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RESEARCH PAPER

Simultaneous Analysis of Quality, Inventory and Maintenance Costs by Considering Rework in Single-Machine Production System

Seyed Mohammad Hassan Hosseini[1]*, Parviz Fattahi[2]

[1]*Associate Professor, Department of Industrial Engineering and Management, Shahrood University of technology, Shahrood, Iran.*
[2]*Professor, Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran.*

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Abstract

Equipment failure leads to non-conforming products and there is usually a direct relation between the failure rates and the amount of the non-conforming products. Inventory is another issue for production managers and so, they should provide an optimum condition between these related issues. This paper aims to investigate the policies of maintenance, quality control, and inventory in a single machine production system simultaneously with the aim of minimizing total costs. To close the problem to practical condition, machine failures are considered under uncertainty with a known distribution probability. The maintenance time is assumed dependent on the maximum non-conforming products produced per unit and the amount of buffer stock. Moreover, the non-conforming products can be reworked in an additional work station. The problem is formulated and modeled under different various scenarios considering three kinds of cost including quality control, inventory, and maintenance. To test the proposed model, four scenarios are investigated for a numerical example consists of: production period without shortage, production period with compensable shortage, production period before the buffer stock with compensable shortage, and time-based cost analysis. Finally, result analysis has been presented that demonstrates proper efficiency of the proposed model in cost reduction.

Keywords: Integrated Production Control Model, Quality Control, Maintenance, Inventory, Rework.

Introduction

The economic production model is used to reduce production and inventory costs of production-inventory systems. In this model, it is assumed that the machines will never be damaged during production, but in practice, this assumption is not always correct, and the machines are faced with a decline in quality and eventually failure. Therefore, maintenance is a key operation in production systems [1]. A set of inspection and maintenance activities of production systems that help reduce the system deterioration and change the system condition to a mode where it can perform the proper operations is called maintenance operations [2, 3]. Although these operations can improve production performance, over-performing them may reduce production. Therefore, production control and maintenance management are interrelated to meet ideal

* Corresponding author: (Seyed Mohammad Hassan Hosseini)
Email: sh.hosseini@shahroodut.ac.ir

goals. Quality control is considered as the third aspect (production, maintenance, and quality control) of production systems [1]. Despite the hidden interactions and commonalities of these three aspects, most of the existing integrated models consider only two aspects simultaneously, and have examined the pairwise interactions of these three aspects (such as[4-6]). Integrated production and maintenance control models have been proposed since the 1980s [7, 8], and have attracted the attention of many researchers in recent years (such as [3],[9-11]). Kuo and Chang (2007) integrated preventive maintenance planning and production planning for a machine under the process of cumulative failure, and examined how interactive maintenance programs and production programs communicate [12]. The integration of production and quality control policies dates back to the 1970s and 1980s [13], [14].

In a small number of studies, the simultaneous integration of quality, inventory, and maintenance aspects have been considered. For instance, Lin et al. (2011) developed an integrated model of production, maintenance, and quality by considering the probabilities of inspection errors, preventive maintenance errors, and minimal repairs for an imperfect production system at an increasing rate of failure. In this study, preventive maintenance is incomplete, and the system cannot be improved as much as the new mode, and may be out of control with a certain probability [15]. Kang and Subramaniam (2018) examined an integrated model of production control and preventive maintenance for a production system with a machine. The machine is subject to accidental damage and loss of quality, maintenance and preventive maintenance performed to maintain the reliability of the machine, while previous studies have generally considered preventive maintenance periodically, which were done only for a certain period of time [11]. Cheng, Zhou, and Li (2018) examined an integrated model of production, quality control, and maintenance and condition-based repairs with respect to quality loss and reliability for an imperfect production system. This system produces a type of product to meet constant demand. Production policy is used to store inventory to protect against uncertainty. Condition-based maintenance policy means inspecting and assessing the condition of the system at the end of the production period.

If the defective rate is detected too high, maintenance is performed. In this case, to determine the proportion of defects, quality control is performed with a 100% inspection policy. Based on qualitative feedback, complete repairs are performed when the ratio of defects during production has reached a certain level. The purpose of this paper is to optimize the batch size, the amount of inventory, preventive maintenance, and the amount of repairs, so that the total cost per time unit is minimized [1]. In fact, in production systems, poor quality products may be produced, some of these products may be modified by reprocessing, hence, the reprocessing can eliminate waste and affect production costs [16]. Chen and Lin (2010) developed an integrated model for determining the amount of economic output, taking into account the deficit and incomplete reprocessing in a production process, which is an increasing rate of failure, and even with periodic preventive maintenance, this production system cannot be improved as much as the original state. Also, a percentage of defective items are discarded and the rest are recycled, and a percentage of recycled items are discarded in the reprocessing process [17]. Wang (2013) developed an integrated model of the amount of economic production and preventive maintenance, to integrate the possibility of minimum repair and rework, which simultaneously determines the number and frequency of inspections, the amount of economic production and the level of preventive maintenance [8]. Chen (2013) considered an integrated model of production, inspection, preventive maintenance, inventory, and determined the optimal period of inspection, inspection frequency, and production value in such a way that the expected profit in an incomplete production process is maximized taking into account the time of inspection and rework. In this model, when the process gets out of control, a percentage of the items are produced defective, which assumes that there is rework [18].

Some new efforts dealt with the integrated optimization problem of production systems considering quality, inventory, maintenance, and scheduling considering operational conditions. Tambe and Kulkarni proposed an integrated procedure considering three core functions of shop floor management including maintenance, production scheduling, and quality with the aim of minimizing the total expected cost of the production system [19]. Pinciroli et al., reviewed integrated optimization approaches considering maintenance within the Industry 4.0 paradigm published until 2020. They also discussed the possible objectives of the optimization, together with the maintenance features to be optimized, such as maintenance periods and degradation thresholds [20]. Shi et al., proposed a new model to optimise production, maintenance, and quality control considering timely replenishment to optimize the production cycle, maintenance frequency, quality inspection cycle, and number of inspections considering the expectation unit cost of the system as the objective function [21]. Finally, Tasias developed an integrated production, quality, and condition-based maintenance model for imperfect processes with multiple out-of-control operating states. The production system is subject to multiple quality disruptions in their study, which affect both the process mean and variance, and failures. The quality of the production process output is monitored by a fully adaptive control scheme, and the state of deterioration is estimated through periodic sampling inspection. The proposed control scheme's operation is modeled through a Markov chain, and the optimal quality control, maintenance, and inventory policy are defined based on an expected total cost minimization criterion [22].

As the summary of the literature review indicates, several works have dealt with the integrated optimization problem of quality, scheduling, maintenance, and inventory. However, none of them have investigated rework condition and shortage costs in their studies. In this way, this study aims to develop the integrated model of quality control, preventive maintenance and production presented by Rezg et al. (2014) under four scenarios in a single machine production system [23]. These scenarios are: production period without shortage, production period with compensable shortage, production period before reaching the buffer stock and without shortage, production period before reaching the buffer stock and with compensable shortage. Inventory cost is calculated according to the scenario, and finally, by minimizing the total cost per time unit, the optimal values of buffer stock (h^*) and the limit value of non-conforming units rate (l_m^*) are determined simultaneously.

The outline of the paper is as follows. Section 1 provided an introduction of the considered problem and a complete survey of works related to this article. Section 2 describes the problem and provides main assumptions. Mathematical modeling of the problem is presented in Section 3 and the cost of inventory under different scenarios has been calculated. Section 4 presents the solution approach of this study. The computational results are presented in Section 5 and at finally, conclusions and suggestions for future studies are presented in Section 6.

Problem Definition

The considered production system includes a machine that must meet demand at a fixed rate. This machine is exposed to breakdown with an incremental rate. Manufactured products may be approved or rejected by the quality control unit. The rate of defective items observed in each batch is compared with a limit value (l_{max}), and based on this, a decision is made whether or not to perform maintenance operations as below:

$$\begin{cases} l_m \le l \le l_{max} & \textit{preventuve repairs} \\ l \ge l_{max} & \textit{corrective repairs} \\ 0 < l < l_m & \textit{no need to repairs} \end{cases}$$

Therefore, the need for preventive maintenance is determined as equation (1) and the probability of the need for major repairs is calculated as equation (2).

$$p(l_{max}) - p(l_m) \tag{1}$$

$$1 - p(l_m) \tag{2}$$

In this system, the possibility of rework is considered once (according to Figure 1). In order to deal with the disruption caused by the cessation of production (during maintenance or correction), there is some buffer stock (h). In addition, it is assumed that rework is allowed once. Items that are found to be defective will be reworked.

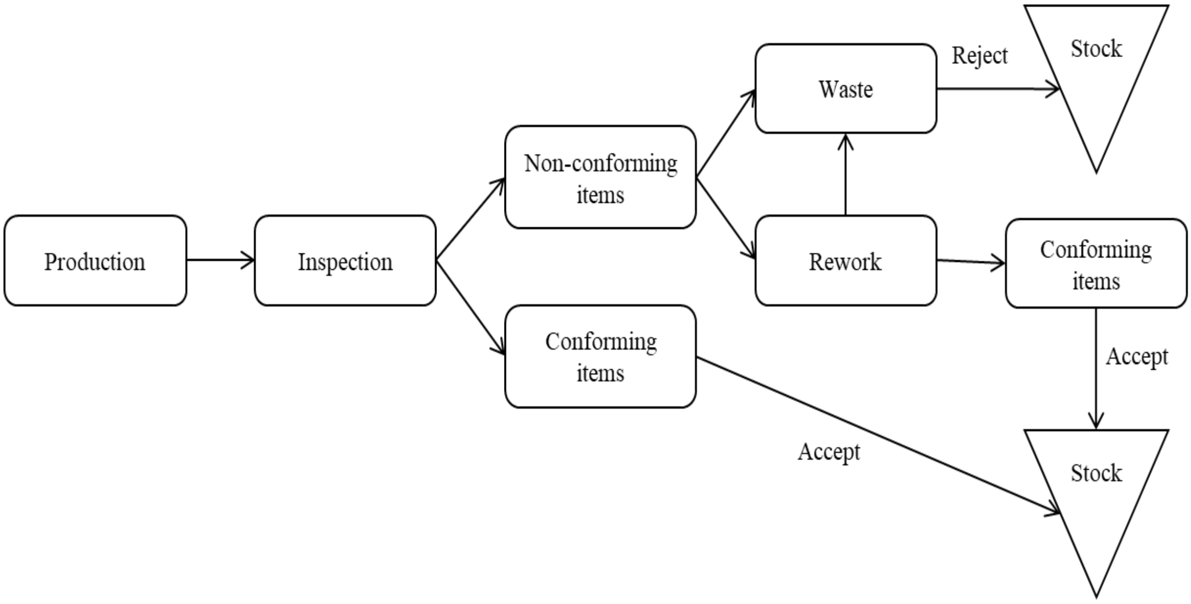

Figure 1. Production system considering one rework stage

To more clarification, the assumptions of the considered problem are presented as below:

1) The function of cumulative life distribution of the production unit is unknown.
2) Cumulative distribution functions related to the time of maintenance operations (both of preventive and corrective actions) are normal.
3) In case of shortage, the demand will be compensated with a delay.
4) Rework is allowed for once.
5) The cost of rework is known.
6) Inspection is done periodically.
7) The batch size is determined and fixed under qualitative inspections.
8) The costs of maintenance, quality, and inventory are known and fixed.
9) The cost of inspection of the quality control unit can be neglected.
10) After maintenance, the system will be as well as a new system.

Mathematical Modeling

As was evident in the problem definition, this study aims to determine the optimal value of buffer stock (h^*) and the limit value of non-conforming units rate (l_m^*) simultaneously in order to minimize total costs of the system. The following analysis expresses the determination of the amount of this cost by considering different scenarios. The total costs consist of a total of three cost including maintenance, inventory, and quality, and finally, the total cost per unit of time is obtained by dividing the total cost by the average production period, which is equal to the average time between two consecutive major repairs. For a production period, according to the occurrence or non-occurrence of deficiency, and the position of the buffer stock, four scenarios are defined. The cost of inventory depends on the scenario that may occur. In all scenarios, it is assumed that the level of the buffer stock is equal to h at the beginning of each period.

Notations:

C_s	Holding cost of each unit of product per unit time;
C_p	Cost of shortage per unit of product (cost of delivery delay);
C_{nc}	Cost of unapproved products per time unit;
h	Size of buffer stock;
M_p	Cost of preventive maintenance action;
M_c	Overhaul costs;
μ_p	Average duration of preventive maintenance action;
μ_c	Average duration of overhaul;
δ_q	Total cost of quality;
δ_I	Total cost of inventory;
d_m	Total cost of maintenance;
d	Demand per time unit;
$p(l)$	probability distribution function related to rejection rate (l);
l_m	Threshold level of non-conforming units' rate;
g_p	Probability density function related to the duration preventive maintenance actions;
u_{max}	Maximum production rate;
(h, l_m)	average total cost of the operation per unit of time;

The total probability of acceptance and the probability of rejection are obtained through relationships (3) and (4), respectively.

$$p(l) + p(l)(1 - p(l)) \tag{3}$$
$$(1 - \mathrm{p(l)})^2 \tag{4}$$

The average rejection rate per time unit during the $\mathrm{T_1}$ period is equal to the rate of production of non-conforming products in $\mathrm{T_1}$ and is calculated according to Equation (5).

$$\overline{\alpha}_{\mathrm{R1}} = \mathrm{u_{max}} \int_0^1 (1 - \mathrm{p(l)}^2)\mathrm{dl} \tag{5}$$

$\mathrm{T_1}$ is the time required to create a buffer stock during which the machine produces in the highest rate ($\mathrm{u_{max}}$). $\overline{\alpha}_{\mathrm{R1}}$ is the average rejection rate per time unit during the $\mathrm{T_2}$ period, which is equivalent to the rate of production of non-conforming products in $\mathrm{T_2}$, and is calculated according to Equation (6). $\mathrm{T_2}$ is a time when the machine only produces to meet demand. Its production rate is equal to $(d + \overline{\alpha}_{\mathrm{R2}})$ in which: $(d + \overline{\alpha}_{\mathrm{R2}}) \leq \mathrm{u_{max}}$. The production rate is calculated as relation (7).

$$\overline{\alpha}_{\mathrm{R2}} = \frac{\mathrm{d} \int_0^1 (1 - \mathrm{p(l)})^2 \mathrm{dl}}{1 - \int_0^1 (1 - \mathrm{p(l)})^2 \mathrm{dl}} \tag{6}$$

$$\begin{cases} \mathrm{u_{max}} & \mathrm{S(t)} < \mathrm{k} \\ \dfrac{\mathrm{d}}{1 - \int_0^1 (1 - \mathrm{p(l)})\mathrm{dl}} & \mathrm{S(t)} = \mathrm{k} \end{cases} \tag{7}$$

Calculating the Average Cost of Inventory in Scenarios

Scenario (1): Production Period without Shortage

In this scenario, the buffer stock is provided before the maintenance operation, and the maintenance operation are completed before the shortage occurs. That is, interruption of service time (D_k) does not exceed the consumable time of the buffer stock, so the shortage does not occur in this scenario as shown in Figure 2.

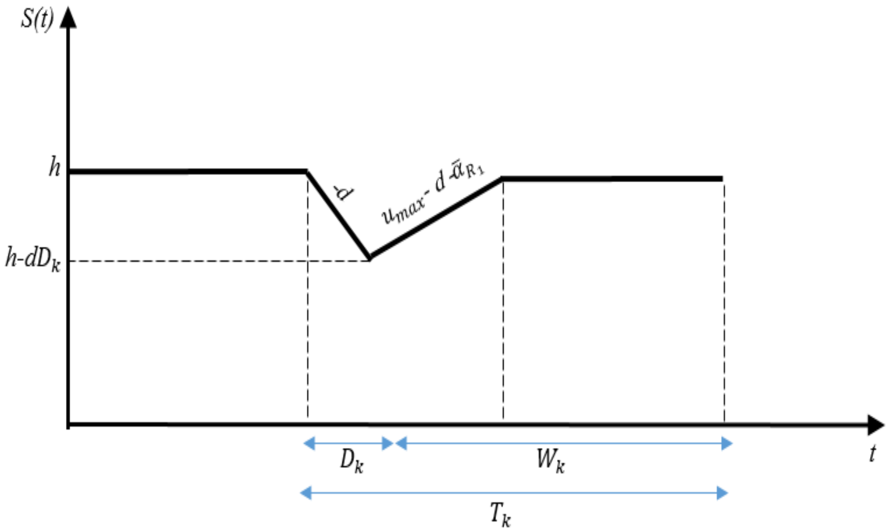

Figure 2. The first scenario without any shortage during the production period

In this case, the average cost of inventory (Γ_{NL}) is calculated as Equation (8).

$$\Gamma_{NL} = C_s[h(W_k + D_k) - \left(\frac{D_k^2 d}{2}\right) + \frac{D_k^2 d^2}{2(\bar{\alpha}_{R_1} + d - u_{max})}] \tag{8}$$

Scenario (2): Production Period with Compensable Shortage

In this scenario, the buffer stock is provided before the maintenance operations. The maintenance time is longer than the first scenario in which the buffer stock is finished before the maintenance completion time. Moreover, it is assumed that the shortage should be compensated after completing the maintenance operation as shown in Figure 3.

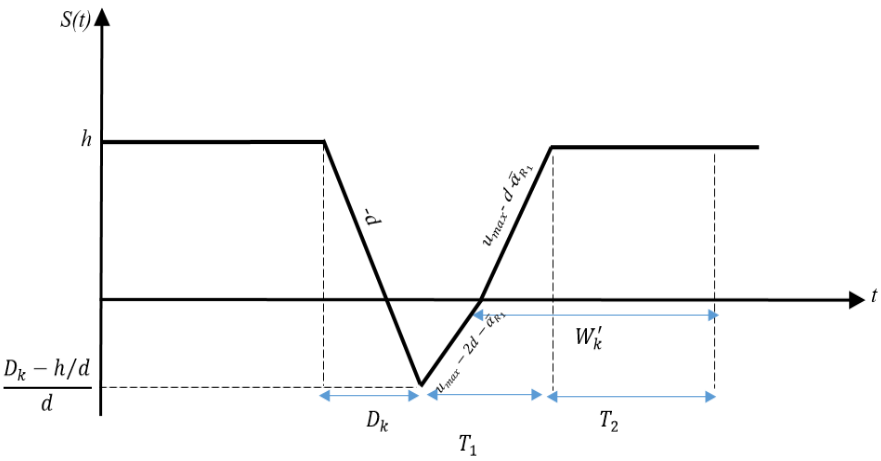

Figure 3. The second scenario with compensable shortage during the production period

In this case, the average cost of inventory (Γ_{WL}) is calculated as Equation (9).

$$\Gamma_{WL} = C_s[h(W_k + \frac{h}{(\bar{\alpha}_{R_1} + d - u_{max})} - \frac{(h - D_k d)}{d^2(\bar{\alpha}_{R_1} + 2d - u_{max})}) - \frac{(h^2)}{2(\bar{\alpha}_{R_1} + d - u_{max})} + \frac{(dh^2)}{2}] - \frac{C_p(h - D_k d)^2}{2d^4(\bar{\alpha}_{R_1} + 2d - u_{max})} \tag{9}$$

Scenario (3): Production Period with Compensable Shortage and Not Reaching the Buffer Stock

In this scenario, also the system shutdown time is longer than the period of consumption of the buffer stock, and a shortage occurs, which is compensable, and before the replenishment of the buffer stock, failure occurs again (as shown in Figure 4).

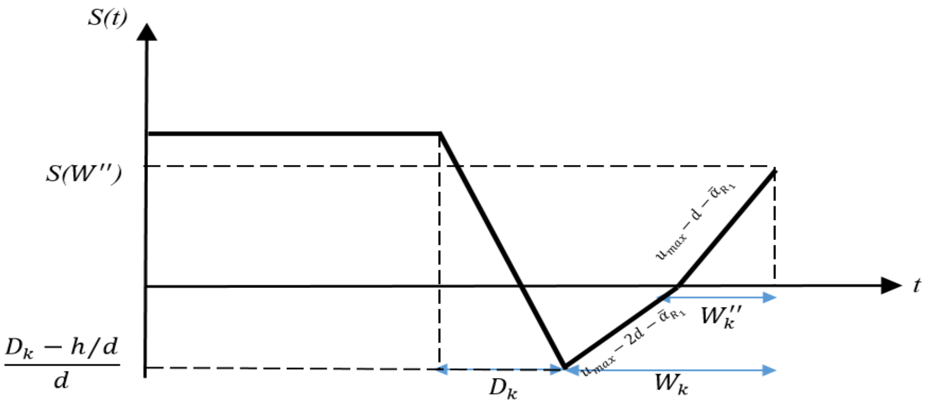

Figure 4. The third scenario: Production period before reaching to buffer stock with compensable shortage

In this scenario, W_k'' indicates the time period after providing shortages, followed by meeting demands and buffer stock. This time period is calculated as the Equation (10) and the related inventory level will be determined as (11). Moreover, the cost average of inventory (Γ_{VL}) is calculated as Equation (12) in this case.

$$W_k'' = W_k - \frac{D_k d - h}{(u_{max} - 2d - \bar{\alpha}_{R_1})d^2} \tag{10}$$

$$S(W_k'') = \frac{d^2 W_k\left(u_{max} - 2d - \bar{\alpha}_{R_1}\right) - (D_k d - h)}{d^2\left(u_{max} - d - \bar{\alpha}_{R_1}\right)\left(u_{max} - 2d - \bar{\alpha}_{R_1}\right)} \tag{11}$$

$$\Gamma_{VL} = \frac{C_s d h^2}{2} - C_s\left(\frac{W_k d^2\left(\bar{\alpha}_{R_1} + 2d - u_{max}\right)^2 - (h - D_k d)}{2d^4\left(\bar{\alpha}_{R_1} + d - u_{max}\right)^3\left(\bar{\alpha}_{R_1} + 2d - u_{max}\right)^2}\right) - \frac{C_p(h - D_k d)^2}{2d^4\left(\bar{\alpha}_{R_1} + 2d - u_{max}\right)} \tag{12}$$

Scenario (4): Production Period Without Shortage and Not Reaching to Buffer Stock

In this case, the breakdown time is less than the consumption period time of the buffer stock and so, the shortage does not occur. However, after restarting, maintenance is required before the buffer stock level reaches to amount of hh (as shown in Figure 5).

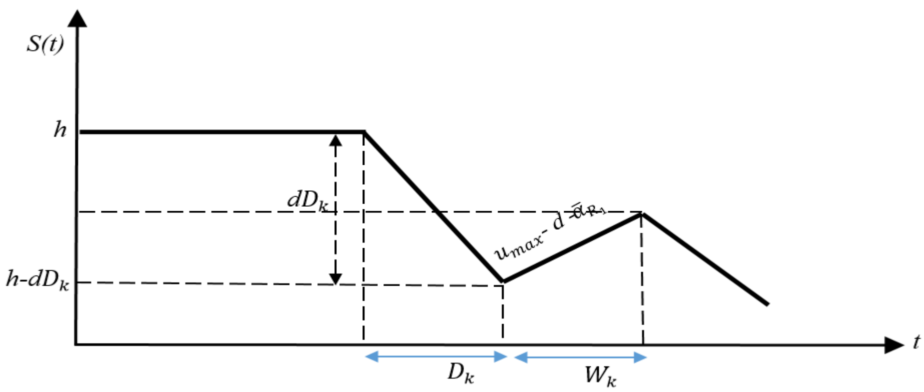

Figure 5: The fourth scenario: Production period before reaching to buffer stock without shortage

In this scenario, the level of inventory recovery between the two intervals of maintenance operations is calculated as Equation (13). Furthermore, the cost average of inventory (Γ_{SW}) is calculated as (14).

Furthermore, the average of total cost of inventory according to four aforementioned scenarios is determined as (15-18). Meanwhile, equations (15) through (17) show how the total inventory cost is obtained. Finally, the average of inventory cost is calculated as (18).

$$S(W_k) = W_k(u_{max} - \overline{\alpha}_{R_1} - d) \tag{13}$$

$$\Gamma_{SW} = C_s\left[D_k h - W_k^2\left(\overline{\alpha}_{R_1} + d - u_{max}\right) - \frac{D_k^2 d}{2} - [\frac{W_k\left(h + D_k d + W_k\left(\overline{\alpha}_{R_1} + d - u_{max}\right)\right)}{2\overline{\alpha}_{R_1} + 2d - 2u_{max}}]\right] \tag{14}$$

$$\Gamma = (\Gamma_{NL} + \Gamma_{SW})(1 - R_D(h/d)) + (\Gamma_{WL} + \Gamma_{VL})R_D(h/d) \tag{15}$$

$$R_D(h/d) = \int_{h/d}^{\infty} g_D(x)dx \tag{16}$$

$$g_D(x) = g_D(x)(1 - P(l_{max})) + g_p(x)(P(l_{max}) - P(l_m)) \tag{17}$$

$$\begin{aligned}\Gamma = R_D(h/d)&\left(C_s h W_k + \frac{C_s h^2}{2\left(\overline{\alpha}_{R_1} + d - u_{max}\right)} - \frac{C_s h(h - D_k d)}{d^2\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)} - \frac{C_p(h - D_k d)^2}{d^4\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)}\right.\\ &\left. + C_s d h^2 - C_s\left(\frac{W_k d^2\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)^2 - (h - D_k d)}{2d^4\left(\overline{\alpha}_{R_1} + d - u_{max}\right)^3\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)^2}\right)\right)\\ &+ \left((C_s h W_k + C_s h D_k) - C_s\left(\frac{D_k^2 d}{2}\right) + \frac{C_s h D_k^2 d^2}{2(\overline{\alpha}_{R_1} + d - u_{max})} + C_s D_k h\right.\\ &\left. - C_s W_k^2\left(\overline{\alpha}_{R_1} + d - u_{max}\right) - \frac{C_s D_k^2 d}{2} - \frac{C_s W_k\left(h + D_k d + W_k\left(\overline{\alpha}_{R_1} + d - u_{max}\right)\right)}{2\overline{\alpha}_{R_1} + 2d - 2u_{max}}\right)\\ &- \left(R_D(h/d)\right)\left(C_s h W_k + 2C_s h D_k - C_s D_k^2 d + \frac{C_s D_k^2 d^2}{2(\overline{\alpha}_{R_1} + d - u_{max})}\right.\\ &\left. - C_s W_k^2\left(\overline{\alpha}_{R_1} + d - u_{max}\right) - [\frac{C_s W_k\left(h + D_k d + W_k\left(\overline{\alpha}_{R_1} + d - u_{max}\right)\right)}{2\overline{\alpha}_{R_1} + 2d - 2u_{max}}]\right)\end{aligned} \tag{18}$$

In this case, the average cost of inventory per unit time (δ_I) is obtained as Equation (19).

$$\begin{aligned}\delta_I = \frac{1}{E(T_k)}&\left[\int_{\frac{h}{d}}^{+\infty}\left(\frac{C_s h^2}{2\left(\overline{\alpha}_{R_1} + d - u_{max}\right)} - \frac{C_s h(h - xd)}{d^2\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)} - \frac{C_p(h - xd)^2}{d^4\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)} + C_s d h^2\right.\right.\\ &\left. - C_s\left(\frac{E(W_k)d^2\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)^2 - (h - xd)}{2d^4\left(\overline{\alpha}_{R_1} + d - u_{max}\right)^3\left(\overline{\alpha}_{R_1} + 2d - u_{max}\right)^2}\right)\right) g_D(x)dx + (C_s h E(W_k))\\ &+ C_s\int_0^{\frac{h}{d}}\left(2hx - x^2 d + \frac{x^2 d^2}{2\left(\overline{\alpha}_{R_1} + d - u_{max}\right)} - \left(E(W_k)\right)^2\left(\overline{\alpha}_{R_1} + d - u_{max}\right)\right.\\ &\left.\left. - \left[\frac{E(W_k)\left(h + xd + W_k\left(\overline{\alpha}_{R_1} + d - u_{max}\right)\right)}{2\overline{\alpha}_{R_1} + 2d - 2u_{max}}\right]\right) g_D(x)\,dx\right]\end{aligned} \tag{19}$$

where $E(T_k)$ demonstrates the average time in a production period that obtained through relations (20) and (21).

$$E(T_k) = E(W_k) + E(D_k) \tag{20}$$

$$E(D_k) = \mu_p[p(l_{max}) - p(l_m)] + \mu_C[1 - p(l_m)] \tag{21}$$

The Cost Average of Maintenance

The expected total cost of maintenance is calculated as relation (22).

$$\delta_M = \frac{1}{E(T_k)}(M_p[P(l_{max}) - P(l_m)] + M_c[1 - P(l_{max})] \quad (22)$$

The Cost Average of Quality

The cost average of quality depends on the cost average of non-conforming products during the production period. This cost element is calculated as (23).

$$E(T_2) = E(W_k) - E(T_1) \quad (23)$$

where we have:

$$W_k = T_1 + T_2 \rightarrow \quad T_2 = W_k - T_1$$

and the expected value of T_1 is determined as (24).

$$E(T_1) = \frac{h}{u_{max} - d - \overline{\alpha}_{R_1}} \quad (24)$$

In additional, the average number of non-conforming products during T_1 and T_2 is obtained as (25) and (26) respectively.

$$Qp_1 = \overline{\alpha}_{R_1} E(T_1) \quad (25)$$
$$Qp_2 = \overline{\alpha}_{R_2} E(T_2) = \overline{\alpha}_{R_2}(E(W_k) - E(T_1)) \quad (26)$$

According to Equations (22-25), the average of total cost of quality is calculated through the Equation (27).

$$\delta_Q = \frac{C_{nc}}{E(T_k)}[Qp_1 + Qp_2] \quad (27)$$

By replacing the Equation (24) in Equation (26), the cost average of quality (δ_Q) can be calculated as equal (28).

$$\delta_q = \frac{C_{nc}}{E(T_k)}[\overline{\alpha}_{R_1} \frac{h}{u_{max} - d - \overline{\alpha}_{R_1}} + \overline{\alpha}_{R_2}(E(W_k) - \frac{h}{u_{max} - d - \overline{\alpha}_{R_1}}) \quad (28)$$

By adding the costs of quality, inventory, and maintenance, the average of total cost per time unit is calculated as Equations (20) and (30).

$$\Pi(h.l_m) = \delta_I + \delta_Q + \delta_M \quad (29)$$

$$\begin{aligned}
\mathbf{\Pi(h.l_m)} = \frac{1}{E(T_k)}\Bigg[\int_{\frac{h}{d}}^{+\infty}\Bigg(&\frac{C_s h^2}{2(\overline{\alpha}_{R_1} + d - u_{max})} - \frac{C_s h(h - xd)}{d^2(\overline{\alpha}_{R_1} + 2d - u_{max})} - \frac{C_p(h - xd)^2}{d^4(\overline{\alpha}_{R_1} + 2d - u_{max})} + C_s dh^2 \\
&- C_s\left(\frac{E(W_k)d^2(\overline{\alpha}_{R_1} + 2d - u_{max})^2 - (h - xd)}{2d^4(\overline{\alpha}_{R_1} + d - u_{max})^3(\overline{\alpha}_{R_1} + 2d - u_{max})^2}\right)\Bigg) g_D(x)dx + (C_s h E(W_k)) \\
&+ C_s \int_0^{\frac{h}{d}}\Bigg(2hx - x^2 d + \frac{x^2 d^2}{2(\overline{\alpha}_{R_1} + d - u_{max})} - (E(W_k))^2(\overline{\alpha}_{R_1} + d - u_{max}) \\
&- [\frac{E(W_k)\left(h + xd + W_k(\overline{\alpha}_{R_1} + d - u_{max})\right)}{2\overline{\alpha}_{R_1} + 2d - 2u_{max}}]\Bigg) g_D(x)\,dx\Bigg] + \frac{1}{E(T_k)}(M_p[P(l_{max}) - P(l_m)] \\
&+ M_c[1 - P(l_{max})] + \frac{C_{nc}}{E(T_k)}[\overline{\alpha}_{R_1}\frac{h}{u_{max} - d - \overline{\alpha}_{R_1}} + \overline{\alpha}_{R_2}(E(W_k) - \frac{h}{u_{max} - d - \overline{\alpha}_{R_1}})]
\end{aligned} \quad (30)$$

To calculate average of total cost per time unit for specific input parameters, it is necessary to determine the value of $E(T_k)$. Based on the proposed strategy, it can be concluded that the end of the operating period (W_k) corresponds to the moment at which the rejection rate is greater than l_m that is denoted by t_m. Suppose $l(t)$ is a continuous and ascending function that expresses the changes of the rejection rate as a function of time. In this case we have (31) and (32) as below:

$$l(t_m) = l_m \quad (31)$$
$$t_m = l^{-1}(l_m) \quad (32)$$

Solution Approach

In this section, the proposed solution approach is presented. The proposed integrated procedure is a numerical iterative technique based on the proposed algorithm in [24]. The proposed method doesn't explore all solution space but it behaves as a numerical optimization procedure and provides near optimal solution. In most numerical methods, the resulting response error can be controlled and reduced to the required amount, and this, in addition to being easier to implement, make numerical algorithms a suitable option for solving NP-hard and non-linear optimization problems. The proposed algorithm is coded with MATLAB R2014a software and some analysis are carried out using MINITAB 16 software. The algorithm was run on a PC with Intel Core i7-8550U CPU, 2.10 GHz and 8GB RAM. In order to evaluate the performance of the proposed algorithms, data of (Montgomery, 2009) was used and new required parameters were added. Moreover, some random instances were used for sensitivity analysis and more investigations.

The average of total cost per time unit as the objective function is calculated via Equation (30). This equation determines the optimal values of the decision variables l_m^* and h^*. As mentioned above, to use Equation (30), the average operating time $E(W_K)$ need to be estimated using Equation (32), which is influenced by the function $l(t)$that represents changes in the rejection rate per time. To estimate the function $l(t)$ for each amount of input data, a model is required for the production process. In the proposed model, a production system must be considered that, along the planning horizon, for the desired cumulative production, it produces non-conforming products according to Equation (33):

$$L(t + \Delta t) = L(t)(1 + \xi) \quad (33)$$

where, $L(t)$ is the cumulative number of non-conforming products at moment t as a random variable between zero and one with the beta distribution function. Moreover, Δt is the time required for accumulation production and the rejection rate is calculated according to Equation (34). In this relation, m and x represents the number of products and the batch size respectively.

$$l(t) = \frac{L(t)}{mx} \quad (34)$$

Procedure of the proposed approach is shown in Figure 6 in detail, in which the following notation has been added:

L°	Number of non-conforming products at the moment $t = 0$
n_{rep}	Number of iterations

The batch size x is generated during period T, then the number of unconfirmed products is randomly calculated using Equation (33) and finally the rejection rate $l(t)$ is obtained using

Equation (34). The approach is repeated for m periods with length T until the rejection rate does not exceed one, and we will have n_{rep} iterations in totally. Finally, the mean value $l_i(t_i)$ for $i = 1,2,\dots,n_{rep}$ is considered to estimate the l_m function using the least squares method.

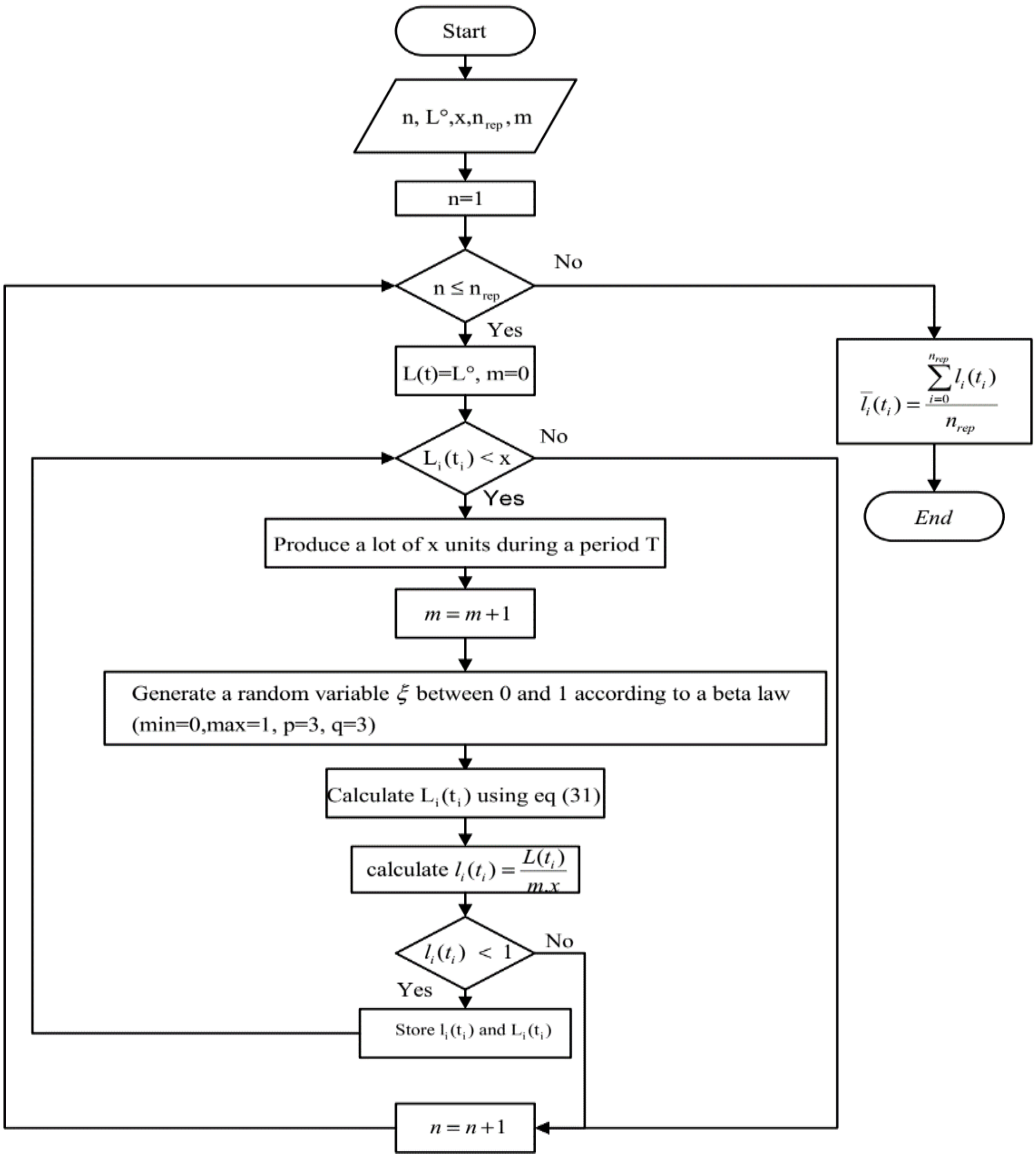

Figure 6. Flowchart of the proposed solving procedure [24]

Computational Results

In order to evaluate performance of the proposed model and solution approach, some test problems are considered to solve and analyze. To this end, the data of Radhavi et al.'s (2010) study are adopted and used in this section (14). Therefore, data and condition of test problems has been defined as below:

The probability distribution function of repair time: N $(\mu = 2.\sigma = 0.5)$;

The probability distribution function of maintenance time and preventive repairs: N $(\mu = 0.5.\sigma = 0.1)$;

The probability distribution function of the rejection rate: Beta $(\alpha = 3.\beta = 3)$;

$l_{max} = 0.8$;

$C_s = 5\$;$
$C_p = 5\$;$
$M_c = 5\$;$
$M_p = 500\$;$
$C_{nc} = 5\$;$
$u_{max} = 100\$;$
$d = 20;$
$l_m = [0.01\ 0.6];$
$h = [10\ 100];$

In additional, values of the input parameters for generating data are considered as below:

- The estimation function: l_t^e;
- The cumulative number of unverified products at the time zero: $L = 0$;
- The batch size: $X = 50$;
- The number of iterations for generating data: $n_{rep} = 10$;
- The number of periods: $m = 7$;

The results of implementation the proposed model for optimizing the problem for 7 periods (shown in rows) and 10 times iteration (columns) has been shown in Table (1). Each cell of Table (1) shows non-conforming unit rate as result for defined period and iteration. As can be seen, with increasing number of periods in each iteration non-conforming unit rate increases.

Table 1. Result of solving the instance for 7 periods in 10 iterations

Period	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6	Iteration 7	Iteration 8	Iteration 9	Iteration 10
T_1	0.142	0.161	0.171	0.144	0.180	0.152	0.181	0.181	0.158	0.183
T_2	0.253	0.287	0.323	0.276	0.321	0.266	0.319	0.293	0.243	0.289
T_3	0.341	0.430	0.440	0.425	0.454	0.377	0.476	0.405	0.346	0.391
T_4	0.441	0.602	0.561	0.591	0.609	0.486	0.627	0.556	0.444	0.490
T_5	0.552	0.788	0.708	0.769	0.777	0.617	0.812	0.755	0.572	0.614
T_6	0.696		0.893	0.974	0.995	0.819			0.745	0.747
T_7	0.866									0.935

Figure 7 shows the shape of the obtained function l_t^e as well as the shape of its estimate using a least square curve fitting tool. The obtained estimated expression has been calculated as equation (36). Hence, using equation (32), for a given l_m, $E[W_k]$ can be found as equation (37).

$$l_t^e = 0.3909\mathrm{Ln}(t) + 0.072 \quad (36)$$

$$E(W_k) = \exp\left(\frac{l_m - 0.072}{0.3909}\right) \quad (37)$$

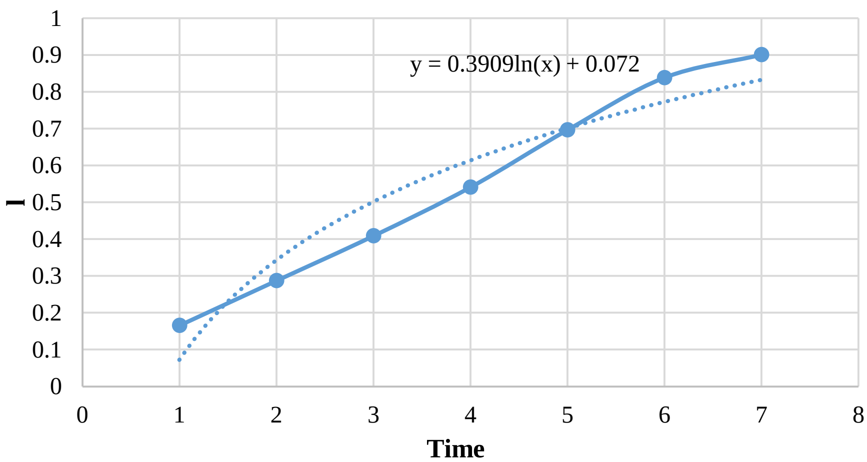

Figure 7. Estimation of rejection rate changes over time

Now that all the terms of the total average cost per time unit (equation (30)) have been defined for any given values of the decision variables h and l_m. Table 2 shows the result of solving the mentioned numerical example using the integrated model. Due to this result, as expected, when the limit value of l_m increases, the average production time $E(W_K)$ increases too.

Table 2. Evaluation of the optimal policy based on the cost of shortage

l_m	h	$E(w_k)$	$E(t_k)$	δ	$\Pi(h.l_m)$
0.05	60	2.502533	3.316689	1210.908	4600.706
0.1	60	2.630841	3.357341	1118.418	4443.149
0.15	60	2.765727	3.369883	970.9142	4249.045
0.2	60	2.907529	3.371529	774.5562	4011.652
0.25	60	3.056601	3.377257	540.2808	3731.711
0.3	60	3.213316	3.399816	285.127	3418.416
0.35	60	3.378067	3.449723	31.87074	3089.358
0.5	10	3.835253	3.924753	882.0638	1364.839
0.6	10	4.337523	4.321523	935.2717	1393.093

Considering the numerical input data presented at the beginning of this section, in this part, there has been used a simple enumeration procedure to find the optimal couple (h^*, l_m^*), which minimizes the total average cost per time unit. Figure 8 shows the contour plot of the response surface of this cost rate $\mathbf{\Pi(h.l_m)}$. In the optimal strategy, the average production time is 8.3 hours and the average cycle time is 9.3. The minimum value of Π (h. l_m) was also obtained in 1364.839.

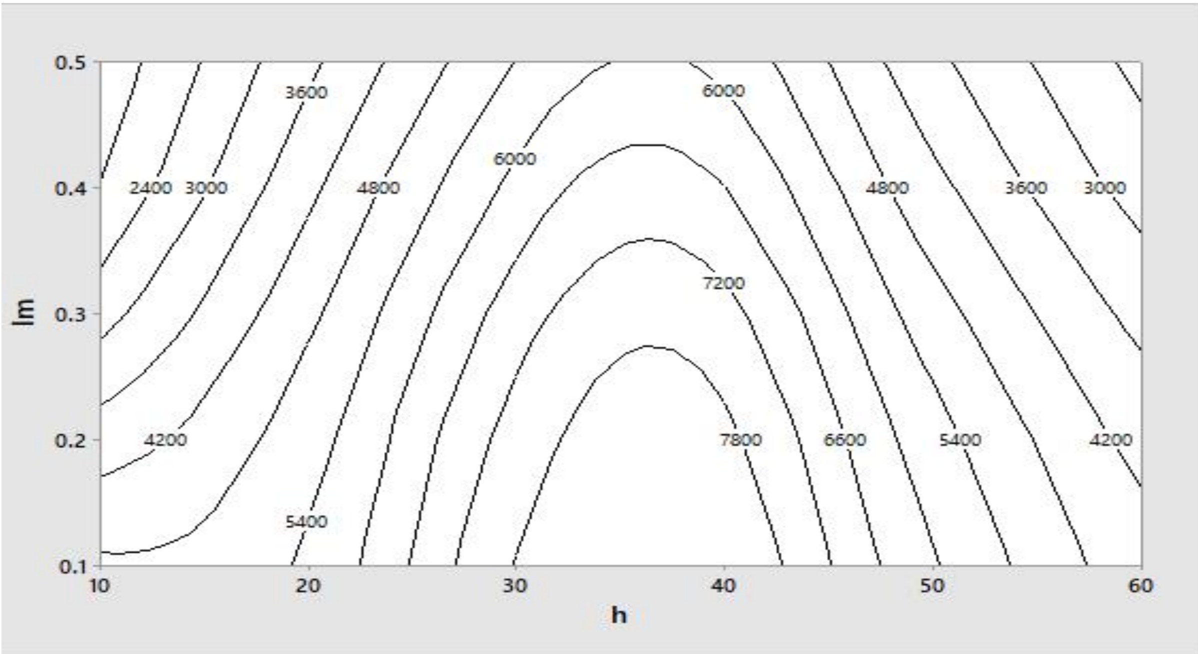

Figure 8: Contour plot for the response surface

Conclusion

In the realm of modern production systems, the interplay between quality management, inventory control, and maintenance scheduling remains a focal point of organizational efficiency. Manufacturing industries are facing difficult challenges and they must achieve a high productivity and a high quality at the same time at the lowest possible cost to maintain their competitiveness and their continuity in today's competitive environment. To reach these objectives, they need to manage successfully several functions such as maintenance, production, quality and inventory. Managers and researchers are increasingly recognized that the survival of any company depends on its management approach. One of the keys of success consists to an effective integration between these different fields which can lead to an excellent manufacturing performance.

In this paper, the policy of quality control, preventive maintenance, production control and simultaneous in a single-machine production system, considering a rework stage, under four scenarios (production period without deficiency, production period with compensable deficiency, production period before reaching saved inventory and without shortage, production period before reaching saved inventory and with compensable shortage) were examined. For generating optimal strategy, a mathematical model has been developed. Non-conforming product rates (l_m^*) and inventory size (h^*) protect the model decision variables, which are determined by an integrated approach. In optimal strategy (l_m^*, h^*) given by modeling and simulation minimizes total average unified cost, that is contained of inventory, maintenance and quality costs.

The aim of this work was to integrate buffer stock sizing, preventive maintenance and quality control issues in a single model. The problem was discussed under four abovementioned scenarios and a numerical instance of a real case study was solved using the proposed solution approach. Findings indicate the performance accuracy of the proposed iterative procedure. Moreover, sensitivity analyses were performed to show the effect of the main parameters and the robustness of the proposed integrated model. Solving the problem under the equal probability of the four scenarios indicated that the total expected cost has been increased compared to the obtained best combination. In addition, the result of the proposed model showed that, with increasing the limit value of l_m, the average production time E (W_K) increases.

In this paper an integrated procedure was proposed that can guide decision-makers in optimizing resource allocation and enhancing operational performance. The result of this study emphasizes the critical importance of integrating quality aspects into overall production strategy. By acknowledging that defects necessitate rework, managers are compelled to adopt a proactive quality assurance framework. Implementing robust quality control measures not only reduces rework rates but also minimizes associated costs, thereby improving overall profitability. This proactive stance should be complemented by systematic training programs to empower employees with the skills necessary to uphold and enhance product quality. Moreover, the article underscores the relationship between inventory management and production efficiency. Maintaining optimal inventory levels is essential to mitigate excess holding costs while ensuring that production runs smoothly without interruptions. The insights provided within the analysis suggest that managers should employ just-in-time (JIT) inventory practices, which could significantly decrease wastage and improve cash flow. Additionally, applying techniques such as demand forecasting and inventory turnover assessments can further refine inventory management processes.

Suggestions for future research include probabilistic demand, limited product storage time, multi-machine modeling, and multi-product modeling. Another suggestion can be modeling with stochastic costs, and define a specific distribution function for cost of inspection.

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