insure the transportation system against disruptions that have not been addressed in the literature. In order to fill this gap, resilience is considered by defining disruption scenarios in our proposed models.
- The environmental factors, in addition to the transportation network efficiency, can decrease the consequences of the negative effects of environmental pollutions caused by traffic congestion. The model proposed in this paper includes two objective functions focusing on the environment and efficiency. On the other hand, the integrated planning of management, assessment, and construction of paths to control the environmental and congestion factors have not been investigated in detail in the literature. In this regard, three programming approaches were developed in this paper. Therefore, our paper focuses on both environmental issues and traffic management and routes construction.

# **Problem statement**

Since the most important problem of urban transportation is the congestion of vehicles, it is necessary to plan to encounter this problem. For this purpose, policymakers and decisionmakers have two solutions for transportation including increasing the construction of lanes and imposing some limitations to use crowded streets. According to the experiences, it was proved that the second solution will provide an appropriate opportunity for solving the problem of traffic. To implement this solution, taxing methods should be used in the form of tolls in which, the travel time should be obtained to determine the toll rates. The travel time can be obtained by Eq. 1.

$$t_{flow} = t_{ft} \left( 1 + \alpha \left( \frac{q_{flow}}{Cap} \right)^{\beta} \right) \tag{1}$$

In this equation,  $t_{flow}$  and  $t_{ft}$  represent, respectively, the travel time based on the flow and the travel time in free mode (without congestion),  $q_{flow}$  and *Cap* represent, respectively, the traffic flow and the capacity of the region under study,  $\alpha$  and  $\beta$  are the experimental parameters based on the expert's viewpoint [28]. The toll rates should be determined based on the penalties for offending the optimal flow. The penalty amount is considered the monetary value of the time for the users in the time unit. Therefore, the toll rate can be obtained by Eqs. 2-4.

$$TT = q_{flow} t_{flow}$$

$$MT = \frac{d(TT)}{dq_{flow}}$$
(2)
(3)

$$TP = TV (MT - TT)$$
<sup>(4)</sup>

Where, *TT*, *MT*, *TP* and *TV* represent the total travel time, marginal time, and paid tolls and the monetary value of the time, respectively. Adding each vehicle to each route will increase travel time where route capacity is limited. The extra time due to the presence of one more vehicle is called marginal time. Therefore, in calculating tolls by Eq. 4, marginal time is taken into account to prevent a sharp increase in travel demand. Eq. 3 is the derivative of Eq. 2 based on the flow, and Eq. 4 is calculated based on the optimal flow. One of the most important gaps in the calculation of tolls is estimating the tolls based on the flows affecting each other. When the flows affect each other, the concept of relative flow is created in which, an optimal flow for a region does not necessarily optimize the flows of other regions. Even, it may decrease the efficiency of the transportation network. An example of this condition is illustrated in Fig. 1.



Fig. 1. A network of highways in Tehran

This figure presents the East-West Niayesh and Hemat highways and their connection by North-South Kordestan, Chamran, and Yadegar-e Emam highways where these connections make the traffic flows affecting each other. Moreover, the relative efficiency of traffic flows of highways should be investigated for determining the toll rates. In the method proposed in this paper, when the toll rate becomes a negative value, it can be considered as subsiding; so that local governments and municipalities can contract with the owners of trucks and semi-trailers based on the subsiding. In other words, in Eq. 4, MT is obtained based on the efficient flow. Therefore, the important issue is finding the efficient flow for the transportation system. DEA is one of the approaches that can be used to obtain efficiency based on a system's inputs and outputs. This approach compares decision-making units (DMUs) based on the efficient frontier and can present the modified efficiency by analyzing the benchmark input and output [29]. In this method, the inputs and outputs of the DMUs may affect each other [30]. In this paper, DEA is used for planning to deal with traffic congestion.

In this paper, three approaches are proposed for investigating the efficient flow.

- 1. In the first approach, the transportation network exists physically and the flow exists in the network, too. The toll rate should be determined such that the flows of the transportation network are efficient.
- 2. In the second approach, the flows emerging from the establishment of each route are determined based on the transportation experts' opinions. Hence, in this approach, the structure of the transportation network is determined.
- 3. In the third approach, the construction of a part of the paths is also possible, and it is not necessary to complete the whole construction project.

Therefore, these three approaches are respectively referred to as, traffic management, construction management, and management and construction. The main reason for this paper for developing three optimization models to measure traffic congestion via efficiency is that traffic congestion affects all three pillars of sustainability. The release of pollutants and noise pollution due to traffic congestion threatens the pillar of the environment. Traffic congestion potentially increases the incidence of accidents and thus threatens the social pillar. Expensive tolls and penalties cancel many essential trips and increase the cost of planning. Thus, the economic pillar is threatened. Supply chains, in particular, use heavy and semi-heavy vehicles, which leads to a severe impact on the sustainability pillars [31].

#### **Traffic management**

Based on the Eqs. 5-11, a model was proposed for traffic flow in the peak period of a transportation network. Several inputs and outputs are considered for such networks. A type of these inputs and outputs is related to the whole transportation network system (a network consisting of several routes) and the other type of inputs and outputs are related to the routes inside the transportation network. Both of these types are investigated together. Input and output refer to the number of transport vehicles that enter into or exit out of the route.

$$\begin{array}{ll}
\text{Max} & \sum_{l} \sum_{j} (1 - d_{lt}) \\
\text{Min} & GC D
\end{array} \tag{5}$$

s.t. 
$$\sum_{r}^{r} u_{rl} y_{lt} + \sum_{m}^{m} w_{ml} z_{lt} - \sum_{n}^{r} w_{nl}' z_{lt}' - \sum_{i}^{r} v_{il} x_{lt} \le 0 \qquad \forall l, t \qquad (7)$$
$$\sum_{n}^{r} w_{nl}' z_{lt} + \sum_{i}^{r} v_{il} x_{lt} \le 1 \qquad \forall l, t \qquad (8)$$

$$\sum_{l} w_{nl}' z_{lt} + \sum_{l} v_{ll} x_{lt} \le 1 \qquad \qquad \forall l, t \qquad (8)$$

$$\sum_{r} u_{rl} y_{lt} + \sum_{m} w_{ml} z_{lt} + d_{lt} = 1 \qquad \forall l, t \qquad (9)$$

$$D \ge d_{lt} \qquad \forall l, t \qquad (10)$$

 $u_{rl}, w_{ml}, w_{nl}', v_{il}, d_{lt} \ge 0$ , D free variable  $\forall l, t, m, n, r, i$ (11)

In which, the decision variables including  $d_{lt}$ , D,  $u_{rl}$ ,  $w_{ml}$ ,  $w'_{nl}$  and  $v_{il}$  represent the extent of the inefficiency of the route l in the period of t, the maximum inefficiency, the  $r^{th}$  output weight for the route l, the  $m^{\text{th}}$  mid-output weight of the route l,  $n^{\text{th}}$  mid-input weight of the route l, and the  $i^{\text{th}}$  input weight of the route l, respectively. The parameters GC,  $y_{lt}$ ,  $z_{lt}$ ,  $z_{lt}$ , and  $x_{lt}$  represent the environmental cost of carbon (and other polluters) emission based on the highest inefficiency which causes congestion, the number of outputs from each route in each period, the number of mid-outputs of each route in each period, the number of mid-inputs to each route in each period, and the number of inputs to each route in each period, respectively. In this model, Eqs. 5 and 6 show, respectively, the objective functions of environment and efficiency in which, Eqs. 5 maximizes the efficiency of routes, while Eq. 6 decreases the cost of carbon (and other polluters) emission. Eq. 6 minimizes the maximum pollution generated by traffic congestion, where traffic congestion is due to routes inefficiencies. Simultaneous consideration of these two objective functions leads to an increase in the power of discrimination in measuring the efficiency of routes [32]. The constraint (7) indicates the ratio of efficiency by all the inputs and outputs, the constraint (8) indicates the maximum weighted sum of the inputs is equal to one, the constraint (9) indicates the efficiency, the constraint (10) indicates the maximum inefficiency, and the constraint (11) represents the system variables.

#### **Construction management**

In this section, it is supposed that activating or blocking the routes is determined based on the flows expected to be created in the routes, and it would be effective in the construction of new (road) paths and adding or removing a path in an online/strategic manner. This approach can be used for decision-making about emergency situations such that, the process of evacuation and refuge will be determined if a path is lost in case of an incident. In this regard, Eqs. 6-8 mentioned in the previous model are changed into Eqs. 12-14 in the following model.

$$\operatorname{Min} \quad \boldsymbol{GC} \, \boldsymbol{D} + \sum_{l} \sum_{t} \boldsymbol{GEC}_{lt} \, \boldsymbol{pl}_{lt} \tag{12}$$

s.t. 
$$\sum_{n} w'_{nl} z_{lt} + \sum_{i} v_{il} x_{lt} \le p l_{lt} \qquad \forall l, t \qquad (13)$$

$$\sum_{l}^{r} u_{rl} y_{lt} + \sum_{m}^{r} w_{ml} z_{lt} + d_{lt} = p l_{lt} \qquad \forall l, t \qquad (14)$$
$$\sum_{l}^{r} p l_{lt} \ge F N_t \qquad \forall t \qquad (15)$$

$$pl_{lt} \in \{0,1\}$$
  $\forall l,t$  (16)

In this model, the objective function (12) minimizes both pollution emissions and costs. Constraints (13) and (14) determine the upper bound of the weighted sum of the inputs and efficiency score, respectively. Constraint (15) determines the number of active routes, and constraint (16) indicates the binary variable of the problem. The binary variable  $pl_{lt}$  is equal to 1 if the route *l* is active in period *t*, and zero, otherwise. Considering the environmental issues including carbon emission, the cost of road construction is indicated by  $GEC_{lt}$  parameter, and the minimum number of the active routes for each period is presented by  $FN_t$ .

#### Management and construction

In this section, some semi-active routes exist in the transportation system; that is, a part of routes that are allowed to be utilized. This model is widely used in the real world in which, at the time of reconstructing the routes, a part of them is active, and the other part is blocked. In some cases, traffic is allowed in semi-constructed routes. For example, in a route with 4 lanes, only 2 lanes may become active for a period of time. These models can also be used to determine

the composition of road building by which the system's total efficiency is maximized. In other words, the volume required for each route is obtained based on this model. Accordingly, some changes are made in Eqs. 5-16, which are presented in Eqs. 17-21.

| $Min \qquad \sum_{l} \sum_{t} (wup  uppl_{lt} + wwp  lwpl_{lt})$  |  | (17)                         |
|---|--|------------------------------|
| s.t. $pl_{lt} - (0 + \varepsilon) \ge uppl_{lt}$ $(1 - \varepsilon) - pl_{lt} \ge lwpl_{lt}$ $pl_{lt} \ge 0$ $lwpl_{lt}, uppl_{lt} \text{ free variable}$ | $ \begin{array}{l} \forall l,t \\ \forall l,t \\ \forall l,t \end{array} $ | (18)<br>(19)<br>(20)<br>(21) |

In which, the penalties for failure to fully establish or fully block a route are presented by wup and wwp, respectively, the values assigned to failure to fully establish or fully block a route are presented by  $uppl_{lt}$  and  $lwpl_{lt}$ , respectively, and  $\varepsilon$  represents a negligible value. In this model, the constraints (18)-(19) represent, respectively, the failure to fully establish or fully block a route. The constraints (20)-(21) show the decision variables. Besides, the objective function in Eq. 17 is aimed at minimizing the values of failure to fully establish or block the routes. According to the three classes of models proposed for transportation networks under the same conditions, the following theorems are defined.

**Theorem 1**: The model including a network of routes does not charge fewer tolls than the models with a single route.

**Proof**: Eqs. 5-11 calculate the efficiency of charging the tolls based on Eqs. 2-4. The more is efficiency, the less will be the tolls. Eqs. 5-11 are separately analyzed for each route in models with a single route. However, in the network model, all the equations are investigated for each route. Therefore, the network model includes more constraints than single route models. In this regard, the feasible region of the network models is never more than the single route models; furthermore, the network efficiency is not higher than the single route efficiency. As a result, the network model can never determine fewer tolls than the single route model.

**Theorem 2**: The tolls resulted from the management model are always equal to, or more than the tolls resulted from the construction model.

**Proof**: In the construction model, some of the routes are only established and hence, fewer constraints are active in the model. Therefore, the feasible region of the problem is never less than the feasible region of the management model. Moreover, the efficiency of the construction model is always equal to or more than the management model, which suggests less or equal tolls in the construction model than the management model.

**Theorem 3**: The tolls resulted from the construction model are always equal to, or more than the tolls resulted from the management and construction models.

**Proof**: The feasible region is defined based on the binary and continuous variables in the construction model. In the management and construction models, all the variables are continuous. Therefore, the feasible region in management and construction models is more than or equal to the feasible region in the construction model. As a result, the toll rate in management and construction models is always less than or equal to the toll rate in the construction model.

# The management of uncertainty

The uncertainty conditions in the real environment lead to failure to determine an exact value for the parameters. When there is an interval for the values of the parameters, the interval grey numbers can be used for solving the problem. Considering the number of input and output vehicles in each route in peak periods under uncertainty conditions, the interval grey numbers are applied in this paper; therefore, Eqs. 7-9 are changed into Eqs. 22-24.

$$\sum_{r} u_{rl}[y_{lt}^{L}, y_{lt}^{U}] + \sum_{m} w_{ml}[z_{lt}^{L}, z_{lt}^{U}] - \sum_{n} w_{nl}'[z_{lt}'^{L}, z_{lt}'^{U}] - \sum_{i} v_{il}[x_{lt}^{L}, x_{lt}^{U}] \le \mathbf{0} \qquad \forall l, t \qquad (22)$$

$$\sum_{n} w_{nl}'[z_{lt}'^{L}, z_{lt}'^{U}] + \sum_{i} v_{il}[x_{lt}^{L}, x_{lt}^{U}] \le 1 \qquad \forall l, t \qquad (23)$$

$$\sum_{r} u_{rl}[y_{lt}^{L}, y_{lt}^{U}] + \sum_{m} w_{ml}[z_{lt}^{L}, z_{lt}^{U}] + d_{lt} = 1 \qquad \forall l, t \qquad (24)$$

In these equations, the upper threshold is presented by U and the lower threshold is presented by L. Then, Eqs. 25-29 are used for crisping Eqs. 22-24.

| $y_{lt} = y_{lt}^L + \delta y_l (y_{lt}^U - y_{lt}^L)$                    | (25) |
|---|------|
| $z_{lt} = z_{lt}^L + \delta z_l (z_{lt}^U - z_{lt}^L)$                    | (26) |
| $z'_{nl} = z'_{lt}{}^{L} + \delta z'_{l} (z'_{lt}{}^{U} - z'_{lt}{}^{L})$ | (27) |
| $x_{lt} = x_{lt}^L + \delta x_l (x_{lt}^U - x_{lt}^L)$                    | (28) |
| $0 \le \delta y_l, \delta z_l, \delta z_l', \delta x_l \le 1$             | (29) |

However, it should be noted that Eqs. 25-29 make the models nonlinear; accordingly, changing the variables is necessary for linearization. These transformations are presented in Eqs. 30-33.

| $u_{rl}\delta y_l = Lu_{rl}, 0 \le Lu_{rl} \le u_{rl}$         | $\forall l, r$ | (30) |
|--|----------------|------|
| $w_{ml}\delta z_l = Lw_{ml}, 0 \le Lw_{ml} \le w_{ml}$         | $\forall l, m$ | (31) |
| $w_{nl}' \delta z_l' = L w_{nl}', 0 \le L w_{nl}' \le w_{nl}'$ | $\forall l, n$ | (32) |
| $v_{il}  \delta x_l = L v_{il}, 0 \le L v_{il} \le v_{il}$     | $\forall l, i$ | (33) |

One of the indicators of resilience in the proposed mathematical model is the positive/negative growth of the route capacity in which, the changes of capacity affect the distance between the upper and the lower thresholds. In other words, a specific increase in capacity can decrease the effects of disruption. Moreover, two methods are proposed for considering the resilience in the transportation network. In the first method,  $RR_{ls}$  parameter is defined to increase the flow passing through route l in scenario s. As a result, the route becomes more congested and its capacity decreases. In the second method, the upper bound of the flow passing through each route is considered as the disruption flow, because it is the maximum flow and leads to traffic congestion in the route.

### Solution approach

In this section, some solution approaches are proposed for solving the models proposed in the previous section.

#### **Multi-objective programming**

One of the interesting approaches for solving multi-objective problems is goal programming. In goal programming, the goals are aimed at reaching their targeted value with the minimum deviation. Considering the difficulty of choosing an aspiration level for each objective, multi-choice goal programming is developed in which, the goals can include more than one level. It should be noted that goal programming may impose many binary variables into the problem in the form of goal levels in which, percentage multi-choice goal programming was developed to solve this problem [33].

In this paper, the mathematical model is first solved based on the objective function of efficiency which leads to obtaining the optimal value of efficiency and the non-optimal values of the environmental factors as the best and the worst goals for efficiency and environment objective functions. Then, the mathematical model is solved based on the objective function of the environment and hence, the optimal value of environmental factors and the non-optimal value of efficiency are obtained as the best and the worst goals for environment and efficiency objective functions. Finally, the single objective model is obtained in accordance with Eqs. 34-38 in which, these changes should be applied in the mentioned models.

$$\operatorname{Min} \quad \sum_{l} \sum_{t} (WEff(de_{lt}^{N} + de_{lt}^{P}) + WEn(De^{N} + De^{P}))$$
(34)

s.t. 
$$\sum_{l} \sum_{t} ((1 - d_{lt}) + de_{lt}^{N} - de_{lt}^{P}) = G^{min} + \gamma_{eff}(G^{max} - G^{min})$$
(35)

$$GC D + De^{N} - De^{P} = GE^{min} + \gamma_{env}(GE^{max} - GE^{min})$$

$$0 \le \gamma_{eff} \le Ex\gamma_{eff}$$
(36)
(37)

$$0 \le \gamma_{eff} \le L \chi_{eff} \tag{37}$$
$$0 \le \gamma_{env} \le E x \gamma_{env} \tag{38}$$

Where, *WEff* and *WEn* represent, respectively, the weight of the objective functions of efficiency and environment. For the objective function of the efficiency, the positive and negative deviations are represented respectively by  $de_{lt}^{P}$  and  $de_{lt}^{N}$ , and for the objective function of the environment, the positive and negative deviations are represented respectively by  $De^{P}$  and  $De^{N}$ . The variables  $\gamma_{eff}$  and  $\gamma_{env}$  can adjust the priority and preference of the objective functions compared to each other based on the parameters of, respectively,  $Ex\gamma_{eff}$  and  $Ex\gamma_{env}$  in which, the less is the value of these parameters, the higher will be the priority. The maximum and minimum values of the objective function of efficiency are presented by  $G^{max}$  and  $G^{min}$ , respectively, and the maximum and minimum values of the objective function (34) minimizes the weighted sum of deviations. Constraints (35) and (36) determine the values of the target. Constraints (37) and (38) indicate the allowable range for values greater than the minimum target value.

## Decision making model for ranking the routes

Considering the solution of the model presented in the previous section, various numerical results are created in different levels of resilience; accordingly, the routes should be ranked based on all the numerical results. For this purpose, a mathematical model is proposed in Eqs. 39-45.

$$\operatorname{Min} \sum_{\substack{s \\ M \text{ or ute}}} \sum_{\substack{route \\ M \text{ or ute}}} (wo \ OIdP_{s,route} + wi \ IIdP_{s,route} + we \ E_{route} + wn \ NA_{Route})$$
(39)

$$DIP_{route} = \sum_{s} (OIdP_{s,route} + IIdP_{s,route}) \qquad \forall route \qquad (42)$$

$$E_{route} \ge \frac{\sum_{s} (D-R)_{s,route}}{\sum_{s} (I-R)_{s,route}} \qquad \forall route \qquad (43)$$

$$NA_{Poute} \ge \frac{DIP_{route}}{\nabla (I-R)_{s,route}} \qquad \forall route \qquad (44)$$

$$NA_{Route} \ge \frac{1}{E_{route}} \qquad \forall route \qquad (44)$$
$$OIdP_{s,route}, IIdP_{s,route}, DIP_{route}, E_{route}, NA_{Route} \ge 0 \qquad (45)$$

In which, *s* shows each scenario of resilience, *route* represent each route, the output and input of each route in each scenario are presented by the parameters  $(O - R)_{s,route}$  and  $(I - R)_{s,route}$ , respectively, the decision variables  $OIdP_{s,route}$ ,  $IIdP_{s,route}$ ,  $DIP_{route}$ ,  $E_{route}$  and  $NA_{Route}$  represent the distance from the goal point for the output, the distance from the goal point for the input, the total distances, efficiency of each route, and the final score of each route, respectively, the maximum output and the minimum input of the routes in each scenario are presented by the parameters of  $Max_{Route}[(O - R)_{s,route}]$  and  $Min_{Route}[(I - R)_{s,route}]$ , respectively, and the weight of each of the decision variables is indicated by parameters wo, wi, we, and wn, respectively. In this model, the objective function (39) minimizes the distances from the goal points for the outputs and inputs, the constraint (42) indicates the total distances from the goal points for each route, the constraint (43) presents the efficiency of each route, the constraint (44) presents the final score of each route, and the route with a lower  $NA_{Route}$  will have a higher rank.

#### **Benders decomposition method**

One of the approaches for solving large-scale mixed-integer programming problems is the Benders decomposition method. By decomposing the problem into the master problem and the sub-problem, this method finds a lower and an upper bound in each iteration. These bounds are compared and, in case of a significant difference between two bounds, a cut that is obtained from the sub-problem is added to the master problem. The master problem is updated again, and the sub-problem is solved. This is repeated until the difference between two bounds becomes so small and insignificant [34]. The general model of an optimization problem is considered based on Eqs. 46-49.

| Min  | $c^T x + b^T$      | (46) |
|------|--------------------|------|
| s.t. | $Ax \ge d$         | (47) |
|      | $Bx + Dy \ge h$    | (48) |
|      | $x \in X, y \ge 0$ | (49) |

Where, x and y are decision variables, x can be a discrete or continuous variable, y is a continuous variable,  $c^T$  and  $b^T$  are the coefficients of the objective function, A, B and D are technological coefficients, and d and h are the right-hand side coefficients. X is a complex variable, because if Eqs. 46-49 are divided into two problems, including a problem for making a decision about x and a problem for making decision about y, then a mixed-integer programming and linear programming problem will be obtained that decreases the complexity of the problem. For this purpose, the master problem in the Benders is presented in Eqs. 50-53.

| Min  | $c^T x + \varphi$                | (50) |
|------|----------------------------------|------|
| s.t. | $Ax \ge d$                       | (51) |
|      | $\varphi_{lb} \leq \varphi$      | (52) |
|      | $x \in X, \varphi$ free variable | (53) |

In which  $\varphi_{lb}$  is a lower bound to prevent the problem from getting unbounded. Eqs. 54-57 are used to calculate the values of  $\varphi$ . These equations present the Benders sub-problem.

| Min  | $\varphi = b^T y$ | (54) |
|------|-------------------|------|
| s.t. | $Dy \ge h - Bx$   | (55) |
|      | $y \ge 0$         | (56) |
|      | $x = x_{mp}$      | (57) |

In Benders sub-problem,  $x_{mp}$  presents the value of x obtained from the master problem. The objective function of the master problem is considered as a lower bound of  $z_{lb}$  and the objective function of Benders sub-problem is an upper bound of  $z_{ub}$  for the general model. If the difference between these two bounds is insignificant, the solution of the general model is obtained; otherwise, based on the dual variables that are corresponding to the equality constraint in the sub-problem, the cut given in Eq. 58 that is known as the Benders' cut should be added to the master problem.

$$\pi^T(h - Bx) \le \varphi \tag{58}$$

Where the dual variable's vector corresponding to the equality constraint is indicated by  $\pi^T$ . Then, the master problem and the sub-problem are solved, and the difference among the bounds is obtained. If the difference is insignificant, Benders decomposition algorithm is finished; otherwise, another cut is added to the master problem. This procedure continues until the difference between the two bounds becomes so small. Accordingly, the Benders master problem, in this paper, is presented in Eqs. 59-62. After the first iteration, Eq. 58 is added to the master problem and updated in each iteration.

$$\operatorname{Min} \quad \sum_{l} \sum_{t} GEC_{lt} \, pl_{lt} + \varphi \tag{59}$$

$$\sum_{l} pl_{lt} \ge FN_t \tag{60}$$

$$pl_{lt} \in \{0,1\} \qquad \forall l, t \tag{61}$$

$$\varphi \ free \ variable \tag{62}$$

Constraints (63) to (71) show the sub-problem where constraint (67) fixes the solutions obtained from the master problem.

$$\operatorname{Min} \quad \sum_{\underline{l}} \sum_{t} (WEff(de_{lt}^{N} + de_{lt}^{P}) + WEn(De^{N} + De^{P})) \tag{63}$$

s.t.  $\sum_{n}^{n} w_{nl}^{\prime} z_{lt} + \sum_{i}^{l} v_{il} x_{lt} \le p l_{lt} \qquad \forall l, t \qquad (64)$  $\sum_{n}^{n} u_{rl} y_{lt} + \sum_{i}^{l} w_{ml} z_{lt} + d_{lt} = p l_{lt} \qquad \forall l, t \qquad (65)$ 

$$\sum_{l=1}^{r} pl_{lt} \ge FN_t \qquad \forall t \qquad (66)$$

$$pl_{lt}\epsilon$$
 master problem  $\forall l, t$  (67)

$$\sum_{l} \sum_{t} ((1 - d_{lt}) + de_{lt}^{N} - de_{lt}^{P}) = G^{min} + \gamma_{eff} (G^{max} - G^{min})$$
(68)

$$GC D + De^{N} - De^{P} = GE^{min} + \gamma_{env}(GE^{max} - GE^{min})$$

$$(69)$$

$$0 \le \gamma_{env} \le Fr\gamma_{env}$$

$$(70)$$

$$J \leq \gamma_{eff} \leq LX\gamma_{eff} \tag{70}$$

$$0 \le \gamma_{env} \le E x \gamma_{env} \tag{71}$$

Benders sub-problem is defined based on the construction management model, which was solved by uncertainty programming and multi-objective programming approaches. The sub-problem considers all deviations in the objective function based on the multi-objective programming sub-section [35]. It is important to note that the decision variables obtained from the master problem were parametrically applied in the sub-problem.

## Numerical examples

In this section, three models proposed in this paper are investigated by a numerical example. In this regard, a transportation network was used which is presented in Fig. 2. This network presents a general structure of the routes, the relationships between the adjacent and non-adjacent routes, and the relationship with outside the network. Since all types of flows are assumed in the proposed transportation network, it is worth examining this numerical example. In this paper, in addition to the case study mentioned in the following sections, the application of the proposed models is demonstrated through a numerical example.



In the numerical example, the results of the management model for 4 routes  $l = \{1,2,3,4\}$  in 10 periods  $t = \{1,2, ..., 10\}$  and for three scenarios of resilience at the levels of 0.01, 1 and 10, are provided in Table 1 in which, the weight of the first and the second objective function is set as 1 (*WEff* = 1) and 0.1 (*WEn* = 0.1), respectively, and the environmental cost is set as 100 (*GC* = 100). In addition, the average, the maximum, and the minimum output flow from the main routes is 1506.25, 5000 and 50, respectively, while the average, the maximum and the minimum input flow to the main routes is 86685, 600000, and 1000, respectively. In this table, the values of the first and the second objective functions (Ov1 & Ov2), total deviations (Total Dev) in addition to the input rate (I-R), and the output rate (O-R) for each route are reported.

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| RR <sub>ls</sub> |             | Period<br>1 | Period<br>2 | Period<br>3 | Period<br>4 | Period<br>5 | Period<br>6 | Period<br>7 | Period<br>8 | Period<br>9 | Period<br>10 | Ov1    | Ov2    | Total<br>Dev |  |  |  |  |  |  |  |  |
|------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------|--------|--------------|--|--|--|--|--|--|--|--|
|                  | 0-R1        | 0.906       | 0.799       | 0.544       | 0.432       | 0.830       | 0.861       | 0.814       | 0.820       | 0.735       | 0.570        |        |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R2</b> | 0.669       | 0.916       | 0.710       | 0.715       | 0.810       | 0.736       | 0.744       | 0.918       | 0.729       | 0.616        |        |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R3</b> | 0.760       | 0.749       | 0.687       | 0.796       | 0.831       | 0.856       | 0.945       | 0.912       | 0.937       | 0.953        |        |        |              |  |  |  |  |  |  |  |  |
| 0.01             | <b>O-R4</b> | 0.627       | 0.623       | 0.709       | 0.682       | 0.695       | 0.794       | 0.468       | 0.656       | 0.299       | 0.830        | 10.216 | 05 270 | 1 702        |  |  |  |  |  |  |  |  |
| 0.01             | I-R1        | 0.437       | 0.327       | 0.676       | 0.681       | 0.761       | 0.409       | 1           | 0.343       | 0.902       | 0.430        | 10.316 | 95.279 | 1.723        |  |  |  |  |  |  |  |  |
|                  | I-R2        | 0.331       | 1           | 0.782       | 0.829       | 0.271       | 0.350       | 0.256       | 0.282       | 0.293       | 0.384        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R3        | 0.519       | 0.509       | 0.313       | 1           | 0.421       | 0.333       | 0.343       | 0.313       | 0.157       | 0.055        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R4        | 0.859       | 1           | 0.291       | 0.470       | 0.305       | 0.441       | 0.540       | 0.579       | 0.701       | 0.959        |        |        |              |  |  |  |  |  |  |  |  |
|                  | 0-R1        | 0.927       | 0.841       | 0.648       | 0.559       | 0.853       | 0.891       | 0.846       | 0.855       | 0.790       | 0.652        | 10 700 |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R2</b> | 0.660       | 0.882       | 0.669       | 0.658       | 0.772       | 0.645       | 0.753       | 0.901       | 0.752       | 0.597        |        |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R3</b> | 0.574       | 0.603       | 0.509       | 0.695       | 0.760       | 0.753       | 0.879       | 0.876       | 0.927       | 0.927        |        |        |              |  |  |  |  |  |  |  |  |
| 1                | <b>O-R4</b> | 0.194       | 0.293       | 0.425       | 0.332       | 0.431       | 0.720       | 0.603       | 0.473       | 0.408       | 0.746        |        | 02 (70 | 0.422        |  |  |  |  |  |  |  |  |
| 1                | I-R1        | 0.456       | 0.375       | 0.796       | 0.880       | 1           | 0.443       | 0.914       | 0.346       | 0.648       | 0.348        | 12.723 | 92.679 | 0.423        |  |  |  |  |  |  |  |  |
|                  | I-R2        | 0.376       | 1           | 0.852       | 0.919       | 0.316       | 0.355       | 0.247       | 0.288       | 0.308       | 0.403        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R3        | 0.831       | 0.716       | 0.491       | 1           | 0.632       | 0.337       | 0.257       | 0.496       | 0.256       | 0.138        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R4        | 0.821       | 1           | 0.575       | 0.810       | 0.569       | 0.692       | 0.599       | 0.527       | 0.592       | 1            |        |        |              |  |  |  |  |  |  |  |  |
|                  | 0-R1        | 0.909       | 0.804       | 0.599       | 0.490       | 0.747       | 0.843       | 0.818       | 0.760       | 0.695       | 0.456        |        |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R2</b> | 0.569       | 0.913       | 0.662       | 0.623       | 0.790       | 0.644       | 0.649       | 0.896       | 0.596       | 0.483        |        |        |              |  |  |  |  |  |  |  |  |
|                  | <b>O-R3</b> | 0.407       | 0.360       | 0.245       | 0.486       | 0.596       | 0.651       | 0.856       | 0.797       | 0.862       | 0.891        |        |        |              |  |  |  |  |  |  |  |  |
| 10               | O-R4        | 0.242       | 0.339       | 0.512       | 0.351       | 0.431       | 0.714       | 0.559       | 0.475       | 0.370       | 0.811        | 15.000 | 01 242 | 2.046        |  |  |  |  |  |  |  |  |
| 10               | I-R1        | 0.607       | 0.376       | 0.875       | 0.925       | 1           | 0.494       | 1.064       | 0.371       | 0.859       | 0.544        | 15.096 | 91.342 | 2.946        |  |  |  |  |  |  |  |  |
|                  | I-R2        | 0.431       | 0.641       | 0.991       | 1           | 0.370       | 0.351       | 0.195       | 0.327       | 0.398       | 0.517        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R3        | 1           | 1.092       | 0.755       | 2.324       | 1.084       | 0.978       | 0.782       | 0.889       | 0.615       | 0.743        |        |        |              |  |  |  |  |  |  |  |  |
|                  | I-R4        | 0.750       | 1           | 0.488       | 0.743       | 0.569       | 0.460       | 0.684       | 0.525       | 0.630       | 1            |        |        |              |  |  |  |  |  |  |  |  |

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Eq. 72 is used for calculating the efficiency and modifying the number of input vehicles of the route.

$$eff_{route} = \frac{1 - (O - R)_{route}}{(I - R)_{route}}$$
(72)

For example, the efficiency of the first route in the first scenario and period 10 that is obtained as  $\frac{1-0.57}{0.43} = 1$ . Based on the decision model proposed in the previous section, route 4 with the score of 8.44, route 2 with the score of 12.09, route 3 with the score of 27.89, and route 1 with the score of 28.84 obtained the ranks 1, 2, 3 and 4, respectively.

Considering the numerical example for the consumption model, in addition to the assumptions of the previous numerical example, routes 1 and 3 are considered as the two candidate routes in which route 1 is selected for establishment. The numerical results are presented in Table 2 where P1 represents the established route 1. Hence, routes 1, 2 and 4 are considered in the transportation network, while route 3 is excluded from the transportation network.

|                  |             |        | Tau    | <b>Die 2</b> . 1 ne | e results o | i the nun | ierical ex | ample to | r constru |        | lei    |        |        |           |  |
|------------------|-------------|--------|--------|---------------------|-------------|-----------|------------|----------|-----------|--------|--------|--------|--------|-----------|--|
| RR <sub>ls</sub> |             | Period | Period | Period              | Period      | Period    | Period     | Period   | Period    | Period | Period | Ov1    | Ov2    | <b>P1</b> |  |
| nnls             |             | 1      | 2      | 3                   | 4           | 5         | 6          | 7        | 8         | 9      | 10     | 011    | 012    | 11        |  |
|                  | 0-R1        | 0.955  | 0.891  | 0.791               | 0.721       | 0.852     | 0.935      | 0.846    | 0.912     | 0.874  | 0.763  |        |        |           |  |
|                  | <b>O-R2</b> | 0.897  | 0.971  | 0.907               | 0.907       | 0.938     | 0.911      | 0.921    | 0.973     | 0.918  | 0.880  |        |        |           |  |
|                  | <b>O-R3</b> | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
| 0.01             | <b>O-R4</b> | 0.870  | 0.782  | 0.755               | 0.913       | 0.883     | 0.830      | 0.440    | 0.748     | 0.283  | 0.789  | 14 042 | 1019 1 | 1         |  |
| 0.01             | I-R1        | 0.334  | 0.413  | 0.755               | 0.829       | 1         | 0.441      | 1        | 0.388     | 0.750  | 0.237  | 14.943 | 4918.4 | 1         |  |
|                  | I-R2        | 0.103  | 1      | 0.289               | 0.392       | 0.073     | 0.211      | 0.222    | 0.128     | 0.082  | 0.121  |        |        |           |  |
|                  | I-R3        | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
|                  | I-R4        | 0.836  | 1      | 0.245               | 0.428       | 0.287     | 0.348      | 0.560    | 0.578     | 0.717  | 0.950  |        |        |           |  |
|                  | <b>O-R1</b> | 0.953  | 0.895  | 0.775               | 0.714       | 0.890     | 0.931      | 0.883    | 0.909     | 0.868  | 0.774  |        |        |           |  |
|                  | <b>O-R2</b> | 0.890  | 0.989  | 0.921               | 0.931       | 0.954     | 0.956      | 0.906    | 0.981     | 0.893  | 0.876  |        | 4918.3 |           |  |
|                  | <b>O-R3</b> | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
| 1                | <b>O-R4</b> | 0.798  | 0.718  | 0.759               | 0.826       | 0.765     | 0.714      | 0.417    | 0.580     | 0.271  | 0.891  | 15 270 |        | 1         |  |
| 1                | I-R1        | 0.358  | 0.378  | 0.748               | 0.849       | 1         | 0.414      | 0.841    | 0.338     | 0.543  | 0.226  | 15.372 |        | 1         |  |
|                  | I-R2        | 0.110  | 1      | 0.329               | 0.439       | 0.081     | 0.221      | 0.229    | 0.143     | 0.107  | 0.157  |        |        |           |  |
|                  | I-R3        | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
|                  | I-R4        | 0.814  | 1      | 0.241               | 0.437       | 0.315     | 0.299      | 0.603    | 0.579     | 0.729  | 0.970  |        |        |           |  |
|                  | 0-R1        | 0.932  | 0.856  | 0.665               | 0.587       | 0.891     | 0.898      | 0.879    | 0.869     | 0.806  | 0.693  |        |        |           |  |
|                  | <b>O-R2</b> | 0.868  | 1      | 0.921               | 0.939       | 0.961     | 0.986      | 0.881    | 0.983     | 0.859  | 0.856  |        |        |           |  |
|                  | <b>O-R3</b> | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
| 10               | <b>O-R4</b> | 0.873  | 0.788  | 0.816               | 0.891       | 0.837     | 0.782      | 0.409    | 0.685     | 0.261  | 0.874  | 15 450 | 4010 2 | 1         |  |
| 10               | I-R1        | 0.400  | 0.401  | 0.782               | 0.853       | 1         | 0.451      | 1        | 0.380     | 0.756  | 0.307  | 15.452 | 4918.3 | 1         |  |
|                  | I-R2        | 0.132  | 1      | 0.447               | 0.579       | 0.104     | 0.250      | 0.249    | 0.187     | 0.178  | 0.264  |        |        |           |  |
|                  | I-R3        | 0      | 0      | 0                   | 0           | 0         | 0          | 0        | 0         | 0      | 0      |        |        |           |  |
|                  | I-R4        | 0.803  | 1      | 0.184               | 0.372       | 0.267     | 0.218      | 0.591    | 0.577     | 0.739  | 0.941  |        |        |           |  |

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In this case, the efficiency of route 1 in the first scenario and period 10 that is obtained as  $\frac{1-0.763}{0.237} = 1$ . Based on the decision model proposed in the previous section, route 4 with the score of 19.53, route 2 with the score of 23.25, and route 1 with the score of 53.86 obtained the ranks 1, 2 and 3, respectively. Applying the Benders decomposition method and setting the value of  $\varphi_{lb} = -100$ , the optimal result is obtained by five iterations. In this case, it is assumed that the algorithm terminates where the difference between the upper bound and the lower bound is less than or equal to 25. Since the number of binary variables is small, this problem was solved using GAMS software to obtain the global optimum. The results obtained from GAMS software showed that the Benders algorithm had achieved the global optimum. The problem was not infeasible in any of the iterations.

Finally, considering the numerical example for the management and construction model, similar to the numerical example of the previous model, routes 1 and 3 are considered as semiactive routes which have establishment permission. Accordingly, the numerical results are presented in Table 3 in which, the rows P1 and P3 represent the activation rate of routes 1 and 3. This model can be used to evacuate vehicles or refuge the vehicles of other routes in emergency conditions such that a part of the routes' input is kept active for keeping the vehicles away from an emergency condition caused by other parts of the transportation system. Similarly, a part of the routes' output is kept active to evacuate the vehicles in the emergency conditions occurred in the route.

Table 2 The

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| RR <sub>ls</sub> |             | Period<br>1 | Period 2 | Period 3 | Period<br>4 | Period<br>5 | Period<br>6 | Period<br>7 | Period<br>8 | Period<br>9 | Period<br>10 | Ov1    | Ov2    |
|------------------|-------------|-------------|----------|----------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------|--------|
|                  | <b>O-R1</b> | 0.486       | 0.463    | 0.345    | 0           | 0.570       | 0           | 0.747       | 0.593       | 0.642       | 0.586        |        |        |
|                  | <b>O-R2</b> | 0.701       | 0.928    | 0.743    | 0.749       | 0.833       | 0.773       | 0.767       | 0.928       | 0.751       | 0.655        |        |        |
|                  | <b>O-R3</b> | 0.249       | 0        | 0        | 0.211       | 0.160       | 0.789       | 0.088       | 0.208       | 0.1         | 0.042        |        |        |
|                  | <b>O-R4</b> | 0.540       | 0.568    | 0.614    | 0.631       | 0.683       | 0.822       | 0.658       | 0.673       | 0.513       | 0.787        |        |        |
| 0.01             | I-R1        | 0.393       | 0.266    | 0.590    | 0.618       | 0.679       | 0.340       | 0.762       | 0.266       | 0.632       | 0.357        | 19.405 | 5022.3 |
| 0.01             | I-R2        | 0.299       | 1        | 0.715    | 0.877       | 0.262       | 0.292       | 0.229       | 0.228       | 0.249       | 0.345        | 19.405 | 3022.5 |
|                  | I-R3        | 0.432       | 0.369    | 0.251    | 0.508       | 0.321       | 0.169       | 0.128       | 0.251       | 0.126       | 0.057        |        |        |
|                  | I-R4        | 0.736       | 1        | 0.386    | 0.565       | 0.317       | 0.383       | 0.378       | 0.420       | 0.487       | 0.650        |        |        |
|                  | P1          | 0.568       | 0.631    | 0.749    | 0.492       | 0.679       | 0.122       | 0.872       | 0.749       | 0.874       | 0.943        |        |        |
|                  | P3          | 0.432       | 0.369    | 0.251    | 0.508       | 0.321       | 0.878       | 0.128       | 0.251       | 0.126       | 0.057        |        |        |
|                  | 0-R1        | 0.330       | 0.481    | 0.284    | 0.240       | 0.429       | 0.747       | 0.789       | 0.512       | 0.611       | 0.555        |        |        |
|                  | <b>O-R2</b> | 0.639       | 0.881    | 0.655    | 0.647       | 0.765       | 0.641       | 0.734       | 0.898       | 0.730       | 0.574        |        |        |
|                  | <b>O-R3</b> | 0.349       | 0        | 0        | 0           | 0.222       | 0           | 0           | 0.253       | 0.108       | 0.044        | 22.249 |        |
|                  | <b>O-R4</b> | 0.196       | 0.295    | 0.428    | 0.333       | 0.431       | 0.720       | 0.601       | 0.473       | 0.407       | 0.748        |        | 4842.8 |
| 1                | I-R1        | 0.371       | 0.233    | 0.525    | 0.534       | 0.577       | 0.308       | 0.719       | 0.243       | 0.637       | 0.360        |        |        |
| 1                | I-R2        | 0.371       | 1        | 0.867    | 0.944       | 0.314       | 0.358       | 0.253       | 0.295       | 0.322       | 0.426        |        | 4042.0 |
|                  | I-R3        | 0.592       | 0.474    | 0.338    | 0.507       | 0.423       | 0.169       | 0.084       | 0.338       | 0.169       | 0.085        |        |        |
|                  | I-R4        | 0.818       | 1        | 0.572    | 0.807       | 0.569       | 0.684       | 0.602       | 0.527       | 0.593       | 1            |        |        |
|                  | P1          | 0.408       | 0.649    | 0.662    | 0.712       | 0.577       | 0.862       | 0.950       | 0.662       | 0.831       | 0.915        |        |        |
|                  | P3          | 0.592       | 0.351    | 0.338    | 0.288       | 0.423       | 0.138       | 0.050       | 0.338       | 0.169       | 0.085        |        |        |
|                  | 0-R1        | 0.788       | 0        | 0.175    | 0           | 0           | 0.752       | 0.775       | 0.486       | 0.541       | 0            |        |        |
|                  | <b>O-R2</b> | 0.451       | 0.849    | 0.507    | 0.508       | 0.673       | 0.531       | 0.581       | 0.859       | 0.562       | 0.360        |        |        |
|                  | <b>O-R3</b> | 0           | 0.422    | 0        | 0           | 0.591       | 0           | 0           | 0.298       | 0.122       | 0.496        |        |        |
|                  | O-R4        | 0.217       | 0.316    | 0.470    | 0.340       | 0.430       | 0.717       | 0.581       | 0.474       | 0.389       | 0.779        |        |        |
| 10               | I-R1        | 0.516       | 0.260    | 0.641    | 0.663       | 0.580       | 0.321       | 0.625       | 0.193       | 0.545       | 0.496        | 23.959 | 4830.8 |
| 10               | I-R2        | 0.174       | 1        | 0.605    | 0.767       | 0.147       | 0.284       | 0.267       | 0.237       | 0.263       | 0.387        | 23.939 | 4030.8 |
|                  | I-R3        | 1           | 0.799    | 0.571    | 0.856       | 0.713       | 0.285       | 0.144       | 0.571       | 0.285       | 0.142        |        |        |
|                  | I-R4        | 0.783       | 1        | 0.530    | 0.776       | 0.570       | 0.570       | 0.644       | 0.526       | 0.611       | 1            |        |        |
|                  | P1          | 0.923       | 0.245    | 0.679    | 0.631       | 0.252       | 0.921       | 0.960       | 0.679       | 0.840       | 0.496        |        |        |
|                  | P3          | 0.077       | 0.755    | 0.321    | 0.369       | 0.748       | 0.079       | 0.040       | 0.321       | 0.160       | 0.504        |        |        |

Again, the efficiency of route 1 in the first scenario and period 10 that is obtained as  $\frac{1-0.586}{0.357} =$ 

1.16 which suggests a rapid traffic relieve compared to the previous state. Based on the decision model proposed in the previous section, route 3 with the score of 0.55, route 1 with the score of 10.43, route 4 with the score of 28.45, and route 2 with the score of 30.16 obtained the ranks 1, 2, 3 and 4, respectively.

## **Management analysis**

In this section, a sensitivity analysis is performed to investigate the effect of the parameters on the performance of three models for the numerical example introduced in the previous section.

For the management model, capacity growth can make effective changes against disruptions. It should be noted that one of the solutions to respond to the disruptions and to increase the resilience is increasing the capacity of routes. In this regard, increasing the capacity of the routes results in a decrease in traffic congestion; accordingly, it leads to a decrease in environmental pollutions. However, the excessive growth of the route's capacity is not acceptable; the excessive growth of capacity leads to a useless capacity that can decrease the routes' efficiency. Fig. 3 illustrates the sensitivity analysis performed on the objective functions of the traffic management, construction management, and management and construction models regarding the resilience scenarios. In this figure, the red and blue lines represent, respectively, the environment and the efficiency objective functions. Considering the fact that the two objective functions do not have the same scale, they were normalized. Based on Fig. 3-a, increasing the resilience in the management model leads to a decrease in the environment objective function, while it results in an increase and then a decrease in the efficiency objective function. Fig. 3-b shows that in the construction management model, the environment objective function is not so

sensitive to resilience, while the behavior of the efficiency objective function is similar to its behavior in the traffic management model. Finally, Fig. 3-c shows that in the construction and management model, the environment objective function has a descending trend, while the efficiency objective function has first a descending trend and then, an ascending trend. Besides, the environment objective function has more effect on resilience before the intersection of two objective functions, while the efficiency objective functions. According to the results, the decision-makers can decide on the appropriate resilience level.



In the following, the variations of the inputs and outputs of routes are investigated so that the routes' efficiency is not changed. According to this sensitivity analysis, the potential resilience of the existing routes is evaluated. Inverse data envelopment analysis (InDEA) is used in this section to answer the question that, if the inputs increase, how much the output should be increased to maintain constant efficiency [36]. Therefore, after solving the mathematical model and fixing the results of the efficiency and objective functions, the extent of changes in the inputs and outputs are determined. In this regard, some decision variables were considered for increasing the inputs and outputs of each route. In the next step, the mathematical model is run to determine changes in inputs and outputs where efficiency is fixed.

Fig. 4 illustrates the increase of the outputs based on an increase in the inputs of the routes for three scenarios of resilience. In this figure, the red and blue lines represent, respectively, the outputs and inputs. Due to the difference between the scales, both the objective functions have been normalized. Fig. 4-a shows that increasing the resilience leads to a decrease in the inputs and an increase in the outputs in the traffic management model. Fig. 4-b demonstrates that in the construction model, the inputs and outputs have a similar behavior under different levels of resilience. Finally, Fig. 4-c shows that an increase in resilience leads to no change in the inputs and outputs in the management and construction model.



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# **Case study**

In this section, the application of the proposed models is illustrated based on a real case in Iran. Considering the fact that the proposed models can be applied to both the urban and interurban road networks, the traffic data related to the Tehran-Fasham, Fasham-Meygun, and Meygun-Shemshak roads in Tehran province, Iran during 2016-2019-March 20 is used in an hour-based manner in this section. The disruption value ( $RR_{ls} = 1.5$ ) was considered 1.5 times higher than the normal demand. The road path is presented in Fig. 5.



Fig. 5. Tehran, Fasham, Meygun, Shemshak road

The results are obtained based on the management model and are reported in Table 4.

| Table 4. The numerical results of the case study |             |       |       |       |      |     |  |  |  |
|--|-------------|-------|-------|-------|------|-----|--|--|--|
| Time interval                                    | <b>O-R1</b> | O-R2  | I-R1  | I-R2  | Ov1  | Ov2 |  |  |  |
| 1  | 0.943       | 0.975 | 0.057 | 0.057 |      |     |  |  |  |
| 2  | 0.952       | 0.976 | 0.094 | 0.048 |      |     |  |  |  |
| 3  | 0.986       | 0.986 | 0.065 | 0.014 |      |     |  |  |  |
| 4  | 0.995       | 0.995 | 0.057 | 0.005 |      |     |  |  |  |
| 5  | 0.991       | 0.995 | 0.052 | 0.009 |      |     |  |  |  |
| 6  | 0.991       | 0.994 | 0.035 | 0.009 |      |     |  |  |  |
| 7  | 0.979       | 0.981 | 0.025 | 0.021 |      |     |  |  |  |
| 8  | 0.930       | 0.965 | 0.109 | 0.070 |      |     |  |  |  |
| 9  | 0.911       | 0.938 | 0.201 | 0.089 |      |     |  |  |  |
| 10   | 0.784       | 0.867 | 0.653 | 0.216 |      |     |  |  |  |
| 11   | 0.684       | 0.819 | 1     | 0.316 |      |     |  |  |  |
| 12   | 0.636       | 0.786 | 0.935 | 0.364 | 6.02 | 100 |  |  |  |
| 13   | 0.711       | 0.857 | 0.950 | 0.289 |      |     |  |  |  |
| 14   | 0.727       | 0.860 | 0.868 | 0.273 |      |     |  |  |  |
| 15   | 0.615       | 0.859 | 0.980 | 0.385 |      |     |  |  |  |
| 16   | 0.584       | 0.840 | 0.931 | 0.416 |      |     |  |  |  |
| 17   | 0.688       | 0.803 | 0.878 | 0.312 |      |     |  |  |  |
| 18   | 0.741       | 0.849 | 0.923 | 0.259 |      |     |  |  |  |
| 19   | 0.754       | 0.870 | 0.903 | 0.246 |      |     |  |  |  |
| 20   | 0.816       | 0.911 | 0.685 | 0.184 |      |     |  |  |  |
| 21   | 0.873       | 0.938 | 0.727 | 0.127 |      |     |  |  |  |
| 22   | 0.884       | 0.935 | 0.680 | 0.116 |      |     |  |  |  |
| 23   | 0.848       | 0.954 | 0.628 | 0.152 |      |     |  |  |  |

**Table 4**. The numerical results of the case study

In this table, the input and output of Fasham region are presented, respectively, by I-R1 and O-R1, and the input and output of Meygun region are presented, respectively, by I-R2 and O-R2. For example, for the 10<sup>th</sup> time interval, the efficiencies of Fasham and Meygun regions are equal to 0.33 and 0.62, respectively. Based on the results,

- Fasham region is efficient in the first time interval.
- Meygun region is efficient in the third and fourth time intervals.
- The efficiency of Fasham and Meygun regions has the least difference in the second time interval; hence, this time interval can be considered a benchmark for other time intervals. This benchmarking is possible via various management methods, including toll payment.

After the second time interval, the least difference in the efficiency of Fasham and Meygun regions belongs to the fifteenth, twenty third, sixteenth, and seventh time intervals, respectively.

## Conclusions

Traffic congestion is considered a major problem in the field of transportation planning. Particularly, traffic congestion causes such negative effects on the environment. Besides environmental issues and fuel consumption, the extension of travel time increases the users' costs. In this way, the sustainability of the transportation system is disturbed. One of the ways of countering traffic congestion is tolling. Although various studies have been done in this area and various models have been proposed for tolling, no comprehensive model with the capacity covering different conditions has been proposed yet.

In the present paper, three mathematical models of transportation planning are proposed. All three models are bi-objective, multi-period, and under uncertain conditions. The first model is developed to manage traffic flow in the transportation network, and the second model is proposed to design an efficient transportation network. The third model is a mixture of the first and second models.

The proposed models are bi-objective. The presented mathematical models are solved through the percentage multi-choice goal programming. Furthermore, the demands have an uncertain range. The resilience of the models under disruption conditions is considered in the form of capacity development. Therefore, since various results are obtained for the models under different conditions, a decision model has been proposed for ranking the routes. On the other hand, a solution has been also proposed for the complexity of the construction model.

Finally, a numerical example has been presented on which, the three models have been implemented. Sensitivity analysis was done on the numerical results, and some managerial analyses were proposed on this basis. Since the proposed models can be implemented on both urban and road transportation systems, a case study was performed on Tehran-Shemshak road. For future studies, it is suggested to implement the issue of delay in the three models. In other words, it is suggested to include the capacity of tracing the outputs created in the transportation network with an interval from the inputs. It is also suggested to independently include the issue of safety in the models. Moreover, different types of uncertainty that can be implemented proportional to other conditions may be investigated. In the end, since the models are based on data analysis, they can be used in an online manner using a digital framework. Hence, it is also suggested to be investigated in future studies.

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