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RESEARCH PAPER

A Data-Driven Pricing Model for Distribution Systems Considering Competition

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Abstract

This paper addresses the need to investigate customer behavior and incorporate competition, given the significant shifts in consumer preferences. To achieve this, mathematical modeling is used to design a distribution system that maximizes profits in a competitive market, comprising a wholesaler and multiple retailers across multiple periods and products. Customer behavior is captured through a behavior-based pricing process at the retail level, with equilibrium values determined using bi-level programming based on Stackelberg modeling, which accounts for asymmetric competition. The model is solved using two distinct approaches: structural modification and data-driven learning models. In the structural modification approach, the bi-level model is linearized and converted into a single-level equivalent. Meanwhile, in the data-driven approach, the pricing process is managed using the CLIQUE clustering method, which helps develop a rule-based pricing system grounded in data extraction. Numerical examples and sensitivity analyses are provided to illustrate the concepts, and the outcomes are compared to highlight managerial implications and avenues for future research.

Keywords:
Data-Mining; Rule-Based Systems; Pricing and Revenue Management; Game Theory; Retail Systems

Introduction

The current world is so competitive, and with the increase of producers in the country, this competition has become more intense (Heidari, Radfar et al. 2022). In the present competitive era, it is essential to have a significant understanding and categorization of customers in order to assess their capabilities, followed by ease of decision-making for managers in controlling various customer groups more than ever before (Su 2007). This perspective on the customer and the product has been identified with various titles, including customer lifetime value, behavior-based price, or customer profitability. 27% of all industries' sales are related to the pricing of new products. Small mistakes in predictions can lead to significant impacts on organizational profits. Thus, it seems necessary to trigger learning process of previous data and try to implement new techniques to be compatible with the new changes in the industries (Ho, Nguyen et al. 2023).

Companies generally use data related to previous sales of their current products to predict future sales or the impact of these products on the market. However, predicting for new products is more complex (Ghomi-Avili, Niaki et al. 2023). Previous studies have shown that companies often do not rely solely on data from previous sales for such products, but instead focus on

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qualitative market research or executive opinions. Today's managers have rightly recognized the importance of customer focus for organizational growth and competition. Customer categorization, market segmentation and efforts to satisfy customers have now become a widespread belief, and many organizations benefit from this approach in practice. However, despite extensive efforts to make the organization customer-centric, desired results are still achieved less frequently. The main reason is a surface-level understanding of the true meaning of customer focus (Ernawati, Baharin et al. 2021). With the expansion of e-business, comparing product features for consumers has become easier. Consequently, product demand has become more erratic, emphasizing the importance of focusing on dynamic pricing and monitoring customer behaviors, as discussed by (Vengerov 2008).

Hence, it appears crucial to first present a pricing model that can be updated and adapted to changes in today's rapidly changing world. Additionally, it is necessary to consider competition among different entities in a market. Above all, another essential aspect to be taken into account is the examination of customer behavior, their characteristics, and market segmentation to create an appropriate pricing model. This necessitates the development of data-driven models or the utilization of data mining tools and the application of learning processes in these areas more than ever before. Therefore, in this study, an attempt has been made to depart from traditional models by combining the issues of competition and behavior-based pricing and present a data-driven pricing model that has the ability to provide dynamic adaptive pricing while considering the mentioned characteristics.

The remainder of this paper is presented as follows. In Section 2, a review of the relevant literature is provided. Section 3 includes the problem definition, assumptions, notations and modeling. Solution methodology by madding a comparison between the traditional and novel approaches are presented in Section 4. Section 5 presents a numerical example and sensitivity analysis to clarify the concepts used, comparing two defined approaches. Finally, Section 6 involves the derived results from the previous sections and suggestions for future research directions.

Literature Review

Various researches have been conducted in the field of competitive pricing, which can be categorized into different types. The first category of research mainly presented competitive pricing in the form of traditional models. Another category includes models that adopt a dynamic and data-driven approach to examine and explore the role of learning concepts in pricing. In the following, articles on competitive pricing will be reviewed, and then the topic of pricing considering customer behavior will be investigated.

Cui and Li (2018), Meng, Li et al. (2021) and Zhang, Li et al. (2023) examined the pricing strategy considering the product's useful life and used this strategy to improve the status of demand fluctuations and adjust the demand peak. Esteves and Cerqueira (2017) investigated the dynamic effects of customer-based pricing in various markets, emphasizing the importance of increasing awareness and demonstrating that discriminatory pricing increases the profits of industrial and commercial entities. Pishvaee, Razmi et al. (2012) and Sabbaghnia, Heydari et al. (2023) highlighted the importance of social responsibilities in industrial centers and business units under study and attempted to reflect the role of social responsibilities in designing the supply chain network and the role of customer behavior in implementing operational decisions. De Nijs and Rhodes (2013) considered a two-period model in which competitors introduce products to the market and sought behavior-based pricing, proving that implementing pricing policies considering customer behavior increases competition intensity. Shen and Miguel Villas-Boas (2018) investigated the advertising effects sent by industrial or commercial centers to their customers and defined the basis for sending these advertisements as customer behavior during the purchase. If the customer's purchasing experience with one product has a positive correlation with another product, advertisements related to the second product will be sent to the customer. Finally, by examining information sharing strategies, they attempted to investigate these effects more accurately. Choe, King et al. (2018) examined the customer pricing process in a dynamic competition, where they collected customer information in the initial purchasing process. They demonstrated how considering pricing strategies will affect the offered prices and the profitability of companies. Ziari and Sajadieh (2021) considered the behavior-based pricing issue by considering horizontal and vertical competition between major and retail centers and addressed the model's solution using the Lagrange method.

In the examination of customer services and analysis of job opportunities, Videla Cavieres (2014), Pei (2020) and Yu, Lu et al. (2021) utilized data mining techniques. Bose and Chen (2010) used clustering techniques, specifically K-Means and KVQ, to discover behavioral patterns of customers who subscribe to mobile services. They utilized knowledge about extensively used features, such as usage, income, services, and user categorization for customer clustering. To compare the distribution of customers among different feature categories, inter-cluster analysis was performed on the clusters obtained from these two techniques.Yaghini and Vard (2012) addressed the automatic clustering of mixed-data using a Genetic algorithm. In real-world clustering problems, we often encounter mixed numeric and categorical datasets for cluster analysis. However, most existing clustering algorithms are only efficient for numerical data instead of mixed datasets. Moreover, traditional methods, such as the K-means algorithm, usually require users to specify the number of clusters. Later, Rana and Oliveira (2015) invetigated the dynamic pricing challenges for perishable goods or services, leveraging a reinforcement learning algorithm to show how reinforcement policies can optimize revenue and enhance pricing strategies. Vamvakas, Tsiropoulou et al. (2018) also investigated behavior-based price discrimination among mobile customers, employing reinforcement learning techniques to have learning automata choose the best wireless internet service provider based on utility functions. In another study, Lu, Hong et al. (2018) implemented an artificial intelligence-based approach to determine optimal dynamic prices in the electricity market, considering demand uncertainties and the flexibility of wholesale prices within a decision-making framework. Similarly, Maestre, Duque et al. (2018) utilized reinforcement learning to make dynamic pricing fairer by incorporating recent experiences and pricing policies to assess market conditions, resulting in significant fairness improvements and revenue changes. More recently, Kong, Kong et al. (2020) introduced an online pricing method that utilizes short and long-term memory networks to analyze the impacts of reinforcement learning and incentive pricing on customer behavior and overall demand levels.

Angeli, Howard et al. (2017) conducted research on the use of data mining techniques in customer relationship management. The focus of this article is mainly on the examination and classification of processes and data mining techniques. The results show that research and guidelines pay the most attention to classification models, customer relationships, and data mining, which provide a valuable means for future research and facilitate knowledge accumulation regarding data mining. Bahari and Elayidom (2015) presented a data-mining framework for customer behavior prediction. In this paper, a model for predicting customer behavior to enhance decision-making processes for retaining valuable customers is presented. Additionally, an efficient data-mining framework and two proposed models, Naïve Bayes and neural networks, are examined, showing that neural networks have relatively better accuracy. Jenabi and Mirroshandel (2014) conducted research on the use of data-mining techniques to improve customer relationship management. The case study of this paper is the automotive repair and maintenance industry, where customers have been segmented through data-mining techniques. The aim is to identify potential customers who are likely to purchase and desire services voluntarily. The research datasets are real-world data from companies, and after preprocessing using the CAID and C5.0 methods, a decision tree was used for customer classification and decision-making assistance. Costa, Fonseca et al. (2017) developed a model for predicting students' failure in their initial terms by using data from Brazilian university students who undertook a preliminary planning course. They employed various classification methods, including Naïve Bayes, J48 decision tree, SVM, and neural networks, and compared their performance using the F-measure in different weeks. Eventually, SVM with the highest effectiveness was introduced. Wang, Ji et al. (2018) examined recent advances in data-mining from various data sources. Due to advances in communication technology, large amounts of data are being generated from various sources, posing challenges in extracting useful information. In this paper, data-mining methods are categorized into four groups: (1) trend analysis, (2) source classification, (3) source categorization, and (4) source dispersion, and suitable methods are mentioned in each group. Sotomayor, Hampel et al. (2018) used the K-means method and genetic algorithm KNN to interpret the complex matrix of water quality. 21 physical, chemical, and microbiological variables from 80 different water quality samples were considered. Initially, the K-means algorithm was used for sample classification, and then the Genetic algorithm KNN was used to classify the results obtained from the K-means algorithm.

More recently, Yang (2019), (Pei 2020), Cong, Luo et al. (2022) and Xu, Hong et al. (2023) presented data-driven pricing models evaluating customer behavior or dynamic nature of markets. Although, great efforts are done in this area, still there are major gaps. In this study, the development of a distribution system is considered, which includes a wholesaler and a set of retailers. The wholesaler and retailers compete on optimal product pricing and desired customer attractiveness in order to attract more customers, increase overall satisfaction, and enhance their profits. To solve the proposed model, structural modifications using K-K-T method were employed, while a data-driven approach was utilized for price optimization with a dynamic perspective to allow for proper comparison. Factors such as competition between wholesalers and retailers, considering customer behavior, and creating two scenarios of structural modifications and data-driven learning methods are among the innovations of this research. It can be said that the present study is among the first research attempts to combine and integrate these concepts towards increasing the efficiency of distribution systems. Finally, an endeavor is undertaken to enrich the existing body of literature by seamlessly integrating pricing strategies with state-of-the-art data-mining methodologies, thereby introducing a pioneering framework that elevates the discourse surrounding pricing decisions. This novel approach meticulously incorporates the fundamental aspect of customer satisfaction by leveraging market-based demand as a guiding principle. Furthermore, the study delves into a practical application within an industrial context, shedding light on a previously proposed case problems to extract invaluable managerial insights of profound significance. The main contributions and features of the paper which highlights it from literature is as follows:

- Proposes a competitive model underpinned by behavior-based demand, offering a nuanced perspective on distribution systems within a competitive landscape.
- Integrates both horizontal and vertical forms of competition with customer behavior considerations, thereby presenting a comprehensive analysis of market dynamics.
- Utilizes the Stackelberg game theory to model the intricate dynamics of uncooperative competition between wholesaler and retailers, elucidating vertical competition in a bi-level structure.
- Addresses the behavior-based price discrimination model through a dual-method approach, encompassing conventional structural modifications alongside an innovative data-driven solution.

Problem Definition

In recent times, there has been a substantial increase in customer orders, accompanied by shifts in customer preferences and the rise of e-tailing. Unfortunately, these changes have resulted in lost demand and inefficiencies for corporations in meeting customer needs. As a result, there is a pressing need for improvements in the existing distribution systems, particularly in traditional formats, to effectively fulfill customer demands while simultaneously reducing associated costs. Furthermore, mathematical models aimed at improving the system should account for customer behavior and their sensitivity towards crucial factors in order to enhance the realism of the model. Additionally, it is important to acknowledge that the entities involved in the distribution system face competition from various sources, including competitors offering similar or substitute products that pose a threat in the market. In light of these challenges, a proposed solution is a multi-level, multi-product distribution system that aims to address the aforementioned gaps with precision.

This paper examines the concept of a distribution channel in the context of determining optimal prices, market share, and demanded quantities in an environment of uneven competition. The analysis focuses on the two primary components of the distribution channel: the wholesaler and the retailers, as illustrated in **Fig 1**. Within this structure, the distribution channel encompasses different levels that cater to customer needs by offering various product types. The wholesaler assumes the role of the leader, with the retailers acting as followers, given the asymmetrical competition dynamic resembling the Stackelberg model. Additionally, there exists vertical competition between the wholesaler and the retailers and horizontal competition among individual retailers, both aimed at attracting more buyers.

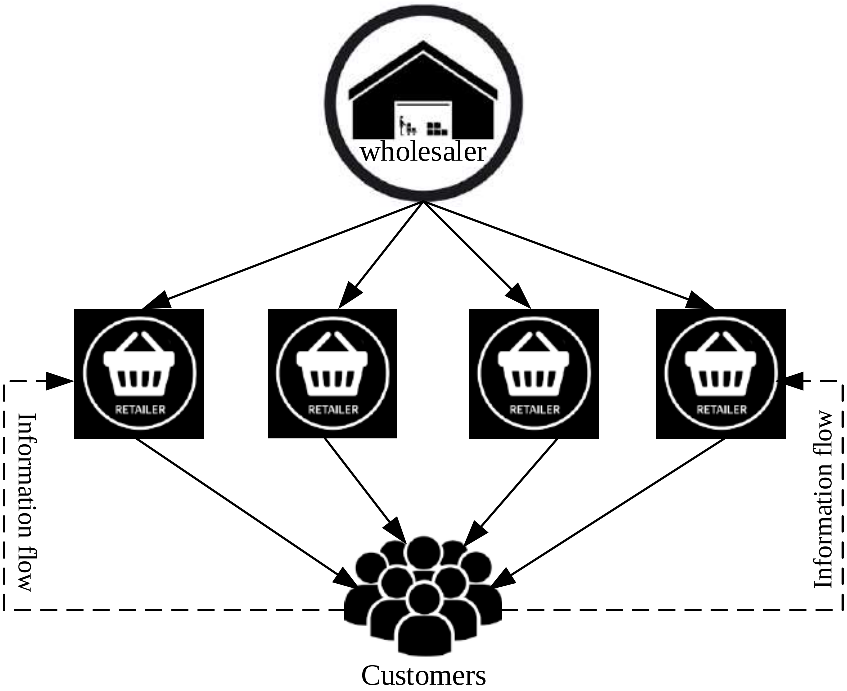

Fig 1. The structure of distribution system

The paper introduces competitive pricing strategies by deploying game theory principles. To elaborate on the Stackelberg competition between wholesalers and retailers, a bi-level programming approach is adopted. The model consists of two levels: the first level encompasses the leader (wholesaler) seeking to maximize their profits, while the second level involves the followers making optimal decisions to maximize their own profits, based on a non-linear demand that is dependent on pricing. Ultimately, this model enables the determination of optimal wholesale and retail prices, the quantity of products purchased by each retailer from the wholesaler, the total demand satisfied by each retailer, and the lost sales of each retailer.

Assumptions

The current paper is based on the following assumptions:

- The distribution system under examination consists of a wholesaler and multiple retailers, functioning to fulfill customer demands for various product types.
- Asymmetric vertical competition between the wholesaler and retailers is analyzed using the Stackelberg competition model.
- Vertical competition among the wholesaler and retailers is modeled using bi-level programming.
- Horizontal competition among the retailers is taken into account as a means to attract a larger customer base.
- Shortage is considered as a possibility within the system.
- The wholesaler experiences pass-off shortage, while the retailers face lost-sale demand.
- All entities within the system compete independently and non-cooperatively.
- The locations and capacities of both the wholesaler and retailers are predetermined and remain fixed.

Mathematical Modelling

Now, the competitive pricing model is formulated in a bi-level form as follows. The following sets, parameters and decision variables are applied through this paper.

Sets	
j	Set of retailers, $j=\{1,...,J\}$
p	Set of products, $p=\{1,...,P\}$
t	Set of time periods, $t=\{1,...,T\}$
Parameters	
cap_p	Wholesale capacity to provide product p
c_p^t	Procurement cost of product p in time period t
π_{pj}^t	Shortage cost of product p for retailer j in time period t
MD_{pj}^t	Market demand of product p for retailer j in time period t
α	Price coefficient in demand function $\in [0,1]$
LP	Lower bound of retail price
UP	Upper bound of retail price
UD	Upper bound for each retailer demand
ε	Desired threshold of clustering entropy
Decision Variable	
w_p^t	Wholesale price of product p in time period t
P_{pj}^t	Retail price of product p proposed by retailer j in time period t
D_{pj}^t	Total demand of product p for retailer j in time period
Q_{pj}^t	Total purchased product p from the wholesaler by retailer j in time period t
b_{pj}^t	Lost sale of product p for retailer j in time period t

Now, the model can be formulated in the following section. To model the competition and pricing procedure in the lower-level problem, the price-dependent demand must be defined, precisely. The customer demand in sensitive to prices and the demand must be derived by the customers utilizations based on proposed retail prices.

Pricng with non-linear price-dependent demand

In the lower-level, retailers compete on prices to seize more customers. Customer demand is sensitive to the proposed retail prices. As there exist many retailers in the market, the utility of customers for each retail price must be determined, then the consequent demand can be derived. Therefore, the utility of a customer trying to buy product p from retailer j, is considered as follows (Sheikh Sajadieh and Ziari 2021):

$$U(P_{Pj}) = u - \frac{P_{Pj}^{1-\alpha}}{1-\alpha} \qquad (\forall p, j) \tag{1}$$

High defined reservation value u in Equation (1) assures that all customers purchase product in these periods. Hence, the non-linear price-based demand function of customers in the market can be derived as follows:

$$D_{Pj} = -\frac{\partial U(P_{pj})}{\partial P_{Pj}} = P_{Pj}^{-\alpha} \qquad (\forall p, j) \tag{2}$$

where $\alpha = -\frac{\partial Q_{Pj}}{\partial P_{pj}} \frac{P_{pj}}{Q_{pj}}$ $, \alpha \in [0,1)$. Different values in range $0 < \alpha < 1$ denote the total demand under different price values (De Nijs and Rhodes 2013). It should be noticed that percent of quantity changes is less than percent of price changes (De Nijs 2017). Now, the mathematical model can be formulated as follows:

Upper Level (as Leader)

$$Max\, Z = \sum_j \sum_p \sum_t (w_p^t - c_p^t) Q_{pj}^t \tag{3}$$

$s.t.$

$$\sum_j Q_{pj}^t \le cap_p \qquad (\forall p, t) \tag{4}$$

$$Q_{pj}^t, w_p^t \in R^+ \qquad (\forall j, p, t) \tag{5}$$

Lower Level (as Followers)

$$Max\, Z_j = \sum_p \sum_t P_{Pj}^t D_{Pj}^t - \sum_p \sum_t (w_P^t Q_{Pj}^t + \pi_{Pj}^t b_{Pj}^t) \tag{6}$$

$s.t.$

$$Q_{pj}^t + b_{pj}^t = D_{pj}^t \qquad (\forall j, p, t) \tag{7}$$

$$\sum_j D_{pj}^t = MD_p^t \qquad (\forall p, t) \tag{8}$$

$$P_{pj}^t \ge w_p^t \qquad (\forall j, p, t) \tag{9}$$

$$D_{pj}^t = (P_{Pj}^t)^{-\alpha} \qquad (\forall j, p, t) \tag{10}$$

$$P_{pj}^t, D_{pj}^t, b_{pj}^t, Q_{pj}^t, w_p^t \in R^+ \qquad (\forall j, p, t) \tag{11}$$

Equations (3) represents the primary profit function, which is obtained from the difference between income and product procurement costs. Constraints (4) indicates the capacity of product supply by the wholesaler for each type of product. Constraints (6) represents the profit function of retailers. Each retailer aims to maximize their profit, which is the difference between income from retail sales, including the cost of purchasing from the wholesaler, and shortage costs. Constraints (7) represents demand equilibrium, and constraint (8) indicates that the total estimated demand by retailers is equal to the total market demand. Constraints (9) and (10) respectively represent the range of retail prices and the price-dependent demand. Constraints (5) and (11) represent the status of decision variables for leader and follower issues.

Solution Approach

In the previous section, a bi-level pricing model was introduced, taking into account customer reactions. Traditionally, solving this model would involve obtaining data through surveys and deriving lower-level optimal equilibriums. Subsequently, these lower-level equilibriums would be integrated into the upper model, resulting in a single-level equivalent form of the problem. By solving this form, optimal solutions could be found. However, an alternative and innovative approach can be employed instead of deriving the lower-level equilibriums. This approach utilizes a data-driven method to evaluate market demand. By analyzing and categorizing market demand, the most suitable prices for each retailer based on their respective customer base can be determined. With these data-based retail prices, the model can be solved to identify the appropriate wholesale price.

The main distinction between the traditional and novel approaches lies in the dynamic nature of the latter. In the novel approach, the system is equipped with a dynamic model that can adapt and deliver new prices in each time period, accounting for even minor changes in customer behavior and purchase histories. This flexibility is not easily achievable with the traditional approach. To illustrate these different approaches, two distinct scenarios will be presented in this section.

Scenario 1: Structural modification method

In literature, there are very few and limited exact methods available for solving mixed-integer linear bi-level problems. However, for bi-level problems where there is no integer variable in the lower-level problem (follower), several methods have been proposed, which can generally be classified into two categories: enumerative methods and structural modification methods (Ziari, Ghomi-Avili et al. 2022). Enumerative methods depend on the type of bi-level problem being examined, where the optimal solution lies at a corner point belonging to a feasible region created by upper-level or lower-level constraints. On the other hand, structural modification methods transform the bi-level problem into a single-level problem. For example, by adding constraints to the lower-level problem based on Karush-Kuhn-Tucker conditions, the lower-level problem becomes bounded by the constraints of the main problem, resulting in a single-level problem (Bialas and Karwan 1978), (Bard 1984), (Chen and Florian 1992). Other research works have also operated based on the enumerative search for optimal points (Gao, Wu et al. 2005), (Allende and Still 2013). Structural modification methods have utilized more than Karush-Kuhn-Tucker optimality conditions to convert the multi-level problem into a single-level problem. Ziari and S Sajadieh (2023) have used K-K-T optimality conditions to transform the multi-level problem into a single-level problem.

In the following section, the bi-level problem is transformed into a single-level problem using Karush-Kuhn-Tucker conditions. By applying Karush-Kuhn-Tucker conditions, the objective function of the follower is replaced with constraints in the alternative problem, which are essentially the same as fixed conditions and complementary conditions added to the constraints of the upper-level problem.

Linearization of the model

As stated in Sections 4.1, the lower-level problem must be merged into the upper-level problem using its equilibriums. To this aim the lower-level must be in a linear form. Due to the non-linear price-dependent demand, the objective function and also the constraints are non-linear.

To deal nonlinearity of the model, McCormick Envelopes method is used to linearize the multiplication of continuous decision variables P_{Pj}^{t} and D_{pj}^{t}. This technique allows for the linearization of these terms and facilitates the optimization process (McCormick 1976), (Ghomi-Avili, Tavakkoli-Moghaddam et al. 2021). To effectively implement this method, it is crucial to establish appropriate lower (LP) and upper (UP) bounds for the variable P_{Pj}^{t}. Additionally, considering the total market demand, it is assumed that D_{pj}^{t} is smaller than UD. With this in mind, suitable bounds can be set for each continuous variable, ensuring the model's feasibility and accuracy. Here the established bounds are derived based on historical data using the average estimator to determine the appropriate limits for each variable. When historical data is not available, an alternate method can be employed. Initially, the basic model should be solved without considering pricing and market-based demand. Subsequently, pricing factors and related constraints are incorporated into the model. Given that the optimal solutions cannot surpass those of the simpler model when additional constraints are introduced, the optimal outcomes of the basic model serve as the boundaries for the newly proposed model. Taking into account these considerations and constraints, the optimization problem can be adjusted and solved effectively. Hence, the following equations are made to linearize the model:

$$LP \le P_{pj}^{t} \le UP \qquad (\forall p, j, t) \qquad (12)$$

$$0 \le D_{pj}^{t} \le UD \qquad (\forall p, j, t) \qquad (13)$$

$$S_{Pj}^{t} = P_{Pj}^{t} \times D_{Pj}^{t} \qquad (\forall p, j, t) \qquad (14)$$

$$UP \times D_{Pj}^{t} + UD \times P_{Pj}^{t} - UP \times UD \le S_{Pj}^{t} \qquad (\forall p, j, t) \qquad (15)$$

$$S_{Pj}^{t} \le UD \times P_{Pj}^{t} + UP \times D_{Pj}^{t} \qquad (\forall p, j, t) \qquad (16)$$

$$L^{P} Q_{ij} \le H_{ij} \le U^{P} Q_{ij} \qquad (\forall p, j, t) \qquad (17)$$

Now, it is possible to substitute the nonlinear term in the objective function. This substitution allows for the inclusion of the term in a linear form, thereby enhancing the tractability of the optimization problem. These constraints also ensure that the linearized representation remains accurate and valid, enabling the successful completion of the overall linearization procedure. By incorporating these additional constraints, the optimization model can be effectively structured and ready for further optimization. Now, the bi-level problem is transformed into a single-level problem using the Karush-Kuhn-Tucker (KKT) conditions. By applying the KKT conditions, the objective function is substituted with constraints in the problem, which are essentially the same as fixed constraints and complementary conditions added to the first-level problem constraints.

Fixed constraint conditions are:

$$\nabla FF\left(P_{pj}^{t}, D_{pj}^{t}\right) - \sum_{j} \lambda_{j} \nabla G_{j} = 0 \qquad (\forall p, j, t) \qquad (18)$$

Complementary constraint conditions are:

$$\lambda_{j} G_{j} = 0 \qquad (\forall j) \qquad (19)$$

As evident from the use of the KKT method, certain nonlinear relationships are added to the problem, which are linearized using the following approach involving a binary variable γ_j.

$$\begin{cases} \lambda_j - M\,\gamma_j \le 0 \\ G_j - M\left(1-\gamma_j\right) \le 0 \end{cases} \quad (\forall j) \tag{20}$$

Now, the unified linear single-level model, incorporating the Karush-Kuhn-Tucker conditions and the linearization approach, can be easily solved using the GAMS. In the next section, the solution outputs of the model will be presented.

Scenario 2: Data-driven learning model

In this section, instead of utilizing structural modification or enumeration methods, which follow a relatively static approach in optimization, and impose numerous constraints on reflecting customer behavior or utilizing their inherent features and examining the impact of these features on the pricing process, it was deemed necessary to introduce and utilize a novel and innovative approach (Videla Cavieres 2015). This approach would be able to identify the characteristics and properties of different demand groups for each retailer based on customer purchase history or demand-related data without any supervision and provide the best suitable price for each group. Hence, by making minor changes in the demand process or even in the face of unforeseen issues such as a pandemic, the model can be utilized, and its effectiveness can be ensured. Another achievement of the model is the presentation of a dynamic pricing system, where an appropriate trend of the pricing process can be observed and good results can be obtained with any change in time. Here we used the CLIQUE algorithm for some major reasons. The CLIQUE algorithm has the capability to identify dense regions within subsets of the primary state space and merge adjacent dense regions to form clusters. One of the key criteria for evaluating the advantage of this method is its ability to handle a large number of samples. It can be applied to samples of large size. It is able to process sets that have not been properly preprocessed and contain noise. It can identify clusters with irregular geometric shapes. CLIQUE algorithm works as follows:

- **Grid-Based Partitioning**: The algorithm divides the dataset into cells using a grid-based approach. Each cell represents a small partition of the data space.
- **Density-Based Approach**: CLIQUE uses a density-based approach to identify clusters. It looks for dense regions in the dataset that satisfy a user-specified density threshold.
- **Identifying Clusters**: CLIQUE identifies clusters by finding dense units in the dataset called "cliques." A clique is a subset of the data where the density of points within a specific region exceeds the given threshold.
- **Combining Cliques**: In the algorithm, neighboring dense units or cliques are combined to form clusters. By merging adjacent cliques, the algorithm can capture the overall spatial distribution of the data points.

To clarify the details of the data-driven learning model in the second scenario, **Fig 2** is presented.

Each step of the solution and optimization process in this approach is summarized and presented as follows:

Step 1: Preprocessing customer demand data and purchase history.

(In this step, it is initially necessary to collect and aggregate all the data related to the purchases or demands of each retailer. This can be easily done based on modern e-commerce and e-tailing methods. After collecting the data, it is necessary to remove noise and errors that are logically incorrect. Furthermore, after eliminating noisy data, it is essential to replace missing information with the average value of the respective element).

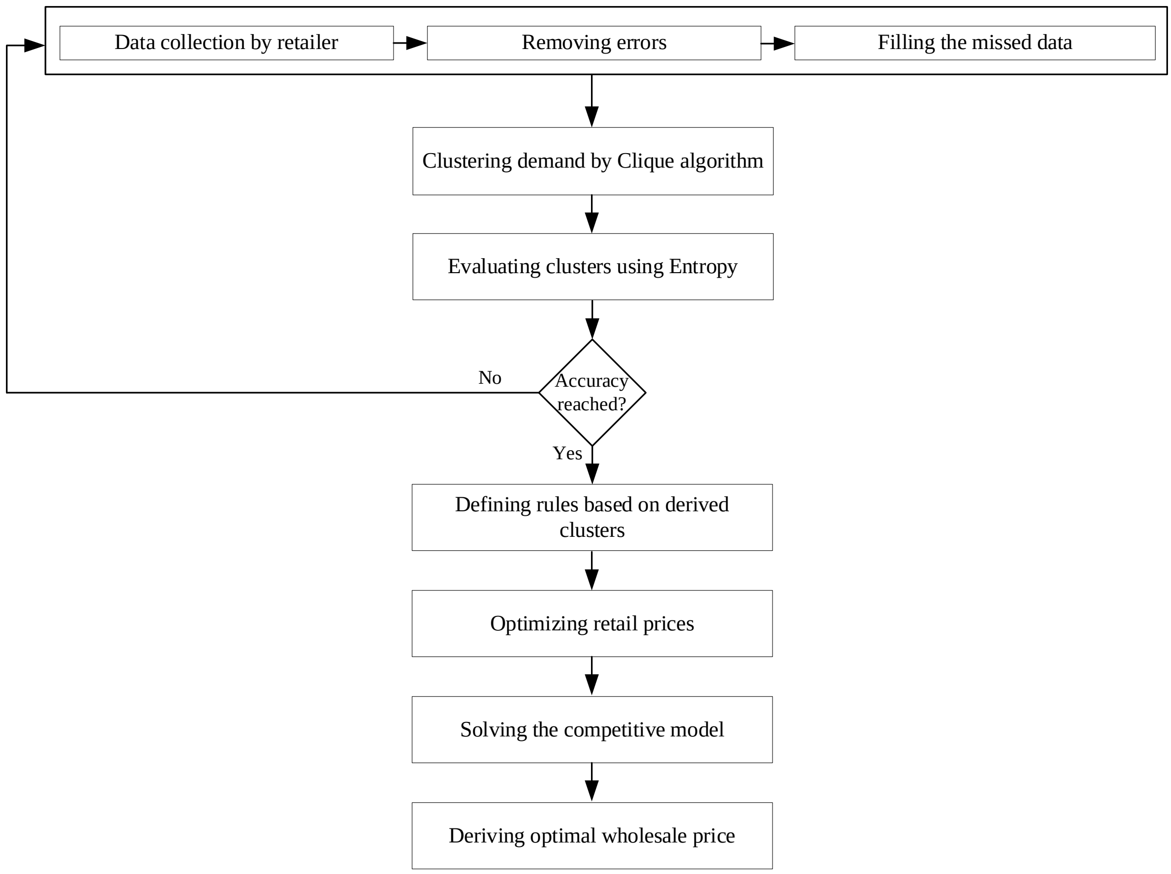

Fig 2. The structure of distribution system

Step 2: Clustering using the CLIQUE algorithm on the cleaned data obtained from Step 1. (The reason for choosing CLIQUE is that it has superior performance in terms of both execution time and accuracy compared to popular algorithms such as K-means, DBSCAN, and density-based approaches (Gramm, Guo et al. 2005). Unlike these algorithms, CLIQUE has the ability to discover clusters of any shape, making it extremely flexible. Additionally, CLIQUE is not limited by a predetermined parameter that specifies the number of clusters to be found or the number of dimensions in the data (Chrobak, Dürr et al. 2020). This means that CLIQUE can effectively identify any number of clusters in datasets of varying dimensions. Another advantage of CLIQUE is its simplicity, which not only facilitates its implementation but also enhances the interpretability of the clustering results (Cao, Hu et al. 2023)).
Step 3: After clustering, it is necessary to examine the quality of the clusters. For this purpose, the entropy measure is utilized, and by comparing the entropy level with the desired threshold ε, decisions about the continuation of the algorithm can be made (Maheshwari, Mohanty et al. 2023). If the entropy of the clusters is below the desired epsilon, proceed to Step 4. Otherwise, the collected data needs to be reviewed in terms of input quality or sample size.

Clusters entropy measure: $E(C) = -\sum_{i=1}^{r} pc_i \log(pc_i)$, where $pc_i = \frac{n_i}{n}$ is the probability of cluster i.
Step 4: Based on the clusters obtained from Step 3, it is necessary to define rules using repetitive patterns and data features within each cluster, determining the appropriate price level for each cluster.
Step 5: Using the rules created in Step 4, the optimal prices for each demand category of the retailers will be determined.

Step 6: Using the optimal values obtained from Step 5, the mathematical competitive model will be solved, and the optimal wholesale price will be optimized as well.

By iterating the algorithm in different time periods, and by revising and solving the model again after minor changes and shifts in demand or customer purchasing behavior, optimal pricing values for the distribution system can be calculated. The model is implemented using the Python programming language, and the relevant results are reported in the next section to allow for a comparison and evaluation of the traditional method's desirability.

Numerical Example and Sensitivity Analyses

In this section, we explore the effectiveness of the proposed approach by implementing a practical case problem. The case involves analyzing the proposed model based on the dataset previously presented by Ali, Rahman et al. (2018) to perform sensitivity analyses. Furthermore, all other parameters can be found in **Table 1**. Subsequently, we subject the developed model to various test scenarios and present comprehensive insights into the results. Solving the modified scenario involved utilizing GAMS 24.3.3 on a Corei5 2.27 GHz system with the CPLEX solver, completing the process within a remarkable 20.71 seconds.

Table 1. Input data

Parameter	Value	Parameter	Value
c_p	Uniform [0.0035, 0.0065] million	π^t_{pj}	Uniform [0.0055, 0.0070] million
cap_p	Uniform [5500, 14500]	MD_{pj}	Uniform [8000, 15000]
LP	0.016 million	UP	0.024 million
α	0.25	UD	14000

The model validation has been conducted using the cost parameter of shortage. In such a way that, by decreasing the cost of shortage, the level of lost sale increases, which has also been observed in this model. The table below summarizes a portion of the conducted experiment information.

Test problems	Shortage cost	Lost sale in each period				Total lost sale
		1	2	3	4	
1	0.0070	871	854	920	892	3537
2	0.0065	958	948	1031	987	3924
3	0.0062	1072	1044	1145	1114	4375
4	0.0060	1183	1139	1257	1235	4814
5	0.0055	1308	1261	1382	1369	5320

Sensitivity analyses on data-driven model

In order to assess the efficacy of structural modification and data-driven approach, the proposed model was applied to a series of test problems including a diverse range of 10 distinct samples. The experimental dataset is systematically created in accordance with the specified case problem, followed by a process of amalgamation aimed at consolidating similar test cases. Ultimately, a refined set of 10 distinct instances is chosen to serve as testbeds for scrutinizing the intricacies of the identified problem domain. Each selected instance is strategically configured to exhibit variations in key parameters, thereby encompassing a diverse range of operational scenarios. For instance, in the event that the initial instance accentuates a high-level impact of the low-price coefficient, subsequent instances are thoughtfully structured to emphasize lower values of this crucial parameter. This deliberate variation in parameter settings among the chosen instances ensures a comprehensive exploration of the problem space, facilitating a comprehensive analysis that explores complexities. This approach aimed to provide a more nuanced understanding of the comparative outcomes between the two scenarios. The findings, depicted in **Fig 3**, unequivocally demonstrate the superiority of the data-driven learning model across all experimental scenarios. However, amidst the triumph of the data-driven approach, a significant challenge takes center stage. While the structural modification method showcases a consistent and steady upward trajectory, the case involving data learning deviates from this anticipated pattern. These results not only underscore the significance of data-driven learning in influencing the outcomes but also emphasize the nature of its impact.

In this context, data-driven pricing is not a short-term optimization strategy but a calculated investment in future success. It involves recognizing the unique preferences and purchasing power of different customer segments, and tailoring pricing structures accordingly. By doing so, businesses can tap into previously untapped market segments and attract customers who might have been deterred by higher prices. This inclusive approach paves the way for higher customer retention rates, improved brand reputation, and ultimately leads to overall growth and profitability.

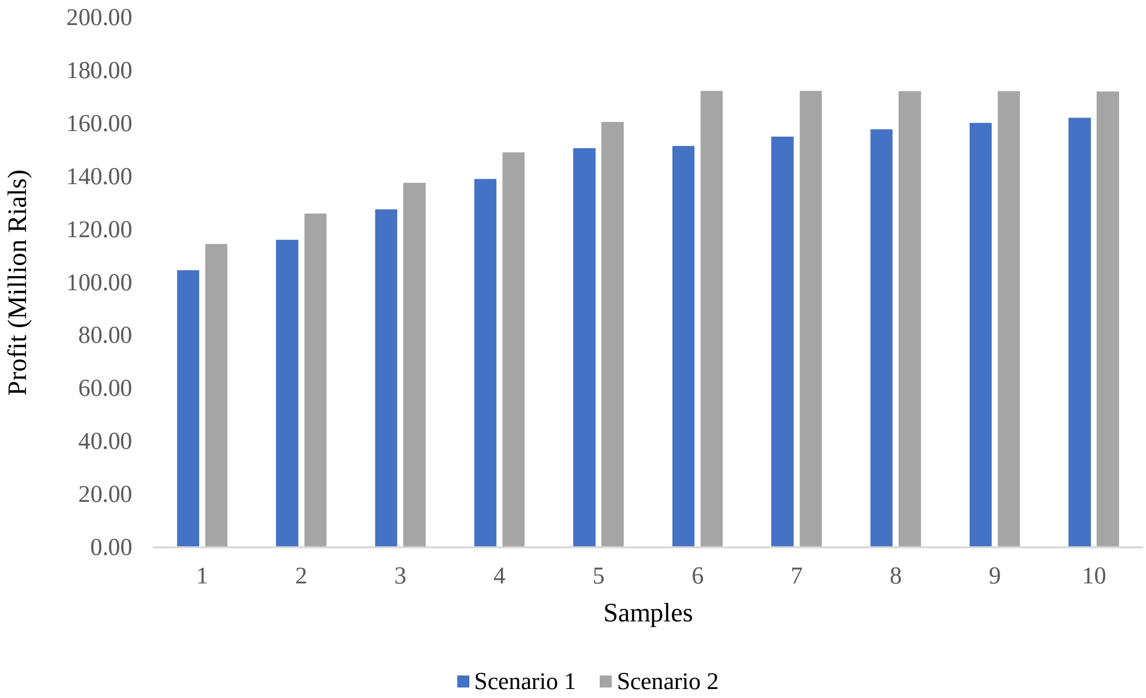

Fig 3. Derived profit in scenarios 1 versus scenario 2 in different samples

Furthermore, valuable insights can be derived from the comprehensive analysis provided in **Table 2** and **Table 3**. These tables shed light on various critical elements, presenting a wide range of factors that have undergone detailed examination. These factors encompass the overall profits of the wholesale section, average wholesale price, average percent of lost sale in the market and average retail prices. By evaluating these tables, one can unearth the important information that uncovers the intricate dynamics at play within the wholesale market, offering suitable insights that can help decision-making in the scope of pricing process in distribution systems.

Table 2. Comparison between Scenarios 1 and 2 in profit and average wholesale prices

Test problems	Total profit (Million Rials)		Average wholesale price (Million Rials)	
	Scenario 1	Scenario 2	Scenario 1	Scenario 2
1	1050.4	1155.6	0.094	0.092
2	1190.2	1333.0	0.085	0.083
3	1270.3	1435.4	0.083	0.081
4	1185.3	1315.6	0.081	0.078
5	1070.8	1220.7	0.091	0.088
6	1161.2	1300.5	0.090	0.089
7	1236.1	1421.9	0.088	0.085
8	1114.7	1270.7	0.091	0.089
9	1086.9	1249.9	0.093	0.092
10	1109.4	1253.6	0.085	0.082

Table 3. Comparison between Scenarios 1 and 2 in average lost sale and retail prices

Test problems	Average lost sale (%)		Average retail price (Million Rials)	
	Scenario 1	Scenario 2	Scenario 1	Scenario 2
1	0.128	0.085	0.1128	0.1094
2	0.132	0.079	0.1045	0.1024
3	0.145	0.100	0.1004	0.0975
4	0.129	0.095	0.1012	0.0991
5	0.136	0.088	0.1128	0.1094
6	0.144	0.091	0.1134	0.1099
7	0.128	0.076	0.1091	0.1049
8	0.139	0.082	0.1119	0.1096
9	0.148	0.101	0.1153	0.1118
10	0.124	0.093	0.1054	0.1022

Sensitivity analysis on the wholesale capacity

Another analysis is carried to investigate the impact of wholesale capacity on the optimization of wholesaler (as leader) objective function. As it is depicted in **Fig 4**, the analysis reveals an interesting trend whereby the profit values exhibit an upward trajectory as the capacity is augmented. It is worth highlighting that the enhancement of wholesale' capacity alone does not necessarily translate into an increase in customer demand.

In conclusion, the efficacy of pricing models within the system highlights the importance of identification and prioritization of key factors that having significant influence over the overall performance of the distribution system. To maximize profitability, managers must diligently direct their attention towards what can data deliver them.

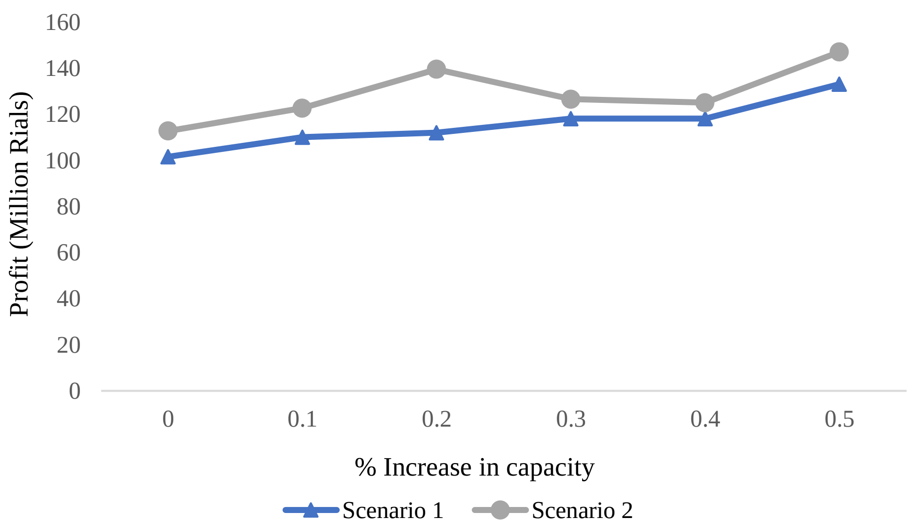

Fig 4. Delivered profit in Scenarios 1 and 2 increasing wholesale capacity

Conclusions and Future Research Directions

This research involves the implementation of the pricing concept, focusing on two crucial factors: competition and customer behavior. The main players in the distribution system, namely wholesaler and retailers, are assumed to engage in a competitive environment. To model this asymmetric competition, the Stackelberg game approach is employed. The study introduces a bi-level model consisting of an upper level and a lower level. The upper level represents the decisions made by the wholesaler acting as leader, while the lower level addresses the concerns of retailers as followers. Wholesaler aims to maximize his profit by offering the most favorable prices to retailers, while the lower-level problem targets around maximizing retailer profit considering customer behavior.

To solve the model, two distinct scenarios are taken into account. The first scenario adopts a traditional survey approach, utilizing structural modifications. The initial step involves linearizing the model, followed by converting the bi-level model into a single-level equivalent using the Karush–Kuhn–Tucker (KKT) conditions. On the other hand, the second scenario proposes a data-driven learning procedure to tackle the model. Specifically, a rule-based pricing model is suggested based on the clusters formed by applying the CLIQUE algorithm. The model is subsequently applied to an industrial case study to validate its effectiveness and extract sensitivity analyses. Additionally, valuable managerial insights are presented based on the outcomes of the sensitivity analyses, offering new ways for further researches. Some critical advantages are derived. By employing the Stackelberg game approach, the study effectively captures the competitive dynamics within the distribution system, providing a strategic framework for wholesalers and retailers to enhance their profitability. The bi-level pricing model allows for a comprehensive analysis of pricing strategies at different levels of the supply chain, enabling stakeholders to make informed decisions based on market conditions and customer preferences. The data-driven learning procedure proposed in the research offers a practical and innovative solution to pricing optimization, leveraging clustering algorithms to derive rule-based pricing models tailored to specific market segments. Some challenges are also existed. The complexity of the bi-level pricing model may pose challenges in real-world implementation and scalability, requiring sophisticated computational resources and expertise for effective application. The reliance on specific clustering algorithms such as the CLIQUE algorithm may limit the generalizability of the pricing strategies proposed in the study, as different market contexts may require alternative data-driven approaches. While the research emphasizes the importance of customer satisfaction and market-based demand in pricing decisions, the impact of external factors and uncertainties on pricing strategies is not explicitly addressed, potentially limiting the model's robustness in dynamic environments. In order to advance studies on competitive pricing with consideration of customer behavior, the utilization of other clustering methods could be beneficial. Furthermore, exploring the inclusion of inherent uncertainties in price coefficients would enhance the attractiveness and applicability of future research. Another challenging area for extending similar pricing models lies in the examination of disruption in different sections.

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