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RESEARCH PAPER

Optimization of a Multi-Item Inventory Model Considering Partial Backordering and Imperfect Products Using Interior-Point, SA and WCA

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Abstract

Developing and optimizing effective inventory systems considering realistic constraints and practical assumptions can help managers remarkably decrease inventory and consequently supply chain costs. In this research, we propose a new variant of the multi-item inventory model taking into account warehouse capacity, on-hand budget constraints, imperfect products in supply deliveries and partial backordering where the products can be converted into perfect products by a local repair shop. To deal with the proposed model, three solution approaches, including interior-point technique, as an exact method, and two metaheuristics based on Simulated Annealing (SA) and Water Cycle Algorithm (WCA), are proposed. Extensive computational experiments are conducted on different sets of instances. Using different measures such as RPD, PRE, and computational time, the performance of the solution approaches is evaluated within different test instances. The results show that the WCA outperforms the two other approaches and leads to the best solutions in the proposed problem.

Keywords:
Inventory, Imperfect Products, Repair, Partial Backordering, Water Cycle Algorithm, Interior-Point Algorithm, Simulated Annealing Algorithm

Introduction

In real-world inventory systems, supply deliveries may contain defective items, leading to additional costs and reduced customer satisfaction. Traditional deterministic models often fail to address the complexities introduced by such defects adequately. This study aims to develop a more sophisticated inventory model that considers partial backordering and imperfect products, extending the existing single-item models to a multi-item formulation. This extension is crucial because it more accurately reflects the complexities of real-world inventory systems, where multiple items with varying defect rates and backordering policies must be managed simultaneously.

The specific problem addressed in this manuscript closely aligns with the issues studied by Khalilpourazari et al. (2019a). However, our research introduces significant new contributions by extending the single-item problem to a multi-item context. While Khalilpourazari et al. (2019b) addressed a multi-item economic order quantity model with imperfect items, our study

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further enhances this by considering partial backordering and the interactions between multiple items, which introduces additional layers of complexity. This multi-item approach, coupled with partial backordering, has not been comprehensively studied in the literature, making our study a novel contribution.

In previous studies, the complexities of this multi-item inventory model, we utilize a combination of Simulated Annealing (SA) and the Water Cycle Algorithm (WCA). Khalilpourazari et al. (2019b) utilized the interior-point method, Grey Wolf Optimizer (GWO), and Moth-Flame Optimization (MFO) for solving their problem. In contrast, our research leverages the complementary strengths of SA and WCA to tackle the optimization challenges effectively. The hybrid application of these methods to a multi-item inventory model considering partial backordering and imperfect products is novel and offers new perspectives on solving such complex problems.

Furthermore, we have introduced modifications to the standard implementations of SA and WCA to enhance their performance specifically for our problem. These modifications include customized cooling schedules in SA and adaptive runoff coefficients in WCA, tailored to better handle the non-linear and multi-modal nature of the multi-item inventory optimization problem. These enhancements ensure more robust and faster convergence to near-optimal solutions compared to traditional implementations. Unlike Khalilpourazari et al. (2019b), who focused on the efficiency of MFO and GWO, our study emphasizes the balance between exploration and exploitation in SA and WCA, making them particularly suited for high-dimensional and complex optimization problems.

The main contributions of this study can be summarized as follows:

1. New Problem Formulation: We present a novel multi-item inventory model that integrates partial backordering and imperfect products, expanding the scope of existing single-item models.
2. Hybrid Optimization Approach: We propose a unique combination of SA and WCA, along with specific modifications to these algorithms, to effectively solve the complex optimization problem.
3. Performance Improvement: Our customized SA and WCA implementations demonstrate superior performance in terms of solution quality and computational efficiency compared to traditional methods.

Comparison with the research of Khalilpourazari et al. (2019b):

In contrast to the work by Khalilpourazari et al. (2019b), which used stochastic operational constraints and focused on exact and meta-heuristic methods like GWO and MFO, our study focuses on the practical implications of partial backordering and defective items in a multi-item setting. While their study performed well in small to medium-sized problems, our research aims to extend this to larger, more complex inventory systems by using the hybrid SA and WCA approach, which provides robust and scalable solutions.

By addressing these new dimensions, our research provides a comprehensive framework for inventory management that better mirrors real-world conditions and offers practical solutions for businesses dealing with multiple items and imperfect supplies. These contributions not only extend the current body of knowledge but also pave the way for further research and practical applications in inventory optimization.

In summary, this study contributes to the literature by not only addressing the complexities of inventory models with partial backordering and imperfect products but also by demonstrating the practical utility of SA and WCA in solving such problems. We believe that the insights gained from this research will pave the way for future studies and practical applications in inventory management, leading to more efficient and cost-effective operations.

Literature Review

Khan et al. (2011) presented an inventory model with defective products, taking into account the imperfectness of the inspection process [4]. They assumed that the operator may commit type I and II errors during the inspection process. They also proposed that the imperfect items have been classified as perfect ones and used to meet the customers' demand, would be immediately returned to the inventory system by the customers. Hsu and Hsu (2013) improved the formulation presented by Khan et al. (2011) by considering the allowability of backorders in the EOQ model [5]. They showed the convexity of the total profit function and presented simple approaches to solve the model. Skouri et al. (2014) extended an inventory model considering imperfect supply lots [6]. They proposed that the supply deliveries are checked upon arrival, and if any defective item is found, the entire lot will be rejected. The rejected batch would be sent back to the supplier, and the supplier should deliver the perfect supply batch in the next period. Taleizadeh et al. (2016) studied the EOQ model in the presence of defective items. They supposed that the defective items could be converted into perfect products by sending them to a local repair shop. They have considered defective products and partial backordering [7]. Khalilpourazari et al. (2019a) developed a new mathematical model for multi-item model considering defective supply batches and partial backordering under uncertainity. The model aims at minimizing the total inventory costs by determining optimal values of the decision variables including time interval between successive perfect supply deliveries. Basic Chance Constraint Programming (BCCP) and Robust Fuzzy Chance Constraint programming (RFCCP) approaches have been utilized to deal with the uncertain parameters of the mathematical model [8]. In another work, Khalilpourazari et al. (2019b) proposed a new version of multi-item EOQ model with imperfect items in supply deliveries and uncertain warehouse capacity and budget constraints. They assumed that the inspection process to classify the items is not perfect and may contain two types of error: Type-I and Type-II. Their proposed model determined the optimal order and back order sizes of the items to achieve maximum total profit. Three different solution methods including the interior-point and two meta-heuristics named grey wolf optimizer (GWO) and moth-flame optimization (MFO) algorithms were utilized to solve the developed constrained nonlinear problem [9].

Taheri-Tolgari et al. (2019) addressed a production system with the defective quality process considering partial backlogging under uncertainiy, inspection errors and preventive maintenance. They considered input parameters as a triangular fuzzy environment, and the output parameters of the model have been solved by the Zadeh's extension principle and nonlinear parametric programming [10].

Nobil et al. (2020) generalized the inventory model presented by Salameh and Jaber (2000) by proposing the optimal reorder point, based on the specifications such that inventory systems do not suffer from shortage. Their model helps managers and researchers to design the inventory systems considering certain rates of imperfect production, lead-time, and system costs to maximize system efficiency and profit [11]. Taheri-Tolgari and Mirzazadeh (2021) represented a multi-item single source production quantity model for random imperfect items with repair failure, inspection errors, sales return, scraps, and backordering. Their study aimed at determining the optimum cycle length and the optimal backordered quantity for each item to minimize the total expected value cost [12].

Table 1 shows some researches focusing on the EOQ models with inperfect products and indicates the place where our research stands in the related literature. In this paper, we introduce a multi-item inventory model with defective products in deliveries considering some realistic operational constraints such as partial backordering, warehouse capacity and on-hand budget constraints.

Many industries face different types of products in real applications. Therefore, considering different items in the mathematical model can significantly improve the model's applicability, which has been considered in this research. Also, inventory systems face many operational and physical constraints, in practice, which can meaningfully reduce the model solution space. Ignoring such constraints may lead to an unrealistic model, and consequently infeasible solutions in real-life cases. This research proposes two significant operational constraints in inventory systems. The first one is the limited capacity of warehouse to keep products in stock. The second one is the on-hand budget constraint where the retailer has a limited budget to purchase items at the beginning of the planning period.

This study is an extension of the work presented by Taleizadeh et al. (2016), in which a multi-item EOQ model is developed with the presence of defective items in supply deliveries, partial backordering, warehouse capacity and on-hand budget constraints [7]. The interior-point technique, as an exact method, and two metaheuristics based on Simulated Annealing (SA) and Water Cycle Algorithm (WCA) have been used. The proposed mathematical model aims at determining the optimal order and back order sizes of the items in order to maximize the total profit.

Table 1 A review of the literature of inventory models

Publication	Author	Multi-Item	Constraint	Back Orders	Inspection errors	Inspection	Remark(s)
[1]	Salameh(2000)	No	No	No	No	Yes	Random defective rate
[22]	Papachristos (2006)	No	No	No	No	Yes	Defective items
[23]	Chung(2006)	No	No	No	No	Yes	Delay in payments
[24]	Wee(2007)	No	No	Yes	No	Yes	Screening cost
[3]	Eroglu(2007)	No	No	Yes	No	Yes	Scrap items
[25]	Konstantaras (2007)	No	No	No	No	Yes	In-house inspection
[26]	Maddah (2008)	No	No	No	No	Yes	Several batches in a lot
[27]	Chung(2009)	No	No	No	No	Yes	Two warehouses
[28]	Lin(2010)	No	No	No	No	Yes	Discount
[29]	Khan(2010)	No	No	No	No	Yes	Learning effect
[30]	Roy(2011)	No	No	No	No	Yes	Partial backordering
[4]	Khan(2011)	No	No	No	Yes	Yes	Returns
[31]	Ouyang(2012)	No	No	No	No	Yes	Permissible delay in payments
[5]	Hsu(2013)	No	No	Yes	Yes	Yes	Returns
[6]	Skouri(2014)	No	No	Yes	No	Yes	Rejection of imperfect lots
[7]	Taleizadeh(2016)	No	No	Yes	No	Yes	Repair
[8]	Khalilpourazari (2019)	Yes	Yes	No	No	No	Disruption
[9]	Khalilpourazari (2019)	Yes	Yes	Yes	Yes	Yes	optimization
[39]	Taheri(2019)	No	No	Yes	Yes	Yes	preventive maintenance
[10]	Taheri(2019)	No	No	No	No	No	Uncertain known price
[32]	Tahami(2019)	No	No	No	No	No	Lead time
[11]	Nobil(2020)	No	No	No	No	No	Reordering
[33]	De(2021)	No	No	No	No	No	Fuzzy reasoning
[34]	Li(2021)	No	No	No	No	No	Customer credit
[35]	Çalışkan(2021)	No	No	No	No	No	Deteriorating items
[36]	Paul(2021)	No	No	No	No	Yes	Selling price-dependent demand
[12]	Taheri(2021)	Yes	No	Yes	Yes	Yes	rework failure
[37]	Taheri(2022)	No	No	No	Yes	Yes	crisp and fuzzy approach
[38]	Masoudi(2022)	No	Yes	No	No	No	Evidence reasoning
[40]	Alamri(2022)	No	No	No	Yes	Yes	Learning Effect
[41]	Asadkhani(2022)	No	No	No	Yes	Yes	Learning Effect
[42]	Kishore(2022)	No	No	No	Yes	Yes	Two-stage credit financing
Current Research	**2024**	**Yes**	**Yes**	**Yes**	**No**	**Yes**	**Repair and partial backlogging**

The mathematical model presented in this research can be applied in any industries dealing with a different number of various products while ensuring the perfectness of the supply deliveries. This paper is organized as follows. The proposed problem is described and formulated in Sections 2. In Section 3, we provide efficient solution methodologies to address the problem. The computational results are represented in Section 4. Finally, the paper ends with conclusions and interesting future research suggestions in Section 5.

Problem definition

Let us consider a buyer who purchases some items from a supplier. The supplied batch (lot) may contain defective products because of quality inspection errors, inappropriate transportation, etc. The buyer inspects the whole lot upon arrival to detect the defective items which should be replaced by perfect ones. Since the supplier is far from the buyer, and due to high ordering costs and lead time, it is impossible to make a new order. Instead of making a new order, the buyer prefers to repair the defective products. In this research, it is assumed that all the defective items can be repaired. The defective items are removed from the batch and sent to a local repair facility. After the reparation period, the buyer will receive the repaired products as a single batch. Also, multiple items and several operational constraints including warehouse capacity and on-hand budget constraints have been considered.

Mathematical modeling

In this section, we present the research methodology employed to address the complexities of multi-item inventory optimization considering defective products and repair processes. Additionally, we provide a detailed description of the mathematical model developed to formulate the problem.

Our research focuses on a scenario where a buyer purchases items from a supplier, and the supplied batch may contain defective products due to various factors such as quality inspection errors and inappropriate transportation. Upon the arrival of the batch, the buyer conducts a thorough inspection to identify defective items, which are then sent to a local repair facility for necessary repairs. It is assumed that all defective items can be repaired, and after the reparation period, the repaired products are returned to the buyer as a single batch.

In addition to defective products and repair processes, our model considers multiple items and several operational constraints, including warehouse capacity and on-hand budget constraints. The inspection process, characterized by a rate denoted as x, is conducted upon arrival by the buyer to identify defective products. The inspection period is determined by $\frac{1}{x}$, and defective goods are subsequently returned to the repair facility. After repair and transportation, the restored products become available to meet customer demands. It is noteworthy that the inspection rate x surpasses the demand rate.

The mathematical model developed to address this scenario incorporates decision variables to represent ordering and backordering quantities for each item, along with binary variables to indicate the presence of defective items. Constraints are formulated to ensure adherence to inventory levels, warehouse capacities, and budget constraints throughout the planning horizon. The objective function aims to maximize total profit, considering costs associated with ordering, holding, repairing defective items, and potential lost sales due to backordering.

To solve the formulated mathematical model, we introduce modifications to the standard implementations of Simulated Annealing (SA) and the Water Cycle Algorithm (WCA). These modifications include customized cooling schedules in SA and adaptive runoff coefficients in WCA, tailored to better handle the non-linear and multi-modal nature of the multi-item inventory optimization problem. By emphasizing the balance between exploration and exploitation in SA and WCA, our study ensures more robust and faster convergence to near-optimal solutions compared to traditional implementations. Figure 1 shows the inventory level over time.

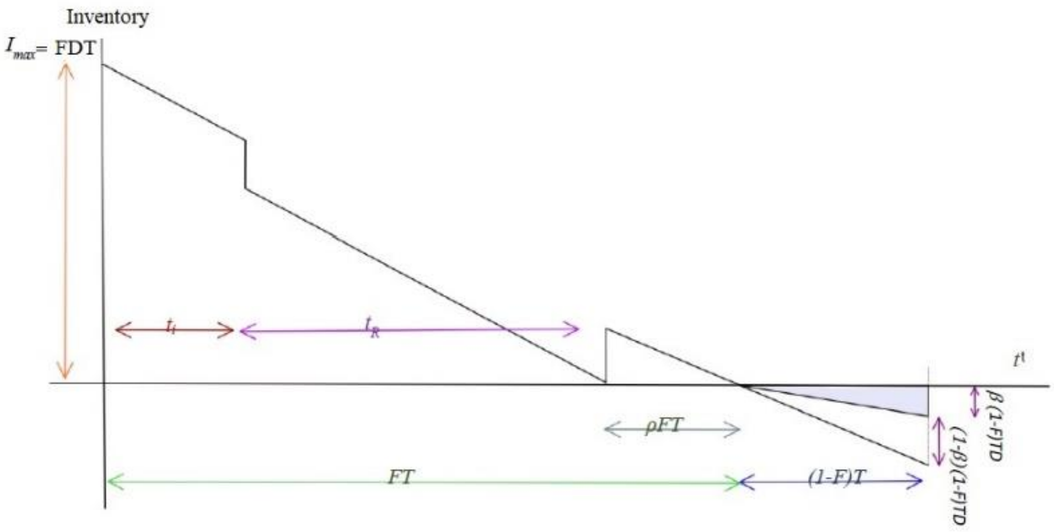

Fig. 1 The inventory level over time Taleizadeh et al. (2016)

Let us introduce the following parameters and decision variables:

Parameters:

t_i : inspection time
t_R : repair time
t_T : transportation time of defective items
D : demand rate
R : reparation rate
x : inspection rate
ρ : the fraction of imperfect products
A : fixed transportation cost
k : buyer's ordering cost
s : repair setup cost
h : holding cost
h' : holding cost at the repair
h_R : holding cost of a renovated item
c_i : unit inspection cost
c_u : unit cost
c_l : material and labor cost to repair a product
c_t : unit transportation cost
c_R : unit repair cost charged to the buyer
g : cost of lost sales
π : backorder cost
P : unit price
m : markup percentage by the repair shop
β : fraction of backordered demand
$f(\rho)$: probability density function of imperfect products (ρ)
$E[x]$: expected value of a random variable

Decision variables:

T : cycle time (time unit)

F : percentage of duration in which inventory level is positive (%)

First, the calculation of the total inventory profit function is presented. At the beginning of the inventory cycle, the maximum inventory level is accrued, which is equal to FTD. In the beginning, the buyer inspects the supplied batch upon arrival at rate *x*. Thus, the inspection time is equal to $t_i = FTD / x$. After the inspection process, ρFTD items are found as defective ones. The defective items are then sent to the reparation site. The repair time can be calculated as $t_R = \rho FTD / R + t_T$. Therefore, the repair cost is equal to $(1+m)\left[\left(\frac{s+2A}{\rho FTD}\right)+(c_l+2c_t+h^{'}t_R\right]$.

Based on the inventory behavior figure, the total holding and backordering costs can be calculated as follows:

$$HC = h\left(\frac{(1-\rho)^2 F^2TD}{2} + \frac{\rho T(FD)^2}{x}\right) + h_R\frac{(\rho F)^2 DT}{2} \tag{1}$$

$$SC = \pi\frac{\beta(1-F)^2 TD}{2} + g(1-\beta)(1-F)D \tag{2}$$

Based on the above-mentioned formulations, the maximization of the total inventory profit is as follows:

$$ETP(T,F) = PD\big(F+\beta(1-F)\big) - \left[\begin{array}{l}\frac{k}{T} + c_u(FD+\beta(1-F)D) + c_iFD + h\left[\frac{E((1-\rho)^2)F^2TD}{2} + \frac{E(\rho)D^2F^2T^2}{x}\right] \\ +(1+m)\left(\frac{s+2A}{T} + E(\rho)FD(c_i+2c_t+h't_T) + \frac{E(\rho^2)h'D^2F^2T}{x}\right) \\ h_R\frac{E(\rho^2)DF^2T}{2} + \pi\frac{\beta(1-F)^2TD}{2} + g(1-\beta)(1-F)D\end{array}\right] \tag{3}$$

Although the above mathematical model is applicable in many industries, it still includes some unrealistic assumptions that can significantly restrict the model's applicability. First, the model considers the single-item EOQ model. However, many industries face different types of products in real applications. Therefore, considering different items in the mathematical model can significantly improve the model's applicability. Second, the presented model did not consider any constraints such as warehouse capacity and economic constraints. For instance, the warehouse capacity is limited in the industry to keep products in stock. Therefore, it is needed to consider the warehouse capacity constraint as an operational constraint in the model. Since these constraints significantly affect the space model, the solutions provided by the model presented by Taleizadeh et al. (2016) may be infeasible in real cases [7]. The second important constraint which highly influences the total order quantity is the on-hand budget constraint. In real-world applications, the retailer has a limited budget to purchase items at the beginning of the planning period. Thus, it is required to reflect this constraint in the formulation to represent a more realistic situation.

In the following, we try to extend the model presented by Taleizadeh et al. (2016) to a multi-item inventory system, which considers defective items in the supply batches and repair options [7]. In this regard, the objective function is modified as:

$$ETP(T_j,F_j) = \sum_{j=1}^{n} P_j D_j \left(F_j + \beta_j(1-F_j)\right) - \left[\begin{array}{l} \sum_{j=1}^{n}\frac{k_j}{T_j} + \sum_{j=1}^{n} c_{uj}\left(F_j D_j + \beta_j(1-F_j)D_j\right) + \sum_{j=1}^{n} c_{ij}F_jD_j + \sum_{j=1}^{n} h_j\left[\frac{E_j\left(\left(1-\rho_j\right)^2\right)F_j^{\,2}T_jD_j}{2} + \frac{E_j(\rho_j)D_j^{\,2}F_j^{\,2}T_j^{\,2}}{x_j}\right] \\ + \sum_{j=1}^{n}(1+m_j)\left(\frac{s_j+2A_j}{T_j} + E_j(\rho_j)F_jD_j\left(c_{ij}+2c_{tj}+h'_j t_{Tj}\right) + \frac{E_j(\rho_j^{\,2})h_j' D_j^{\,2}F_j^{\,2}T_j}{x_j}\right) \\ + \sum_{j=1}^{n} h_{Rj}\frac{E_j(\rho_j^{\,2})D_jF_j^{\,2}T_j}{2} + \sum_{j=1}^{n}\pi\frac{\beta_j(1-F_j)^2T_jD_j}{2} + \sum_{j=1}^{n} g(1-\beta_j)(1-F_j)D_j \end{array}\right] \quad (4)$$

where index j is related to different items and n indicates the total number of items.

To consider the operational constraints, we need to derive the warehouse capacity constraint. As it is clear from Figure 1, the maximum inventory level for each product is equal to $F_jD_jT_j$. Let suppose that each item requires η_j units of warehouse space for keeping the product in stock. Therefore, the total warehouse capacity constraint can be presented as follows:

$$\sum_{j=1}^{n}\eta_j F_j D_j T_j \le Cap \quad (5)$$

where Cap indicates the total existing warehouse capacity.

The second operational constraint is the on-hand budget constraint. From Figure 1, it is clear that the retailer receives a single batch for each item, including $F_jD_jT_j + \beta_j(1-F_j)D_jT_j$ units. Therefore, considering the price c_{ui} for each item-unit, the total on-hand budget constraint can be formulated as follows.

$$\sum_{j=1}^{n} c_{ui}\left(F_jD_jT_j + \beta_j(1-F_j)D_jT_j\right) \le B \quad (6)$$

where the parameter $Budget$ presents the total on-hand budget amount to purchase items at the beginning of the planning cycle. The developed multi-item EOQ model with defective items and repair options in a multi-item inventory system with the proposed operational constraints can be presented as follows:

$$Max\ z = \sum_{j=1}^{n} P_jD_j\left(F_j + \beta_j(1-F_j)\right) - \left[\begin{array}{l} \sum_{j=1}^{n}\frac{k_j}{T_j} + \sum_{j=1}^{n} c_{uj}\left(F_jD_j + \beta_j(1-F_j)D_j\right) + \sum_{j=1}^{n} c_{ij}F_jD_j \\ + \sum_{j=1}^{n} h_j\left[\frac{E_j\left(\left(1-\rho_j\right)^2\right)F_j^{\,2}T_jD_j}{2} + \frac{E_j(\rho_j)D_j^{\,2}F_j^{\,2}T_j^{\,2}}{x_j}\right] \\ + \sum_{j=1}^{n}(1+m_j)\left(\frac{s_j+2A_j}{T_j} + E_j(\rho_j)F_jD_j\left(c_{ij}+2c_{tj}+h'_j t_{Tj}\right) + \frac{E_j(\rho_j^{\,2})h_j' D_j^{\,2}F_j^{\,2}T_j}{x_j}\right) \\ + \sum_{j=1}^{n} h_{Rj}\frac{E_j(\rho_j^{\,2})D_jF_j^{\,2}T_j}{2} + \sum_{j=1}^{n}\pi\frac{\beta_j(1-F_j)^2T_jD_j}{2} + \sum_{j=1}^{n} g(1-\beta_j)(1-F_j)D_j \end{array}\right] \quad (7)$$

Subject to

$$\sum_{j=1}^{n}\eta_j F_j D_j T_j \leq Cap \quad (8)$$

$$\sum_{j=1}^{n} c_{ui}\left(F_j D_j T_j + \beta_j (1 - F_j) D_j T_j\right) \leq B \quad (9)$$

$$0 \leq F_j \leq 1 \quad (10)$$

$$T \geq 0 \quad (11)$$

Solution methods

Interior-Point method

To optimally solve the proposed problem, an exact solution approach, called the interior-point technique, is applied. This method is one of the most frequent solution approaches in resolving challenging Non-Linear Programming (NLP) models (Byrd et al. (2000), Byrd et al. (1999) and Waltz et al. (2006) [13-15]).

Simulated annealing

Simulated annealing (SA), proposed by Kirkpatrick et al. (1983), is an eminent metaheuristic algorithm to solve challenging optimization problems [16]. Tang (2004) and Yang (2010) claimed that SA achieves excellent solutions for many problems [17,18]. The pseudo-code of the SA is as follows:

Algorithm 1 Simulated Annealing algorithm

```
Set the parameters of Simulated Annealing
Create an initial solution
it ⟵ current iteration
Maxit ⟵ maximum number of iterations
Ct ⟵ Current temperature
Ft ⟵ Final temperature
while Ct>Ft and it<Maxit
    Update the position using Neighborhood
    C=Calculate the change in objective function value
    if the new solution is better
        Accept the new solution
    end if
    p = exp[−C/Ct] > rand(0,1)
    if p > rand(0,1)
        Accept the new solution
    end if
    Update the best solution
    Iteration=iteration+1
end while
```

Water cycle algorithm

Eskandar et al. (2012), for the first time, suggested the Water cycle algorithm (WCA) which inspires the water cycle and the flow of streams and rivers to the sea [19]. This approach starts by creating an initial population. Each solution in the WCA is called a stream or raindrop. In this research, a raindrop is defined as $Raindrop = [T_1, ..., T_n, F_1, ..., F_n]$.

After creating the initial population, the WCA sorts the raindrops according to their fitness value. Then, the best solution is considered as the sea. The (Nsr-1) of the sorted population are regarded as rivers and streams. We note that Nsr is a parameter of the WCA. The WCA updates the location of the solutions to the position of rivers and the sea, respectively. To perform updating, the WCA uses the following operator.

$$x_{str}^{i+1} = x_{str}^{i} + R \times C \times (x_{riv}^{i} - x_{str}^{i}) \tag{12}$$

where R and C are coefficients and x_{str}^{i} and x_{str}^{i+1} show the location of the stream before and after the update, respectively. Eskandar et al. (2012), Sadollah et al. (2015) showed that the WCA could achieve better solutions than the other algorithms in various unconstraint and constrained optimization problems [19, 20]. The pseudo-code of the utilized WCA of this paper to solve the problem at hand is as follows.

Algorithm 2 Water cycle algorithm (WCA)

```
set the parameters of WCA
for i=1:number of raindrops
    Create a raindrop
    Calculate the objective function
end for
sort the raindrops in non-decreasing order of fitness
Sea ← the first raindrop
Rivers ← n_sr - 1
Stream ← n_pop - n_sr
Determine the flow intensity of streams to rivers
While  iteration < max it
        Updating process
        Fval-stream=obj_new stream
        for each raindrop
            if  Fval-stream < Fval-river
                River= the new stream
                if  Fval-stream < Fval-sea
                    Sea= the new stream
                end if
            end if
            if  Fval-river < Fval-sea
                Sea=river
            end if
        end for
        for  each river
            if  the distance between the sea and river < d_max
                create new streams
            end if
        end for
        Decrease the d_max
end while
```

In response to the insightful suggestion by the reviewer, we recognize the potential value in reframing the problem as a stochastic optimization problem. This approach allows for the integration of probabilistic elements such as demand variability and lead time uncertainty into the decision-making process, thereby enhancing the robustness and adaptability of inventory management policies.

To address the stochastic nature of the problem, we propose to extend the current deterministic optimization framework to incorporate probabilistic constraints and objectives. Specifically, we intend to model demand fluctuations and lead time uncertainties using probability distributions, such as normal or exponential distributions, to capture the inherent variability in real-world inventory systems.

Building upon existing optimization methods, both exact and approximate, we aim to adapt these techniques to solve the stochastic optimization problem effectively. For instance, by relaxing capacity or budget constraints, we can explore the application of Chance-constrained approaches or Lagrangian dual relaxation methods. These methods offer systematic ways to handle uncertainty while optimizing inventory policies, leveraging the rich literature on stochastic optimization techniques.

The proposed extension of the problem to a stochastic optimization framework represents a significant research contribution to the Inventory and Operations Management (OM) literature. By investigating new solution methodologies tailored to stochastic environments, we can provide practical insights and tools for decision-makers to optimize inventory policies under uncertainty. Furthermore, this research direction aligns with the growing demand for robust and adaptive inventory management strategies in dynamic and unpredictable business environments.

In conclusion, the integration of stochastic optimization techniques into our research framework offers a promising avenue for future exploration and contribution to the field. By leveraging probabilistic models and innovative solution methods, we can address the inherent uncertainties in inventory systems and enhance decision-making capabilities for practitioners in various industries.

Performance evaluation and numerical examples

In this paper, we consider three criteria utilized to assess the efficiency of the proposed algorithms. For small-size instances, the best solution is determined using the interior-point method. In this case, the Percentage Relative Error (*PRE*) shows the gap between the solutions obtained by metaheuristics and the exact method as follows [21]:

$$PRE = \frac{Alg_{sol} - O}{O} \times 100, \tag{13}$$

where Alg_{sol} is the solution of the metaheuristic algorithms, and O is the optimal solution.

Since the interior-point method is not able to obtain the optimal solution in a practical computational time for large-size instances, the Relative Percentage Deviation measure (*RPD*) is used to compare the efficiency of the algorithms.

$$RPD = \frac{A_{sol} - Min_{sol}}{Min_{sol}} \times 100, \tag{14}$$

where A_{sol} is the solution obtained by WCA and SA, and Min_{sol} is the best solution determined among these two algorithms.

The other important measure to evaluate the effectiveness of the proposed approaches is the computational time (CPU-Time). In the next section, various test problems with different sizes are solved to evaluate the performance of the algorithms using the above-mentioned measures.

Small-size test problems

In the following, a different number of products are considered. In each problem (with a specific number of products), four randomly generated test problems are considered. Then, each problem is resolved by interior point to find the optimal solution. Also, each problem is resolved ten times (repetition) using WCA and SA. Table 2 presents the computational results.

As it can be seen in Table 2, the two meta-heuristic algorithms find near-optimal solutions. Although the results show that the two algorithms can efficiently solve the problem, more examination is required to draw a consistent conclusion. First, by increasing the number of products in the problem, the interior-point method needs more computation time. It means that solving the constrained non-linear model of the problem using this method becomes too hard for large sizes.

Table 2 Computational results of the small instances

# of items	Run	Interior-Point		SA					WCA				
		Opt Sol	CT	PRE_{Avg}	Best	worst	Std Dev	CT_{Avg}	PRE_{Avg}	Best	worst	Std Dev	CT_{Avg}
5	1	26901330.1	19.664	1.31E-01	-7.43E-09	6.74E-01	0.253	0.415	1.34E-01	-7.43E-09	6.71E-01	0.26854	0.422
	2	22092182.4	27.248	3.79E-01	-1.69E-14	1.27E+00	0.577	0.408	-1.69E-14	-1.69E-14	-1.69E-14	3.16E-30	0.413
	3	25664638.3	23.813	1.76E-01	-2.90E-14	1.51E+00	0.447	0.410	-2.90E-14	-2.90E-14	-2.90E-14	0	0.412
	4	24370478.8	22.478	6.17E-01	-3.06E-14	2.80E+00	0.928	0.414	-3.06E-14	-3.06E-14	-3.06E-14	0	0.412
10	1	49545409	35.382	1.30E+00	2.36E-02	3.58E+00	0.990	0.886	8.65E-0	-1.61E-09	1.87E+00	0.635492	0.789
	2	59493183	63.512	-4.92E-01	-1.04E+00	3.15E-01	0.389	0.816	-6.80E-01	-1.01E+00	1.01E-01	0.377833	0.751
	3	51172780	60.305	-1.42E+00	-2.06E+00	-5.90E-01	0.401	0.835	-1.36E+00	-2.20E+00	-2.13E-01	0.720938	0.748
	4	48695794	28.502	1.98E+00	6.79E-01	4.38E+00	1.206	0.809	1.11E+00	4.90E-01	2.37E+00	0.55198	0.736
15	1	74103921	26.805	2.45E+00	-2.07E-01	3.99E+00	1.303	1.505	-7.26E-01	-1.13E+00	3.62E-01	0.429634	1.137
	2	61731276	94.001	3.00E+00	5.70E-01	5.54E+00	1.396	1.444	6.78E-01	-2.49E-01	1.28E+00	0.434238	1.119
	3	79856807	82.591	1.97E-01	-1.23E+00	8.74E-01	0.643	1.434	7.63E-01	-7.47E-01	2.93E+00	0.919529	1.093
	4	75098433	67.516	1.12E+00	-7.44E-01	3.01E+00	1.283	1.523	-8.62E-01	-1.95E+00	6.83E-01	0.876919	1.144
20	1	84562497	339.25	1.23E+00	1.02E-01	2.06E+00	0.587	2.083	8.96E-01	-5.57E-01	1.68E+00	0.658868	1.558
	2	93604784	354.80	8.98E-01	5.56E-01	1.43E+00	0.253	1.953	6.39E-01	1.53E-01	1.09E+00	0.291159	1.484
	3	83432678	104.23	1.91E+00	5.98E-02	3.76E+00	1.367	1.931	2.50E-01	-5.14E-01	8.59E-01	0.435	1.456
	4	1.09E+08	248.30	9.79E-01	5.99E-01	1.40E+00	0.259	2.006	6.35E-01	9.02E-02	1.01E+00	0.308	1.494
25	1	1.21E+08	385.61	7.46E-01	4.81E-01	1.04E+00	0.171	2.951	7.06E-01	4.79E-02	1.29E+00	0.350	1.875
	2	1.2E+08	817.52	1.56E+00	9.98E-01	2.42E+00	0.473	2.847	9.39E-01	4.97E-01	1.56E+00	0.285	1.833
	3	1.21E+08	410.74	8.26E-01	2.25E-01	1.45E+00	0.403	2.860	5.19E-01	-2.72E-02	1.02E+00	0.302	1.823
	4	1.43E+08	476.04	3.51E-01	1.48E-02	8.25E-01	0.254	2.891	1.76E-01	-3.11E-01	5.54E-01	0.275	1.841
30	1	1.48E+08	397.74	1.09E+00	4.95E-01	2.85E+00	0.618	3.552	8.43E-01	1.46E-01	1.23E+00	0.298	2.229
	2	1.54E+08	1101.8	3.89E-01	1.05E-02	7.34E-01	0.228	3.470	2.99E-01	-9.08E-02	8.63E-01	0.293	2.200
	3	1.38E+08	721.70	1.37E+00	2.53E-01	3.42E+00	1.353	3.449	3.54E-01	-9.04E-02	7.29E-01	0.201	2.196
	4	1.63E+08	689.83	7.10E-01	1.60E-01	1.11E+00	0.328	3.532	-8.20E-02	-3.88E-01	4.25E-01	0.254	2.242
35	1	1.83E+08	2337.3	1.53E+00	2.85E-01	2.08E+00	0.545	4.308	3.32E-01	-1.83E-01	8.70E-01	0.338	2.645
	2	1.89E	2228.2	5.13E-	2.73E-01	7.79E-	0.163	4.06	2.65E-	-	6.23E-	0.17	2.554

	+08		01		01		3	01	7.07E-02	01	7	
3	1.45E+08	2228.2	1.31E+00	5.03E-01	3.26E+00	1.016	4.047	7.64E-01	1.20E-01	1.65E+00	0.397	2.554
4	1.83E+08	1339.8	9.50E-01	3.42E-01	1.12E+00	0.252	4.088	2.65E-01	6.14E-02	4.97E-01	0.124	2.521

To make the results more comprehensible, the schematic view of the performance of the algorithms is presented in figures 2-5.

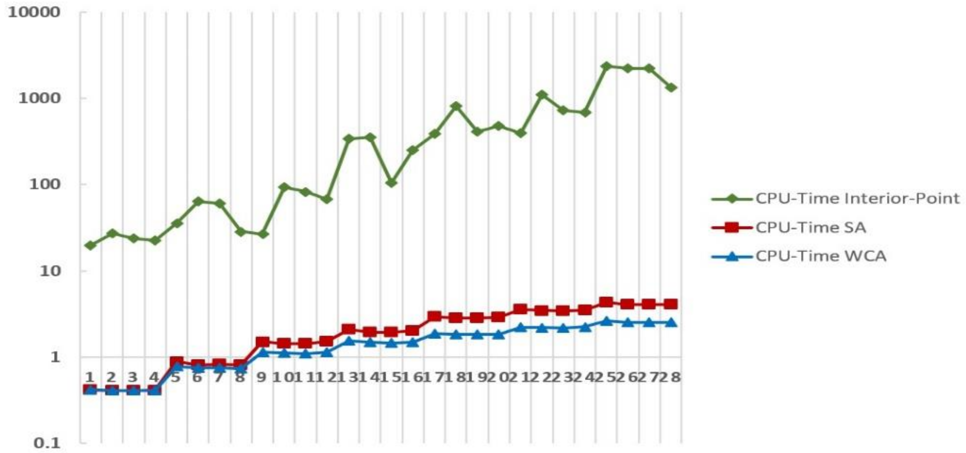

Fig. 2 CPU-Time of the solution methods

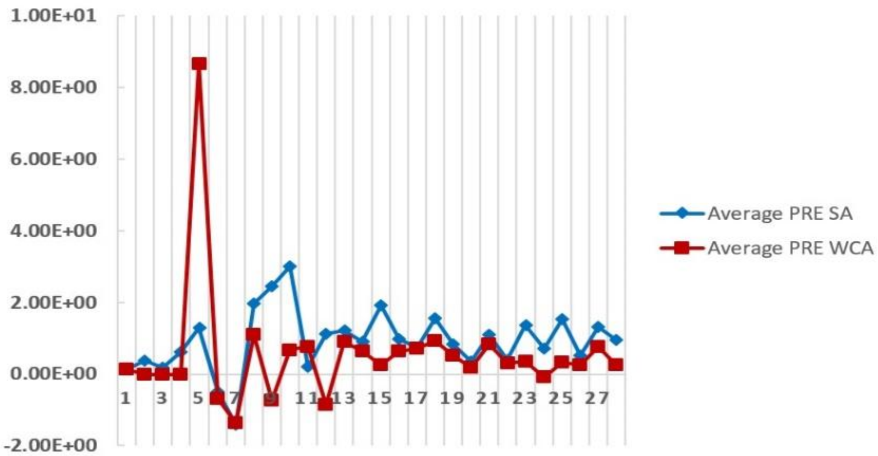

Fig. 3 Average PRE of the SA and WCA

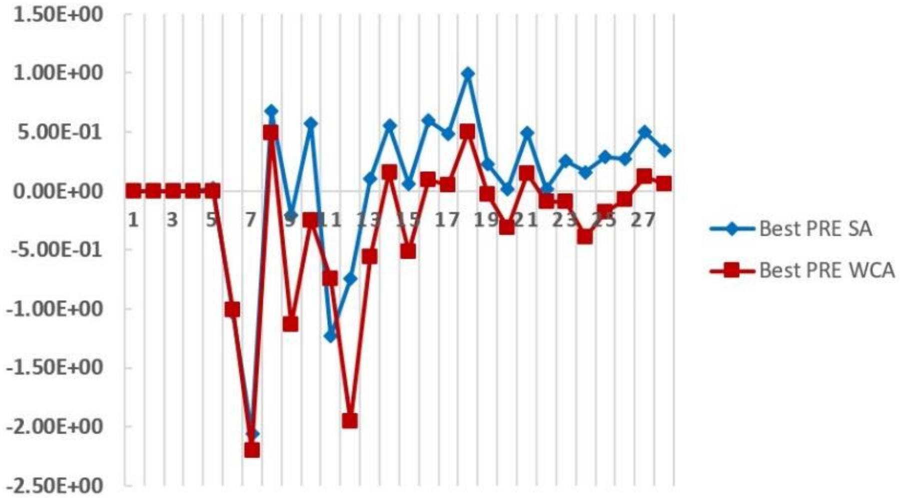

Fig. 4 Best PRE of the SA and WCA

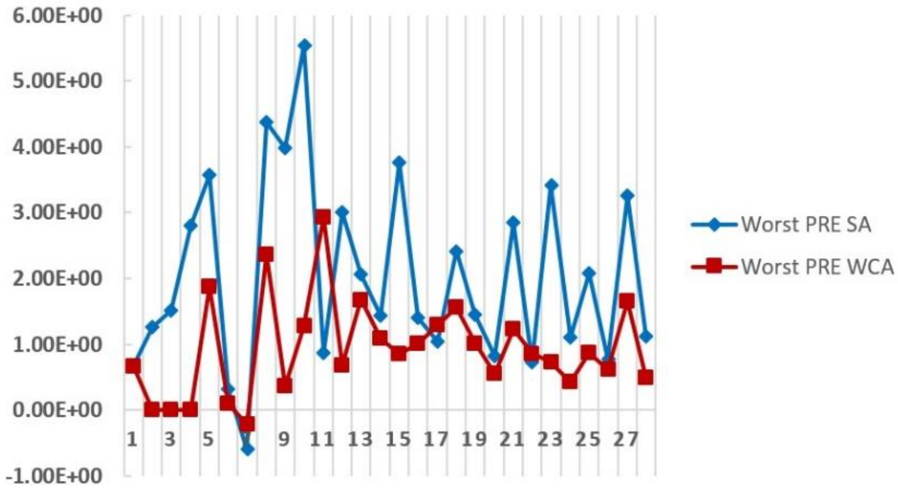

Fig. 5 Worst PRE of the SA and WCA

One of the most commonly used approaches to find a significant difference between metaheuristics and exact methods is to use single-factor ANOVA. Therefore, in this research, single-factor ANOVA is employed to disclose significant variances among algorithms. For this purpose, first, the CPU-Time measure is considered, and the ANOVA test is carried out to compare the average computation times. Table 3 shows the outcomes of the ANOVA for the CPU-Time measure.

Table 3 Results of ANOVA for CPU-Time measure

Groups	Count	Sum	Average	Variance		
Interior-Point	280	147333.5	526.191	479149.9		
SA	280	609.4477	2.176599	1.656705		
WCA	280	416.9529	1.489117	0.524729		
Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Between Groups	51324335	2	25662167	160.6724	8.8E-60	3.00648
Within Groups	1.34E+08	837	159717.4			
Total	1.85E+08	839				

Since the p-value is smaller than 0.05, there is an essential variance between the utilized methods considering computation time measure with a 95% confidence level. Since there are more than two groups in the test, a post hoc analysis is needed to find out which algorithms are performing significantly different. For this purpose, Tukey's multiple comparison test is utilized to find significant differences. Table 4 presents the results of Tukey's HSD.

Table 4 Results of Tukey's HSD test for CPU-Time measure

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
SA - Interior-Poi	-524.0	33.8	(-603.1, -445.0)	-15.51	0.000
WCA - Interior-Poi	-524.7	33.8	(-603.8, -445.6)	-15.53	0.000
WCA - SA	-0.7	33.8	(-79.7, 78.4)	-0.02	1.000

From Table 4, it is evident that the interior-point method performs meaningfully diverse from SA and WCA in terms of computation time measure. Considering the average computation time of the algorithms from Table 3, it is clear that the SA and WCA can perform significantly better than the interior-point method in the resolution of the complex mathematical model in less computational time. Also, the WCA with less average and variance computation time performs slightly better than the SA since there is no significant difference between SA and WCA (refer to Table 4). Figures (6-7) show detailed information about Tukey's HSD test.

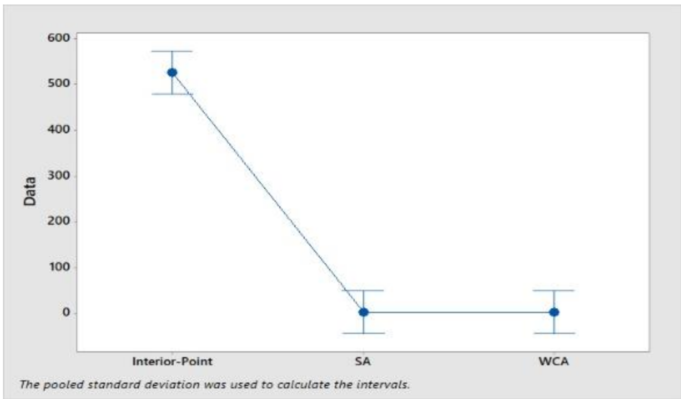

Fig. 6 Boxplot of the CPU-Times

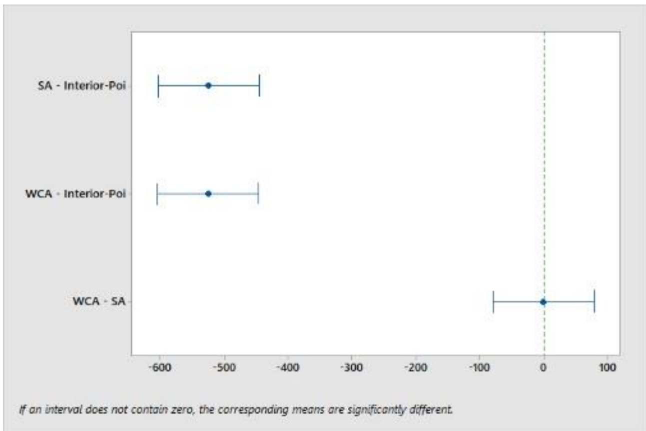

Fig. 7 Results of the Tukey's HSD for CPU-Times

The other important measure that shows the SA and WCA's efficiency and effectiveness compared to the interior point is the PRE measure. It is essential to mention that in some cases, the average and best PRE of the SA and WCA are negative. In these cases, the SA and WCA were able to find a better solution than the interior-point method. This is due to the complexity and existence of many local optima's in the problem, which significantly decrease the efficiency of the interior-point method. In this section, single-factor ANOVA is applied to discover important variances between SA and WCA at a 95 percent confidence level considering the average PRE measure. Table 5 presents the results.

Table 5 Outcomes of ANOVA for PRE

Groups	Count	Sum	Average	Variance		
Average PRE SA	280	258.0277	0.921528	1.332824		
Average PRE WCA	280	77.17516	0.275626	0.531715		
Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Between Groups	58.4065	1	58.4065	62.6498	1.34E-14	3.858178
Within Groups	520.2064	558	0.932269			
Total	578.6129	559				

From the results, it is evident that the WCA performs significantly better than the SA in solving the problem and finding a very near-optimal solution since the p-value of the AVONA test is less than 0.05, which shows a significant difference between algorithms. The WCA with an average of 0.275 and a variance of 0.531 performs significantly better than SA. Although the above-mentioned results show the superiority of the WCA in solving the problem, it is worthwhile to consider the best and worst-case analyses of the algorithms. In this regard, single-factor ANOVA is used to find significant differences considering the best and worst PRE. Table 6 presents the results.

Table 6: Results of ANOVA for best and worst PRE measures

Groups	Count	Sum	Average	Variance	P-value
Worst PRE SA	28	57.091	2.038964	2.043585	0.000584
Worst PRE WCA	28	26.004	0.928714	0.540294	
Best PRE SA	28	1.3487	0.048168	0.406541	0.056304
Best PRE WCA	28	-7.9126	-0.28259	0.398565	

In the best-case analysis, the p-value of the test is higher than 0.05; thus, there is no significant difference between SA and WCA. However, the WCA with a better average performs slightly better. In the worst-case analysis, the value of the p-value in the ANOVA test reveals that the WCA achieves better solutions compared to the SA in obtaining near-optimal solutions. This shows that the WCA can avoid trapping in local optima.

Large-size test problems

In this section, we consider large-size test problems (more than 40 products). In each problem (with a specific number of products), four randomly generated test problems are considered. Then, each test problem is solved using SA and WCA algorithms ten times (repetition). Table 7 presents the computational results.

Table 7 Computational results of large instances

# of items	Run	SA					WCA				
		RPD Avg	Best	worst	Std Dev	CT_{Avg}	RPD_{Avg}	Best	worst	Std Dev	CT_{Avg}
40	1	0.385	0	0.903	0.269	6.823	0.017	0	0.169	0.051	3.638
	2	0.602	0	0.963	0.316	6.475	0.007	0	0.075	0.022	3.624
	3	0.316	0	0.749	0.237	6.425	0.006	0	0.061	0.018	3.499
	4	0.455	0	0.841	0.236	6.372	0.007	0	0.074	0.022	3.48
45	1	0.145	0	0.409	0.156	7.179	0.118	0	0.31	0.098	3.848
	2	0.118	0	0.401	0.157	7.013	0.057	0	0.201	0.072	3.797
	3	0.697	0.176	1.119	0.338	7.035	0	0	0	0	3.753
	4	0.596	0.205	1.156	0.264	7.136	0	0	0	0	3.859
50	1	0.127	0	0.468	0.167	8.179	0.079	0	0.386	0.119	4.332
	2	0.097	0	0.445	0.148	7.819	0.072	0	0.212	0.076	4.192
	3	0.04	0	0.172	0.058	7.738	0.039	0	0.123	0.045	4.11
	4	0.278	0	0.532	0.189	7.851	0.015	0	0.153	0.046	4.243
55	1	0.144	0	0.373	0.147	9.795	0.065	0	0.377	0.13	5.128
	2	0.057	0	0.21	0.063	8.515	0.116	0	0.755	0.233	4.554
	3	0.071	0	0.337	0.103	8.562	0.025	0	0.102	0.039	4.618
	4	0.179	0	0.453	0.188	8.81	0.053	0	0.239	0.081	4.726
60	1	0.07	0	0.227	0.074	10.103	0.034	0	0.221	0.068	5.273
	2	0.221	0	0.515	0.186	9.307	0.022	0	0.101	0.037	4.909
	3	0.213	0.014	0.496	0.159	9.303	0	0	0	0	5.005
	4	0.057	0	0.153	0.058	9.481	0.005	0	0.033	0.01	5.036
65	1	0.076	0	0.249	0.077	10.94	0.047	0	0.241	0.079	5.724
	2	0.026	0	0.079	0.03	10.038	0.02	0	0.078	0.031	5.299
	3	0.139	0	0.325	0.124	10.225	0.034	0	0.209	0.062	5.438
	4	0.119	0	0.439	0.127	10.121	0.018	0	0.127	0.038	5.416
70	1	0.008	0	0.062	0.019	11.434	0.039	0	0.114	0.034	5.917
	2	0.053	0	0.221	0.069	10.922	0.041	0	0.162	0.055	5.805
	3	0.017	0	0.064	0.021	10.88	0.036	0	0.168	0.062	5.776
	4	0.123	0	0.285	0.115	11.01	0.013	0	0.123	0.037	5.864

From Table 7, we can infer that the two algorithms perform very competitively. In other words, in some cases, the SA can find a better solution, and in some cases, WCA outperforms. To present a graphical representation of the outcomes, the following figures are presented.

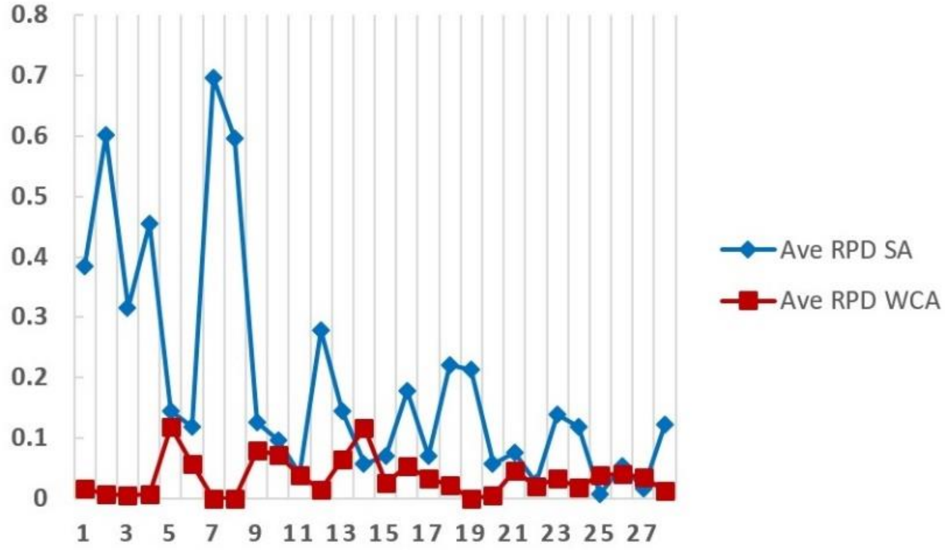

Fig. 8 Average RPD of the SA and WCA

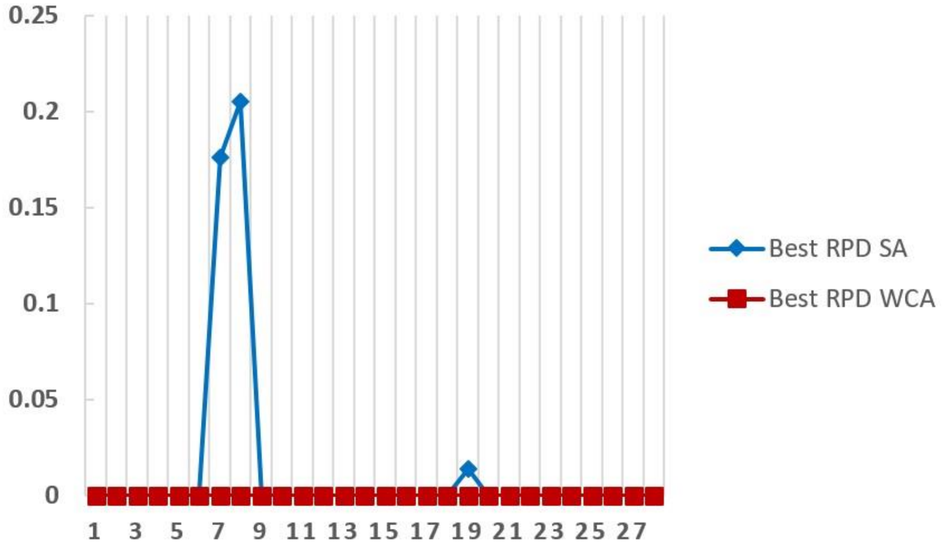

Fig. 9 Best RPD of the SA and WCA

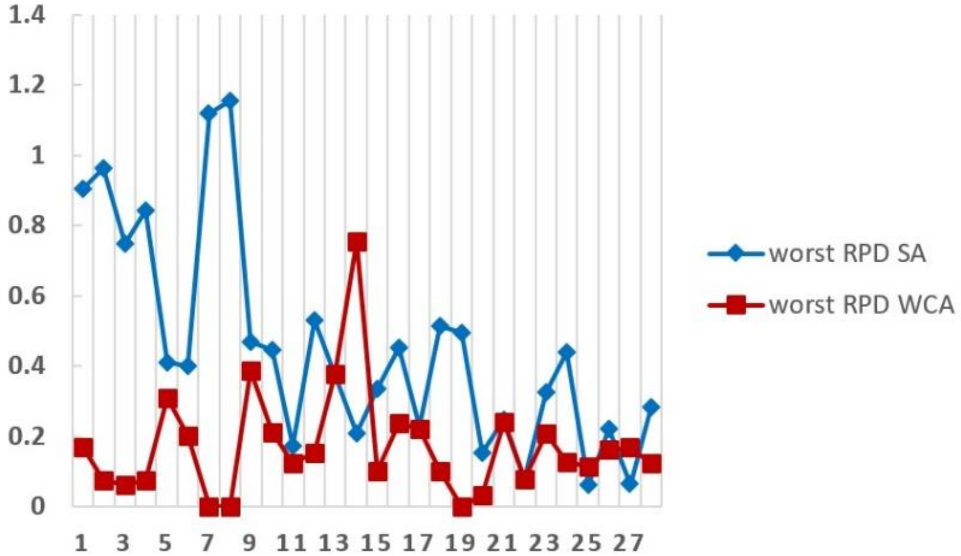

Fig. 10 Worst RPD of the SA and WCA

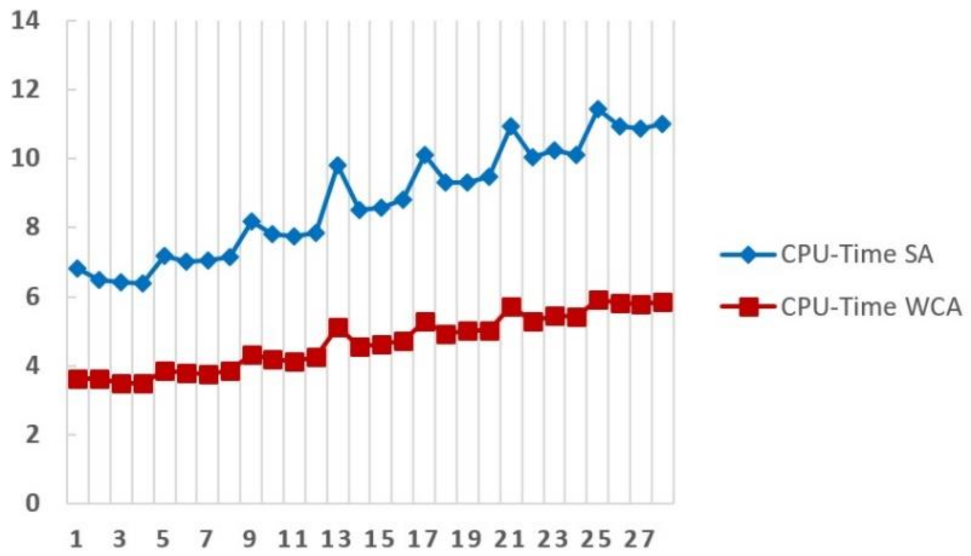

Fig. 11 CPU-Time of the solution methods

Since the performance of the two algorithms is competitive, more analyses are needed. As in the previous section, the ANOVA test on average is performed for each measure at a 95 percent confidence level to find significant differences among algorithms. Table 8 presents the results of the ANOVA test of different measures.

Meanwhile, the p-value on average RPD is smaller than 0.05 which means a substantial difference between SA and WCA in solving the composite model. Considering the average and variance of the average RPD measure, it is clear that the WCA can discover better solutions than SA. In the best RPD measure, the p-value is higher than 0.05, which shows that both algorithms can find near-optimal solutions to the problem. Considering the results, the WCA achieves better solutions compared to SA. In addition to the above criteria, it is vital to assess the robustness of the algorithms. For this purpose, we consider the worst RPD and std of the RPD measures. These measures show which algorithm can obtain near-optimal solutions in all the repetitions. Based on the results, the p-value of the ANOVA test of both measures shows significant differences among algorithms. Therefore, we can infer that the WCA is significantly more robust than the SA in avoiding trapping in local optima. The results show that the WCA uses updating operators to update the particles' position in the solution space, which enables the WCA to perform very well in both exploration and exploitation phrases. Based on the results, the WCA is the best approach to solve the problem in near optimality in small and large test problems. In addition to the measures mentioned above, the CPU-Time measure has its importance. The ANOVA test results reveal that the WCA can solve the problem in large sizes significantly better than the SA. Based on the results, the WCA with less average and variance of the CPU-Time criterion achieves better results than SA in terms of computation time.

Table 8: Results of ANOVA for different measures

Measure	Groups	Sum	Average	Variance	P-value
Average RPD	Ave RPD SA	5.429	0.193892857	0.035811581	5.5752E-05
	Ave RPD WCA	0.985	0.035178571	0.001024152	
Worst RPD	worst RPD SA	12.646	0.451642857	0.094022238	6.73346E-05
	worst RPD WCA	4.814	0.171928571	0.02337318	
Best RPD	Best RPD SA	0.395	0.014107143	0.002504618	0.141627752
	Best RPD WCA	0	0	0	
Std RPD	Std RPD SA	4.095	0.14625	0.007568935	1.25569E-05
	Std RPD WCA	1.565	0.055892857	0.002316099	
CPU-Time RPD	CPU-Time SA	245.491	8.767535714	2.601514925	7.34746E-17
	CPU-Time WCA	130.86	4.673678571	0.657857634	

Conclusion

The conclusion of the research highlights the development of a novel formulation of the Economic Order Quantity (EOQ) model to address the presence of defective products in supply deliveries and partial backordering, where defective items can be repaired locally. This study aimed to address gaps in the literature by incorporating realistic assumptions into the mathematical model. By extending the model to a multi-item inventory system and considering various operational constraints such as warehouse capacity and on-hand budget constraints, the research enhances the model's applicability.

The first significant contribution lies in extending the EOQ model to accommodate imperfect products and partial backordering, which is crucial given the prevalence of such scenarios in inventory management. The inclusion of these factors results in a Constrained Non-Linear Programming (CNLP) model, reflecting the complexity of real-world inventory systems.

To effectively solve the developed mathematical model, three solution approaches were proposed: an exact method and two meta-heuristics. The performance of these approaches was evaluated using various measures, including Relative Percentage Deviation (RPD), Percentage Relative Error (PRE), and computational time, across different test instances.

The findings indicate that the Water Cycle Algorithm (WCA) outperforms the other two approaches, demonstrating superior efficiency in addressing the complexities of the proposed problem. This underscores the potential of meta-heuristic algorithms, particularly the WCA, in optimizing multi-item inventory systems with imperfect products and partial backordering.

In conclusion, this research contributes to the advancement of inventory management by introducing a comprehensive EOQ model capable of handling real-world complexities. The findings provide valuable insights for practitioners in optimizing inventory systems and lay the groundwork for future research directions in this domain.

Based on the results obtained from the research, a promising avenue for future investigation could be the development and validation of hybrid optimization techniques for multi-item inventory systems with imperfect products and partial backordering.

Firstly, hybridization involves combining different optimization algorithms or techniques to leverage their respective strengths and mitigate their weaknesses. In the context of inventory management, hybrid meta-heuristic algorithms could be developed by integrating the Water Cycle Algorithm (WCA) with other efficient optimization methods, such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), or Simulated Annealing (SA).

Secondly, the performance of these hybrid algorithms could be evaluated using a comprehensive set of test instances representing diverse real-world scenarios. This evaluation process should consider various performance metrics, including solution quality, convergence speed, computational efficiency, and robustness.

Furthermore, the impact of different problem parameters and constraints on the performance of the hybrid algorithms could be investigated through sensitivity analysis. This analysis would provide valuable insights into the behavior of the algorithms under different operating conditions and help identify critical factors influencing their performance.

Additionally, future research could explore the application of machine learning techniques, such as reinforcement learning or deep learning, in optimizing multi-item inventory systems. These techniques have shown promise in solving complex optimization problems and could potentially enhance the efficiency and effectiveness of inventory management strategies.

Overall, the proposed research direction aims to advance the state-of-the-art in inventory management by developing innovative optimization approaches tailored to address the challenges associated with imperfect products and partial backordering. By combining insights from operations research, optimization theory, and machine learning, researchers can contribute to the development of more robust and adaptive inventory management systems capable of meeting the demands of modern supply chain environments.

References

[1] Salameh, M.K., Jaber, M.Y.: Economic production quantity model for items with imperfect quality. International Journal of Production Economics, 64, 59–64 (2000)
[2] Cárdenas-Barrón, L.E.: Observation on: Economic production quantity model for items with imperfect quality. International Journal of Production Economics, 67, 201–201 (2000)
[3] Eroglu, A., Ozdemir, G.: An economic order quantity model with defective items and shortages. International Journal of Production Economics, 106, 544–549 (2007)
[4] Khan, M., Jaber, M.Y., Bonney, M.: An economic order quantity (EOQ) for items with imperfect quality and inspection errors. International Journal of Production Economics, 133, 113–118 (2011)
[5] Hsu, J.T., Hsu, L.F.: An EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales returns. International Journal of Production Economics, 143, 162–170 (2013)
[6] Skouri, K., Konstantaras, I., Lagodimos, A.G., Papachristos, S.: An EOQ model with backorders and rejection of defective supply batches. International Journal of Production Economics, 155, 148-154 (2014)
[7] Taleizadeh, A. A., Khanbaglo, M. P. S., Cárdenas-Barrón, L. E.: An EOQ inventory model with partial backordering and reparation of imperfect products. International Journal of Production Economics, 182, 418-434 (2016)
[8] Khalilpourazari, S., Teimoori, S., Mirzazadeh, A., Pasandideh, S. H. R., Ghanbar Tehrani, N.: Robust Fuzzy chance constraint programming for multi-item EOQ model with random disruption and partial backordering under uncertainty. Journal of Industrial and Production Engineering, 36(5), 276-285 (2019a)
[9] Khalilpourazari, S., Pasandideh, S.H.R., Niaki, S.T.A.: Optimizing a multi-item economic order quantity problem with imperfect items, inspection errors, and backorders. Soft Comput 23, 11671–11698 (2019b)
[10] Taheri-Tolgari, J., Mirzazadeh, A.: Determining Economic Order Quantity (EOQ) with Increase in a Known Price under Uncertainty through Parametric Non-Linear Programming Approach. Journal of Quality Engineering and Production Optimization, 4(1), 197-219 (2019)
[11] Nobil, A. H., Sedigh, A. H. A., Cárdenas-Barrón, L. E.: Reorder point for the EOQ inventory model with imperfect quality items. Ain Shams Engineering Journal, 11(4), 1339-1343 (2020)
[12] Taheri-Tolgari, J., Mirzazadeh, A.: Multi-item single-source production quantity model for imperfect items with rework failure, inspection errors, scraps, and backordering. International Journal of Industrial and Systems Engineering, 38(3), 291-327 (2021)
[13] Byrd, R.H., Gilbert, J.C., Nocedal, J.: A trust region method based on interior point techniques for nonlinear programming. Mathematical Programming, 89(1), pp.149-185 (2000)
[14] Byrd, R.H., Hribar, M.E., Nocedal, J.: An interior point algorithm for large-scale nonlinear programming. SIAM Journal on Optimization, 9(4), pp.877-900 (1999)
[16] Kirkpatrick, S., Gelatt Jr, C.D., Vecchi, M.P.: Optimization by simulated annealing. Science 220: 671–680 (1983)
[17] Tang, O.: Simulated annealing in lot sizing problems. International Journal of Production Economics, 88: 173-181 (2004)
[18] Yang X.-S.: Engineering Optimization: An Introduction with Metaheuristic Application. Wiley USA (2010)
[19] Eskandar, H., Sadollah, A., Bahreininejad, A., Hamdi, M.: Water cycle algorithm–A novel metaheuristic optimization method for solving constrained engineering optimization problems. Computers & Structures 110-111 (2012)
[20] Sadollah, A., Eskandar, H., Kim, J. H.: Water cycle algorithm for solving constrained multi-objective optimization problems. Applied Soft Computing, 279-298 (2015)
[21] Khalilpourazari, S., Pasandideh, S. H. R., Niaki, S. T. A.: Optimization of multi-product economic production quantity model with partial backordering and physical constraints: SQP, SFS, SA, and WCA. Applied Soft Computing, 49, 770-791 (2016)
[22] Papachristos, S., Konstantaras, I.: Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics. 100 (1),148–154 (2006).
[23] Chung, K.J., Huang, Y.F.: Retailer's optimal cycle times in the EOQ model with imperfect quality and a permissible credit period. Quality and Quantity, 40(1), 59–77 (2006)
[24] Wee, H.M., Yu, J., Chen, M.C.: Optimal inventory model for items with imperfect quality and shortage backordering. Omega, 35(1),7–11 (2007)
[25] Konstantaras, I., Goyal, S.K., Papachristos, S.: Economic ordering policy for an item with imperfect quality subject to the in-house inspection. International Journal of Systems Science, 38(6), 473–482 (2007)
[26] Maddah, B., Jaber, M.Y.: Economic order quantity for items with imperfect quality: revisited. International Journal of Production Economics112(2), 808–815 (2008)
[27] Chung, K.J., Her, C.C., Lin, S.D.: A two-warehouse inventory model with imperfect quality production

processes. Computers and Industrial Engineering, 56(1), 193–197 (2009)
[28] Lin, T.Y., An economic order quantity with imperfect quality and quantity discounts. Applied Mathematical Modelling, 34(10), 3158–3165 (2010)
[29] Khan,M., Jaber, M.Y., Wahab, M.J.M.: Economic order quantity for items with imperfect quality with learning in inspection. International Journal of Production Economics, 124(1), 87–96 (2010)
[30] Roy, M.D., Sana, S.S., Chaudhuri, K.: An economic order quantity model of imperfect quality items with partial backlogging. International Journal of Systems Science, 42(8), 1409–1419 (2011)
[31] Ouyang, L.Y., Chang, C.T., Shum, P.: The EOQ with defective items and partially permissible delay in payments linked to order quantity derived algebraically. Central European Journal of Operations Research, 20, 141-160 (2012)
[32] Tahami, H., Mirzazadeh, A., Gholami-Qadikolaei, A.: Simultaneous control on lead time elements and ordering cost for an inflationary inventory-production model with mixture of normal distributions LTD under finite capacity. RAIRO-Operations Research, 53(4), 1357-1384 (2019)
[33] De, S. K.: Solving an EOQ model under fuzzy reasoning. Applied Soft Computing, 99, 106892 (2021)
[34] Li, R., Yang, H. L., Shi, Y., Teng, J. T., Lai, K. K.: EOQ-based pricing and customer credit decisions under general supplier payments. European Journal of Operational Research, 289(2), 652-665 (2021)
[35] Çalışkan, C. (2021). A simple derivation of the optimal solution for the EOQ model for deteriorating items with planned backorders. Applied Mathematical Modelling, 89, 1373-1381.
[36] Paul, A., Pervin, M., Roy, S. K., Weber, G. W., Mirzazadeh, A.: Effect of price-sensitive demand and default risk on optimal credit period and cycle time for a deteriorating inventory model. Rairo-Operations Research, 55, S2575-S2592 (2021)
[37] Taheri-Tolgari J., Mirzazadeh A.: Optimization of inventory system with defects, rework failure and two types of errors under crisp and fuzzy approach. Journal of Industrial and Management Optimization, 18, 2289-2318 (2022)
[38] Masoudi, S., Mirzazadeh, A.: The new evidence reasoning based pharmaceutical inventory models with stochastic deterioration rates and lead times using PSO and GA. International Journal of Computer Mathematics, 99(4), 771-790 (2022)
[39] Taheri-Tolgari J., Mohammadi M., Naderi B., Arshadi-Khamseh A., Mirzazadeh A.: An inventory model with imperfect item, inspection errors, preventive maintenance and partial backlogging in uncertainty environment. Journal of Industrial & Management Optimization, 15(3): 1317-1317 (2019)
[40] Alamri, O. A., Jayaswal, M. K., Khan, F. A., Mittal, M.: An EOQ Model with Carbon Emissions and Inflation for Deteriorating Imperfect Quality Items under Learning Effect. Sustainability, 14(3), 1365 (2022)
[41] Asadkhani, J., Mokhtari, H., Tahmasebpoor, S.: Optimal lot-sizing under learning effect in inspection errors with different types of imperfect quality items. Operational Research, 22(3), 2631-2665 (2022)
[42] Kishore, A., Cárdenas-Barrón, L. E., Jaggi, C. K.: Strategic decisions in an imperfect quality and inspection scenario under two-stage credit financing with order overlapping approach. Expert Systems with Applications, 195, 116426 (2022)