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RESEARCH PAPER

Availability analysis for general repairable series and parallel systems with repair time threshold

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Abstract

One of the most important topics in system reliability and performance measurements is availability analysis. In this paper, the availability of both series and parallel systems is addressed considering repair time threshold. A threshold is considered for the repair time, and if the repair time is less than the threshold, the system can be considered as working, and the repair time can be ignored. On the contrary, if the repair time is longer than the threshold, then the system is considered as not working from the beginning of the system failure until the repair time exceeds the threshold. Both constant and random repair time thresholds are considered. Also, to investigate the instantaneous availability of the series and parallel systems, both identical and non-identical components are incorporated. In addition, user-observed and perceived systems are incorporated and analyzed. Numerical analysis is conducted, and the results suggest an increase in the instantaneous and steady-state availability, especially in the series systems. Based on the results, neglecting repair time threshold can lead to a significant difference in the system's availability, which can have a substantial impact on maintenance plans and company costs.

Keywords:
Availability Analysis, Series and Parallel Systems, Repairable Systems, Repair Time Threshold.

Introduction

Due to the importance of system performance and reliability, examining the reliability and availability of systems has become a significant concern for researchers. (Sharma and Misra, 1988; Cui and Xie, 2005). To examine the performance of a system, various modes and assumptions will be considered, which can affect the system's availability. One assumption that is of interest to researchers is the exclusion of minor system repairs. In this case, a repair time threshold is set, and if the repair time is less than the threshold, the repair time can be disregarded, and the system can be considered operational. (Qiu and Cui, 2018; Zheng et al., 2007; Zheng, 2006; Yang et al., 2009)

The idea of setting a threshold for repair time is based on real-world issues, such as the concrete water supply system for an urban area (Zheng et al., 2006; Bao and Cui, 2010). In this system, if there is a failure in the water supply, and the repair time is short, the presence of water in the tanks ensures an uninterrupted water supply for the city. In this case, despite the

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failure, the system continues to operate. However, if the repair time is long, the water in the tanks will run out, and the system will be considered out of order. Another example is the failure in the electrical system of a factory or a hospital (Du et al., 2017). In the event of a minor failure, the emergency power system is activated, ensuring no malfunctions. However, if the repair time is long, the system will fail.

In this study, the availability of multi-component systems is investigated in both series and parallel configurations, taking into account a repair time threshold. If the repair time is less than the threshold, the system is assumed to be operational, but if the repair time exceeds the threshold, the system is considered out of order. This assumption is examined under two scenarios: a fixed threshold value and a random threshold value. The research aims to answer the following questions:

1- What is the availability of a series and parallel system with similar and dissimilar components and a constant repair threshold?
2- What is the availability of a series and parallel system with similar and dissimilar components and a random repair threshold?

To answer these questions, the reliability and availability formulations for each of the scenarios are presented. This research contributes to the literature by considering a general distribution for the time to failure (TTF) of components and their repair time, and formulating the reliability and availability of series and parallel systems.

The paper is organized as follows: Section 1 provides an overview of the relevant literature on this subject. In Section 3, the proposed models are presented. Section 4 includes a numerical example, and finally, conclusion and future research directions are presented in Section 5.

Literature Review

The availability of repairable systems with general failure and repair time distributions has been studied extensively in the literature. For example, Cui and Xie (2001) studied two repairable systems in which failures were identified only by inspection. Cui et al. (2004) investigated five inspection models under periodic inspection rules. In another study, Cui and Li (2004) examined a system where the number of allowed inspections was limited. Papageorgiou and Kokolakis (2007) and Smidt-Destombes et al. (2007) considered a system with two active components, where in case of failure of any component, one of the (n-2) standby components was replaced, and examined its availability. They also studied the availability of k out of n systems that included N identical and repairable components.

Some studies have examined the availability of single-component systems, while others have focused on series and parallel systems. Yang et al. (2009) calculated the availability of a single-component system by establishing a threshold for the repair time, with the repair time and time to failure (TTF) following a general distribution. In this system, if the repair time exceeds the threshold, the entire repair time is considered as downtime. Qiu and Cui (2018) considered a one-component system with TTF and repair time following a general distribution and a threshold for repair time. In this system, if the repair time is less than the threshold, the system is considered to be in a working state; otherwise, the system is considered to be down for the time it takes to be repaired. Bao and Cui (2010) analyzed a Markov system in which TTF and repair time followed an exponential distribution and had a threshold for repair time. They calculated the availability for this system in both series and parallel modes. As mentioned in the previous studies, some research has considered a threshold for the repair time, and if the repair time is less than this threshold, the system is considered to be working even though it is being repaired. Furthermore, this threshold can be either a fixed or a random value (Bao and Cui, 2010; Qiu and Cui, 2018).

Sana's model (2022) extended previous frameworks by addressing lead time shortages and underscoring the importance of proactive maintenance schedules. This approach aims to detect issues early, sustaining consistent production, reducing costs, enhancing product quality, and ensuring an uninterrupted supply. However, the model's constraints, including fixed production rates and key parameter values, highlight opportunities for future iterations exploring stochastic or fuzzy variables. Ahmadi et al.'s (2023) delved into a unique maintenance strategy for parallel systems, centering on a mean remaining time process (SDMRL) that integrates historical failure data. Their approach distinguished itself by relying on the time taken to reach a defective state rather than merely on system failures. Su et al. (2022) presented a new strategy for optimizing selective maintenance in series-parallel systems. The mathematical model is designed to simultaneously enhance system reliability while minimizing overall maintenance expenses. Each component in the system offers a diverse array of maintenance actions, ranging from minor repairs to complete replacements, encompassing incomplete maintenance methodologies. Shivani et al. (2023) concluded that addressing and mitigating the system's heightened sensitivity can be achieved through the reduction of failure rates and strategic control over service costs. These measures are anticipated to culminate in the development of a more profitable system.

Research on availability can be categorized based on conditions such as the inspection and reparability of system components. System components can be either repairable or non-repairable, and inspections can be either perfect or imperfect. Inspections can occur non-periodically, where if a component fails, it is immediately detected (Taghipour and Banjevic, 2012; Hajipour and Taghipour, 2016), or periodically, where failures are detected at predetermined times (Peng et al., 2009; Liu et al., 2013). Each type of inspection has its own advantages. For instance, non-periodic inspections detect failures faster, while periodic inspections impose less cost on the system. Therefore, the choice of inspection type should be made based on the system being reviewed (Qiu et al., 2018).

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Uncertain repair time and repair time thresholds are also investigated in the literature. Kumar et al. (2024) discussed queueing modeling of machine repair problems under threshold recovery policy (Q), server unreliability and k-type phase repairs. The service facility may experience a partial or complete breakdown while providing service and needs to be repaired in required phases under the threshold recovery policy. Li et al. (2024) investigated a cold standby repairable system in a time-varying environment based on different operational levels and maintenance strategies, in which repairman's vacation and maintenance rules vary with environments. Tavakoli Kafiabada et al. (2024) proposed a novel approach to embedding uncertain repair times of faulty components. A multi-stage stochastic programming model is developed for integrated production and workforce planning in such facilities under independent random repair time.

In the landscape of repairable systems and their maintenance times, this paper sets itself apart by focusing explicitly on the distinct configurations of parallel and series systems. While the existing literature generally addresses repairable systems as a whole, it often overlooks the critical differences between these specific configurations. This study pioneers a comparative availability analysis by integrating both constant and random downtime thresholds, tailored specifically for parallel and series systems. The innovation of this research lies in its ability to bridge the identified gap, providing deeper insights into the unique performance dynamics of these configurations incorporating user-observed and perceived reliability. This contribution not only enhances the theoretical understanding of system availability but also offers practical guidance for optimizing maintenance strategies in real-world applications.

Modeling

The following notations are used in the proposed model.

Notations	
X	Lifetime of the system
$F_x(x)$	Distribution function of X
$R_x(x)$	Survival function of X with single component
$\dot{R}_{x,M}(x)$	Survival function of X in the series case with with M components
$\ddot{R}_{x,M}(x)$	Survival function of X in the parallel case with M components
Y	Downtime of the system
$F_Y(y)$	Distribution function of Y
$R_Y(y)$	Survival function of Y
τ	Constant down time threshold
$X_0(t)$	Stochastic process of the user-observed system
$A_0(t)$	Instantaneous availability of the user-observed system
$\dot{A}_{0,M}(t)$	Instantaneous availability of the series user-observed system with M components
$\ddot{A}_{0,M}(t)$	Instantaneous availability of the parallel user-observed system with M components
$X_N(t)$	Stochastic process of the user-perceived system
$A_N(t)$	Instantaneous availability of the single user-perceived system
$\dot{A}_{N,M}(t)$	Instantaneous availability of the series user-perceived system with M components
$\ddot{A}_{N,M}(t)$	Instantaneous availability of the parallel user-perceived system with M components
$N(t)$	Number of renewals in (0,t)
$S_{N(t)}$	Time of the last renewal prior to or at time t
s	A realization of $S_{N(t)}$
ω	Duration of a repair since the last failure
X_n	Lifetime of the system in the nth renewal cycle
Y_n	Down time of the system in the nth renewal cycle
$F_{X+Y}^n(t)$	Distribution function of the sum of n
$\tilde{\tau}$	Random down time threshold
$\tilde{A}(t)$	Probability that the user-perceived system is in up-state while the user-observed System is in down state
$F_{\tilde{\tau}}(\tau)$	Distribution function of $\tilde{\tau}$
$R_{\tilde{\tau}}(\tau)$	Survival function of $\tilde{\tau}$

In this section, user-perceived system's availability is discussed. In order to formulate user-perceived system availability, the initial step should be obtaining user-observed system availability. We define $X_0(t)$ as follows:

$$X_0(t) = \begin{cases} 0 & the\ original\ system\ is\ in\ faiure\ state\ at\ time\ t \\ 1 & the\ original\ system\ is\ in\ working\ state\ at\ time\ t \end{cases}$$

Thus, the user-observed system availability can be formulated as:

$$A_{0ld}(t) = P(X_0(t) = 1) \tag{1}$$

$A_{0ld}(t)$ in Eq.(1) is defined as follows:

$$A_{old,i} = R_{X,i}(t) + \sum_{n=1}^{\infty} \int_0^t R_{X,i}(t-s) dF_{X+Y,i}^n(s) = R_{X,i}(t) + \sum_{n=1}^{\infty} F_{X+Y,i}^n(t) * R_{X,i}(t) \tag{2}$$

Obviously, can be expressed $X_{New}(t)$ as follows:

$$X_{New}(t) = \begin{cases} 0 & \text{the new system is in failure state at time } t \\ 1 & \text{the new system is in working state at time } t \end{cases}$$

The availability of the new system is formulated as:

$$A_{New,i} = A_{old,i}(t) + P(X_{New,i} = 1, X_{old,i} = 0) \tag{3}$$

Obviously, the availability of the new system is higher than the old system due to neglecting down time, and the second term of the above formula represents the probability that the user-perceived system is in working state while the user-observed system is in failure state.

Availability analysis of a series system

In this section, the availability of the series system is formulated in identical and non-identical cases.

Identical components case

Availability of the user-observed system in a series system with *m* identical components is formulated as:

$$\begin{aligned} \dot{A}_{old,M}(t) = {} & \dot{R}_{X,M}(t) + \left(\dot{R}_{X,M-1}(t). \sum_{n=1}^{\infty} F_{X+Y,1}^n(t) * R_{X,i}(t) \right) \binom{m}{1} \\ & + \left(\dot{R}_{X,M-2}(t). (\sum_{n=1}^{\infty} F_{X+Y,1}^n(t) * R_{X,i}(t))^2 \right) \binom{m}{2} + \cdots \\ & + \left(\dot{R}_{X,1}(t). (\sum_{n=1}^{\infty} F_{X+Y,1}^n(t) * R_{X,i}(t))^{m-1} \right) \binom{m}{m-1} \end{aligned} \tag{4}$$

$\dot{A}_{old,M}(t)$ in Eq.(4) can be defined as follows:

$$\dot{A}_{old,M}(t) = \prod_{i=1}^{m} A_{old,i} = {A_{old,i}}^m \tag{5}$$

Availability of the user-perceived system in series case can be defined as:

$$\dot{A}_{New,M}(t) = \dot{A}_{old,M}(t) + P(X_{New,i} = 1, X_{old,i} = 0) \dot{A}_{old,M-1}(t). \binom{m}{1} \tag{6}$$

Non-identical components case

When the components of a series system are not identical, the availability of the user-observed system can be formulated as:

$$\dot{A}_{old,M}(t) = \prod_{i=1}^{m} A_{old,i} \tag{7}$$

Also, the availability of the user-perceived system is defined as:

$$\dot{A}_{New,M}(t) = \dot{A}_{old,M}(t) + \sum_{i=1}^{m} P(X_{New,i} = 1, X_{old,i} = 0)\dot{A}_{old,M-1}(t) \tag{8}$$

Availability analysis of a parallel system

In this section, the availability of the parallel system will be formulated in identical and non-identical cases.

Identical components case

In the parallel case, it is known that the old system would be in the failure state if all the components be in the failure state, thus user-observed system availability can be expressed as:

$$\ddot{A}_{old,M} = 1 - \prod_{i=1}^{m}\left(1 - A_{old,i}\right) = 1 - (1 - A_{old,i})^{m} \tag{9}$$

Let $\tilde{A}_i(t) = P(X_{New,i} = 1, X_{old,i} = 0)$.

The availability of the new system is formulated as:

$$\ddot{A}_{New,M} = \ddot{A}_{old,M} + \tilde{A}_i(t)\left(1 - \ddot{A}_{old,M-1}\right)\left(1 - \ddot{A}_{New,M-1}\right)\binom{m}{1} + \tilde{A}_i(t)^2\left(1 - \ddot{A}_{old,M-2}\right) \times (1 - \ddot{A}_{New,M-2})\binom{m}{2} + \cdots + \tilde{A}_i(t)^{m-1}\left(1 - \ddot{A}_{old,1}\right)(1 - \ddot{A}_{New,1})\binom{m}{m-1} + \tilde{A}_i(t)^m \tag{10}$$

Non-identical components case

The availability of the old system with *m* non- identical components is given by:

$$\ddot{A}_{old,M} = 1 - \prod_{i=1}^{m}(1 - A_{old,i}) \tag{11}$$

Also, the availability of the new system with *m* non-identical components is formulated as:

$$\begin{aligned}\ddot{A}_{New,M} = \ddot{A}_{old,M} &+ \sum_{i=1}^{m}\left(\tilde{A}_i(t)\left(\prod_{j=1,j\neq i}^{m}\left(1 - A_{old,j}\right)\right)\right)\left(1 - \ddot{A}_{New,M-1}\right) \\ &+ \sum_{i_1=1}^{m}\sum_{i_2=i_1+1}^{m}\left(\tilde{A}_{i_1}(t).\tilde{A}_{i_2}(t)\left(\prod_{j=1,j\neq i_1,i_2}^{m}\left(1 - A_{old,j}\right)\right)\right)(1 - \ddot{A}_{New,M-2}) + \cdots \\ &+ \sum_{i_1=1}^{m}\sum_{i_2=i_1+1}^{m}\cdots\sum_{i_{m-1}=i_{m-2}+1}^{m}(\tilde{A}_{i_1}(t).\tilde{A}_{i_2}(t)\ldots\tilde{A}_{i_{m-1}}(t)\left(\prod_{j=1,j\neq i_1,i_2,\ldots,i_{m-1}}^{m}\left(1 - A_{old,j}\right)\right))(1 \\ &- \ddot{A}_{New,1}) + \prod_{i=1}^{m}\tilde{A}_i(t)\end{aligned} \tag{12}$$

Calculating the instantaneous availability for the user-perceived system

Two different scenarios are considered to calculate $\tilde{A}_i(t)$. Based on Qui & Cui (2018), $\tilde{A}_i(t)$ with a constant repair time threshold is given by:

$$\tilde{A}_i(t) = \begin{cases} 1 - A_{old,i}, & t < \tau \\ \int_0^{\tau} R_Y(w)dF_x(t-w) + \int_0^{t}\int_0^{(t-w)\wedge\tau} R_Y(w)dF_x(t-s-w)\sum_{n=1}^{\infty} dF_{X+Y}^{n}(s), & t \geq \tau \end{cases} \tag{13}$$

Also, $\tilde{A}_i(t)$ with a random repair time threshold is derived as:

$$\tilde{A}_i(t) = \int_0^t \int_0^\tau R_Y(w) dF_x(t-w) dF_\tau(\tau) + \int_0^t R_Y(t-x) dF_x(u) R_\tau(t) + \int_0^t \int_0^t \int_0^{(t-w)^\tau} R_{Y(w)} dF_x(t-s-w) \sum_{n=1}^{\infty} dF_{X+Y}^n(s) dF_\tau(\tau) + \int_0^t \int_0^{t-s} R_Y(w) dF_x(t-s-w) \sum_{n=1}^{\infty} dF_{X+Y}^n(s) R_\tau(t) \tag{14}$$

Since the exponential distribution assumes a constant failure and repair rate, its impact on system availability is more predictable. The calculations can be simplified by using this distribution. However, it is applicable in specific situations where fatigue is not a factor and can be utilized in various real-world applications.

The case is considered when the lifetime of a component and its repair time follow an exponential distribution. It is supposed that $F_X(x) = 1 - e^{-\lambda x}$ and $F_Y(y) = 1 - e^{-\mu y}$, $x, y > 0$. Based on Qui & Cui (2018), $\tilde{A}_i(t)$ with a constant repair time threshold is given by:

$$\tilde{A}_i(t) = \frac{\lambda e^{-\lambda t}}{\mu - \lambda}\left(1 - e^{(\lambda-\mu)\tau}\right) + \frac{\lambda e^{-\lambda t}\left(1 - e^{(\lambda-\mu)\tau}\right)[\mu\left(e^{\lambda(t-\tau)} - 1\right) + \lambda(e^{-\mu(t-\tau)} - 1]}{(\mu - \lambda)(\mu + \lambda)} + \frac{\left(\mu - \mu e^{-\lambda\tau} - \lambda + \lambda e^{-\mu\tau}\right) + e^{-(\mu+\lambda)t}\left(-\mu + \mu e^{\lambda\tau} + \lambda - \lambda e^{\mu\tau}\right)}{(\lambda - \mu)(\mu + \lambda)} \tag{15}$$

Also, $\tilde{A}_i(t)$ with a random repair time threshold is derived as:

$$\tilde{A}_i(t) = \int_0^t \frac{\lambda e^{-\lambda t}}{\mu - \lambda}\left(1 - e^{(\lambda-\mu)\tau}\right) dF_{\tilde{\tau}}(\tilde{\tau}) + \int_0^t \frac{\lambda e^{-\lambda t}\left(1 - e^{(\lambda-\mu)\tilde{\tau}}\right)[\mu\left(e^{\lambda(t-\tilde{\tau})} - 1\right) + \lambda(e^{-\mu(t-\tilde{\tau})} - 1]}{(\mu - \lambda)(\mu + \lambda)} dF_{\tilde{\tau}}(\tilde{\tau}) + \int_0^t \frac{\left(\mu - \mu e^{-\lambda\tilde{\tau}} - \lambda + \lambda e^{-\mu\tilde{\tau}}\right) + e^{-(\mu+\lambda)t}\left(-\mu + \mu e^{\lambda\tilde{\tau}} + \lambda - \lambda e^{\mu\tilde{\tau}}\right)}{(\lambda - \mu)(\mu + \lambda)} dF_{\tilde{\tau}}(\tilde{\tau}) \tag{16}$$

Numerical Example

The role of ventilator systems in ensuring safety in coal mines cannot be overstated. These systems are responsible for circulating fresh air in the underground mining sites (Jia, 2004). Down time is critical for this system, and therefore the plant management decides that the maximum acceptable downtime for the system is 4 hours. This is based on the machine's importance to production schedules and the associated costs of downtime. In the event of a ventilator failure, it is crucial to allow for a delay period to allow for maintenance while ensuring the safety of underground workers (Zhou and Wang, 2013). If the system is repaired within a critical value, the ventilation system is still considered to be in the up state, and the effect of failure is either neglected or delayed. Therefore, conducting an availability analysis on ventilator systems is of significant importance in ensuring the normal production of the coal mine industry. This section presents a practical scenario where the neglect or delay of downtime in coal mine ventilator systems can have severe consequences, as illustrated in the previous sections. If the downtime of the ventilator surpasses τ, the system remains operational within the interval τ. It is important to note that if $t < \tau$, the effects of ventilator failures are neglected, while $t \geq \tau$ implies that the effects of ventilator failures are delayed. As suggested by Zhou and Wang (2013), τ is set at 0.1.

Let's consider two ventilators operating in both a parallel and series system. The lifetime of each ventilator is exponentially distributed, with a failure rate of λ. Any ventilator failure is immediately detected, and the corresponding repair time is a non-negative exponential random variable with a parameter of μ. Consistent with the methodology of Zhou and Wang (2013), we have chosen the values of λ and μ to be 1 and 2, respectively. It is important to note that prior to applying the new settings, the results are compared with those of the study by Zhou and Wang (2013) to validate the proposed model. Additionally, to assess the modified version of the model in light of the research contributions, a sensitivity analysis will be conducted in the following section. Table 1 presents availability of series and parallel systems with constant threshold and identical components.

Table 1: Series and Parallel System Availability with constant threshold and Identical Components (τ =0.1)

t	A_{New}	$A_{Old}(series)$	$A_{Old}(parallel)$	$A_{New}(series)$	$A_{New}(parallel)$
0	1.00000	1.00000	1.00000	1.00000	1.00000
0.2	0.92926	0.72182	0.97738	0.85719	0.98542
0.3	0.87686	0.64350	0.96087	0.76331	0.97008
0.5	0.80928	0.54914	0.93294	0.65029	0.94434
0.7	0.77220	0.50053	0.91443	0.59210	0.92724
0.9	0.75184	0.47481	0.90332	0.56133	0.91695
1	0.74543	0.46684	0.89967	0.55180	0.91356
1.2	0.73715	0.45667	0.89487	0.53963	0.90910
1.5	0.73118	0.44939	0.89134	0.53092	0.90581
1.7	0.72933	0.44715	0.89024	0.52825	0.90479
1.9	0.72832	0.44593	0.88963	0.52678	0.90422
2	0.72800	0.44554	0.88943	0.52632	0.90404
3	0.72713	0.44449	0.88891	0.52507	0.90355
4	0.72709	0.44444	0.88889	0.52501	0.90353
5	0.72709	0.44444	0.88888	0.52500	0.90353
6	0.72709	0.44444	0.88888	0.52500	0.90353
7	0.72709	0.44444	0.88888	0.52500	0.90353
8	0.72709	0.44444	0.88888	0.52500	0.90353
9	0.72709	0.44444	0.88888	0.52500	0.90353
10	0.72709	0.44444	0.88888	0.52500	0.90353

Based on the figures 1 and 2, in the parallel system, the steady-state availability increases from 0.88 to 0.90. Similarly, in the series system, the steady-state availability increases from 0.44 to 0.52. Therefore, there is an increase of 2.27% and 18.18% in availability in the parallel and series systems, respectively.

In addition to the previous scenario, it is considered a case where the two components are non-identical. Again it is considered two ventilators in both a parallel and series system. The lifetime of each ventilator follows an exponential distribution with a failure rate of λ. Any ventilator failure is immediately detected, and the corresponding repair time is a non-negative exponential random variable with a parameter of μ. However, this time the values of λ and μ are chosen to be 1 and 2, respectively, for the first ventilator, and 2 and 3, for the second ventilator, respectively. The results are presented in Table 2 and Figure 3.

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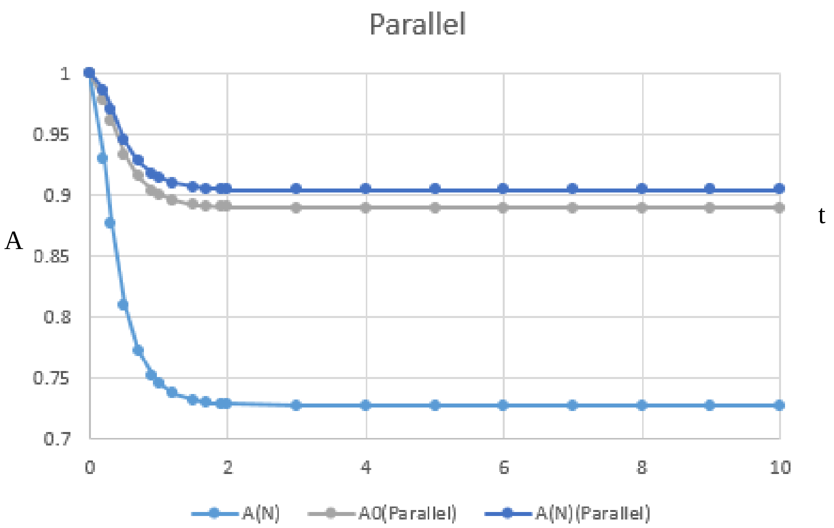

Figure 1: Parallel System Availability with Constant Threshold and Identical Components (τ =0.1)

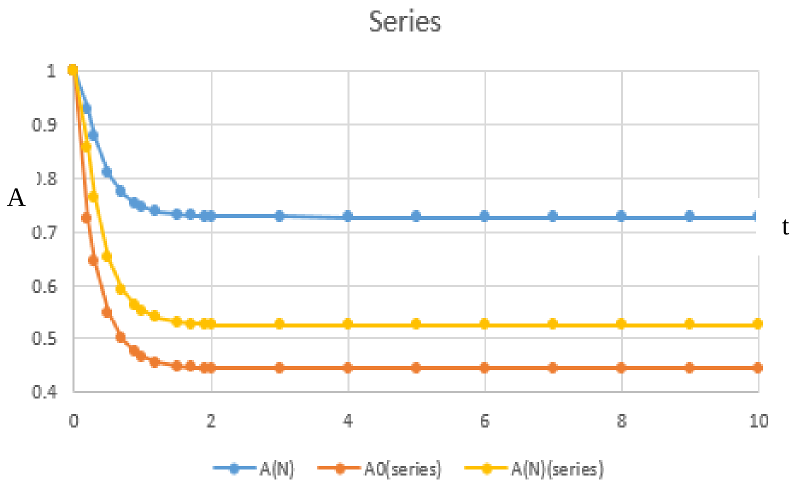

Figure 2: Series System Availability with Constant Threshold and Identical Components (τ =0.1)

Table 2: Series and Parallel System Availability with constant threshold and Non-Identical Components (τ =0.1)

t	$A_{Old}(series)$	$A_{Old}(parallel)$	$A_{New}(series)$	$A_{New}(parallel)$
0	1.00000	1.00000	1.00000	1.00000
0.2	0.72182	0.97738	0.90526	0.98985
0.3	0.64350	0.96087	0.80243	0.97419
0.5	0.54914	0.93294	0.68193	0.94860
0.7	0.50053	0.91443	0.62156	0.93192
0.9	0.47481	0.90332	0.59019	0.92199
1	0.46684	0.89967	0.58056	0.91875
1.2	0.45507	0.89411	0.56645	0.91383
1.5	0.44939	0.89134	0.55969	0.91137
1.7	0.44715	0.89024	0.55704	0.91039
1.9	0.44593	0.88963	0.55559	0.90986
2	0.44554	0.88943	0.55514	0.90969
3	0.44449	0.88891	0.55390	0.90923
4	0.44444	0.88889	0.55384	0.90921
5	0.44444	0.88888	0.55384	0.90921
6	0.44444	0.88888	0.55384	0.90921
7	0.44444	0.88888	0.55384	0.90921
8	0.44444	0.88888	0.55384	0.90921
9	0.44444	0.88888	0.55384	0.90921
10	0.44444	0.88888	0.55384	0.90921

From figure 3, it is concluded that in the parallel system, the steady-state availability increases from 0.88 to 0.90 and in series system, the steady-state availability increases from 0.44 to 0.53. Therefore, there is a 2.27% and a 20.45% increase of availability in parallel and series system.

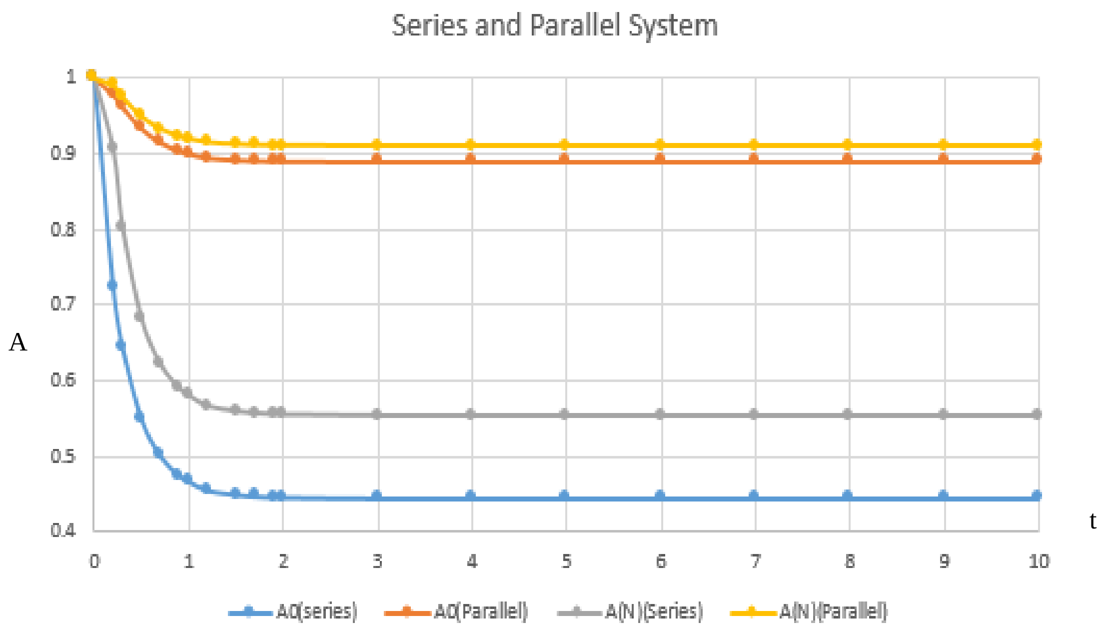

Figure 3: Series and Parallel System Availability with Constant Threshold and Non-Identical Components (τ =0.1)

Table 3 presents availability for series and parallel systems with random threshold and identical components. In the case of random repair time threshold, it is supposed that τ is a random variable with distribution function $F_\tau(\tau) = 1 - e^{-3\tau}$, $\tau > 0$. Two identical ventilators are considered working in parallel and series systems. The values of λ and μ are respectively chosen to be 1 and 2.

Table 3: Series and Parallel System Availability with random threshold and Identical Components (τ =0.1)

t	$A_{Old}(series)$	$A_{Old}(parallel)$	$A_{New}(series)$	$A_{New}(parallel)$
0	1.00000	1.00000	1.00000	1.00000
0.2	0.83467	0.99253	0.97143	0.99828
0.3	0.72182	0.97738	0.91628	0.99171
0.5	0.64350	0.96087	0.85874	0.98224
0.7	0.54914	0.93294	0.76537	0.96277
0.9	0.50053	0.91443	0.70525	0.94788
1	0.47481	0.90332	0.66941	0.93816
1.2	0.46684	0.89967	0.65762	0.93483
1.5	0.45348	0.89334	0.63700	0.92884
1.7	0.44939	0.89134	0.63042	0.92689
1.9	0.44715	0.89024	0.62675	0.92580
2	0.44593	0.88963	0.62472	0.92519
3	0.44554	0.88943	0.62407	0.92500
4	0.44449	0.88891	0.62231	0.92447
5	0.44444	0.88889	0.62222	0.92444
6	0.44444	0.88888	0.62222	0.92444
7	0.44444	0.88888	0.62222	0.92444
8	0.44444	0.88888	0.62222	0.92444
9	0.44444	0.88888	0.62222	0.92444
10	0.44444	0.88888	0.62222	0.92444

From the figures 4 and 5, it can be seen that similar to the foregoing case, the user-perceived availability is higher than the user-observed availability in the case of random repair time threshold for both series and parallel cases.

A sensitivity analysis was performed to assess the impact of model parameters on the availability of the user-perceived system and the results are presented in Table 4. When the critical repair time is larger than $\tau = 0.1$, i.e., $\tau = 0.2$, the availability of the user-perceived system increases. This is because the probability of completing the repair procedure within time τ increases. For instance, in the case of identical components and a constant repair time, when τ increases from 0.1 to 0.5, the corresponding steady-state availability in the parallel system increases from 0.88 to 0.95, indicating an 8.23% increase. Similarly, in the series system, the steady-state availability increases from 0.44 to 0.72, a 63.63% increase.

Based on the figure 6, it can be seen that in the parallel system, the steady-state availability increases from 0.88 to 0.92. Similarly, in the series system, the steady-state availability increases from 0.44 to 0.62. Therefore, there is an increase of 4.54% and 40.9% in availability in the parallel and series systems, respectively.

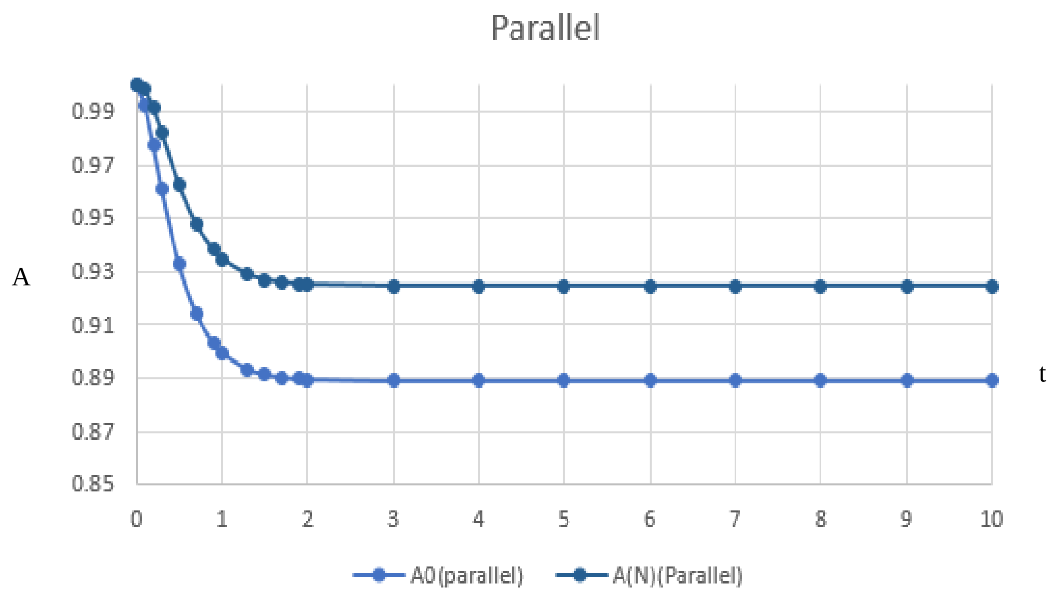

Figure 4: Parallel System Availability with Random Threshold and Identical Components (τ =0.1)

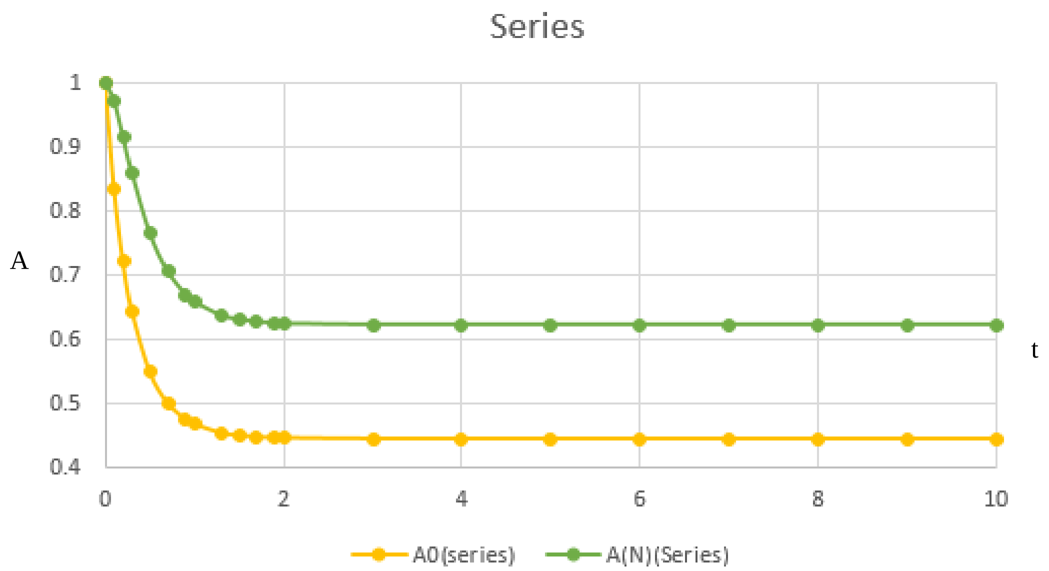

Figure 5: Series System Availability with Random Threshold and Identical Components (τ =0.1)

Table 4: Series and Parallel System Availability with Constant Threshold and Identical Components (τ =0.2)

t	$A_{Old}(series)$	$A_{Old}(parallel)$	$A_{New}(series)$	$A_{New}(parallel)$
0	1.00000	1.00000	1.00000	1.00000
0.2	0.72182	0.97738	1.28151	1.06814
0.3	0.64350	0.96087	1.12261	1.03813
0.5	0.54914	0.93294	0.93294	1.00000
0.7	0.50053	0.91443	0.83614	0.97837
0.9	0.47481	0.90332	0.78522	0.96605
1	0.46684	0.89967	0.76949	0.96209
1.2	0.45667	0.89487	0.74943	0.95691
1.5	0.44939	0.89134	0.73511	0.95313
1.7	0.44715	0.89024	0.73072	0.95195
1.9	0.44593	0.88963	0.72831	0.95130
2	0.44554	0.88943	0.72755	0.95109
3	0.44449	0.88891	0.72549	0.95054
4	0.44444	0.88889	0.72539	0.95051
5	0.44444	0.88888	0.72538	0.95051
6	0.44444	0.88888	0.72538	0.95051
7	0.44444	0.88888	0.72538	0.95051
8	0.44444	0.88888	0.72538	0.95051
9	0.44444	0.88888	0.72538	0.95051
10	0.44444	0.88888	0.72538	0.95051

It is important to note that increasing the critical repair time threshold (τ) significantly enhances the availability of systems with identical components, particularly in parallel configurations. As τ rises from 0.1 to 0.5, parallel systems see an availability boost of 8.23%, while series systems experience a notable 63.63% increase. This underscores the importance of minimizing repair time variability and considering system design, as parallel systems offer greater resilience against failures. Managers should strategically invest in maintenance and repair processes to improve operational efficiency, leveraging the quantifiable gains in availability to justify resource allocation and prioritization of initiatives.

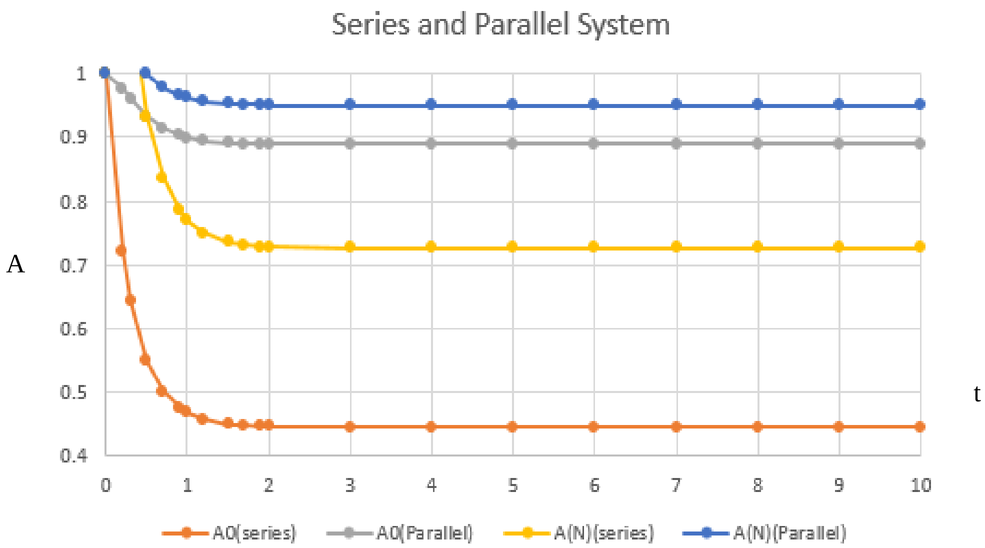

Figure 6: Series and Parallel System Availability with Constant Threshold and Identical Components (τ =0.2)

Conclusion

This research conducted an availability analysis for repairable systems in both series and parallel configurations, taking into account the threshold for repair time and the potential neglect or delay of downtime. Both constant and random downtime thresholds were considered, and analytical results were derived for system availability. The results indicate that this model is more effective for series systems, as the increase in availability was higher in series systems in all cases. Additionally, the value of τ is of critical importance for evaluating the user-perceived availability. As τ increases, the user-perceived availability also increases.

It is essential for managers and decision-makers who rely on repairable systems with a repair threshold, such as air distribution systems, water pumps, and emergency power systems, to analyze their system's availability while taking into account the neglected time caused by the repair threshold. Neglecting this factor can lead to a significant difference in the system's availability, which can have a substantial impact on maintenance plans and company costs. In fact, the availability of the system may be higher than what the manager assumes. Therefore, by calculating the threshold correctly, a better understanding of the system's performance can be achieved, leading to more accurate decision making. The results underscore the importance of considering repair time thresholds in real-world systems such as urban water supplies and industrial electrical systems. By integrating repair time thresholds into maintenance planning, organizations can achieve higher availability and optimize costs. Future research should focus on developing practical tools for implementing these thresholds and exploring additional factors affecting system reliability.

It should be noted the results are so far limited to the series-parallel cases where:

1- Failures are detected immediately.
2- Standby systems were not considered.
3- The down time caused by preventive maintenance and corrective maintenance were not considered.
4- The failure mode of the system is assumed to be single in this research.

Addressing these limitations can be considered as an interesting direction for the future studies.

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