



An Interval-Valued Fuzzy Group Decision-Making Model Based on Two New Developed IVF-LBWA and IVF-MAIRCA Methods for Sustainable Project Selection

Sajjad Karami¹ and Seyed Meysam Mousavi^{2*} 

¹Ph.D. Candidate, Department of Industrial Engineering, Shahed University, Tehran, Iran.

²Professor, Department of Industrial Engineering, Shahed University, Tehran, Iran.

Received: 10 December 2024, Revised: 21 March 2025, Accepted: 27 March 2025

© University of Tehran 2025

Abstract

In an era of rapid change, complexity, and uncertainty, organizations must rely on sustainable project portfolio management to achieve long-term objectives. In project-oriented environments, selecting the most suitable project portfolio remains a critical challenge. To address this, advanced decision-making approaches, particularly multi-criteria decision-making (MCDM) techniques, have been developed to support well-informed and dependable choices. This study develops a new synergistic integration of Interval-Valued Fuzzy Level-Based Weight Assessment (IVF-LBWA) and Interval-Valued Fuzzy Multi-Attribute Ideal Real Comparative Analysis (IVF-MAIRCA), to improve decision-making in uncertain environments. In contrast to traditional methods, these approaches utilize interval-valued fuzzy numbers, thereby increasing the precision of project ranking and selection. An application example involving five projects and six evaluation criteria is provided to demonstrate the practical application of these methods. The results indicate that IVF-LBWA and IVF-MAIRCA yield stable and consistent project rankings, reinforcing their applicability in real-world scenarios. A sensitivity analysis was performed across 40 different criteria weighting scenarios to evaluate the impact of weight variations on project rankings. The results demonstrate that the proposed integrated approach preserves ranking stability, reflecting decision-makers' priorities and the relative importance of each criterion. These findings validate its effectiveness in managing uncertainty and supporting reliable decision-making. The findings confirm that this approach provides a systematic and reliable framework for sustainable project portfolio selection. By enhancing decision accuracy and strengthening resilience to uncertainty, it enables decision-makers to align project selection with long-term sustainability, resource efficiency, and strategic objectives.

Keywords:

Group Decision-Making, Interval-Valued Fuzzy Sets, LBWA, MAIRCA, Sustainable Project Selection.

Introduction

Organizations are increasingly situated within environments characterized by continuous and rapid change, escalating complexity, and heightened uncertainty. Consequently, a significant challenge is presented to most organizations, requiring them to adapt and advance at an unprecedented pace to sustain readiness for future demands. An increasing number of organizations are adopting a project-based approach to address the pressing demand for

* Corresponding author: (Seyed Meysam Mousavi)

Email: sm.mousavi@shahed.ac.ir

continual change. By utilizing projects, these organizations enhance their capacity for flexibility, equipping them to effectively manage complex and uncertain conditions [1].

Strategic planning and management are essential for organizations to align with their mission and long-term objectives, often manifesting through projects in the execution phase. In projectized organizations, one of the most critical challenges is selecting the optimal portfolio of projects [2]. Successfully navigating this challenge can provide a considerable competitive edge in the marketplace [3]. Consequently, the process of selecting a project portfolio must involve a comprehensive assessment of all pertinent criteria to achieve the most advantageous outcomes.

The swift growth of the global population, combined with resource constraints and the significant threats of climate change, has given rise to a broad spectrum of environmental and societal challenges. As a result, sustainability has become more crucial than ever before. Financial success alone is no longer sufficient for business growth, and sustainability is increasingly recognized as a critical priority [4]. Organizations must, therefore, move beyond a narrow focus on financial success and embrace sustainability factors such as social responsibility and environmental impact in their long-term strategies to maintain a competitive advantage. In projectized organizations, one of the key challenges is to integrate sustainability into the core processes of project management.

Selecting a sustainable project portfolio represents a complex MCDM problem that involves assessing numerous projects across various criteria to pinpoint those that best match an organization's sustainability goals, strategic vision, and resource constraints. MCDM methods provide a rigorous, systematic framework for this process, enabling decision-makers to assess and prioritize projects based on a diverse set of sustainability, financial, and operational criteria. By following this systematic approach, organizations can guarantee that chosen projects not only achieve financial goals and align with the organization's overall strategic vision, but also meet essential sustainability targets.

Extensive research has demonstrated the significant effectiveness of MCDM methods in addressing challenges associated with project portfolio selection, including its sustainable variants. As reviewed by Kandakoglu [5], a diverse array of MCDM techniques, such as AHP, ANP, DEMATEL, and WASPAS, are frequently employed, both individually and in various combinations, to effectively manage the complexities inherent in project portfolio selection problems.

A significant share of the uncertainties in project portfolio selection stems from the intrinsic vagueness in decision-makers' perceptions or from employing linguistic variables to evaluate project performance, especially concerning sustainability criteria. Fuzzy theory provides a suitable framework for addressing these uncertainties. By incorporating fuzzy sets, decision-making processes are better equipped to handle vague and imprecise information, enabling more accurate modeling of subjective judgments. This approach proves especially beneficial in the context of sustainability, where evaluations often involve complex and uncertain criteria.

This research focuses on addressing the complexities of selecting an effective and sustainable project portfolio, a process that involves the assessment of multiple, often conflicting, criteria. To improve the decision-making process, the study proposes two innovative modifications to the IVF-LBWA and IVF-MAIRCA methods. These adjustments aim to enhance the methods' capability to handle uncertainty and imprecision, which are often present in such evaluations. By incorporating fuzzy logic, the approach more effectively addresses the inherent vagueness in the criteria and their respective weights. Ultimately, this work aims to offer a more dependable and transparent framework for selecting a sustainable portfolio of projects.

The structure of this paper is organized as follows: The next section presents a comprehensive review of relevant literature. Section 3 offers a concise introduction to Interval-

Valued Fuzzy Sets (IVFSs) and their associated operators. In Section 4, the integrated IVF-LBWA and IVF-MAIRCA model is introduced and discussed in detail. Section 5 provides a numerical example. The assessment of the results, along with sensitivity analysis and managerial insights, is presented in Section 6. Finally, the paper concludes in Section 7, summarizing the key findings and implications.

Literature Review

Existing Studies and Approaches

Project portfolio selection has received significant attention, particularly with the application of single or hybrid MCDM methods to navigate complex sustainability objectives and select appropriate projects [2, 6, 7]. Zhu et al. [8] provided a foundational approach by integrating green supplier management into project portfolio selection, using the ANP to develop a systematic three-step methodology. This approach evaluated suppliers by assessing their influence, power, and performance, incorporating environmental factors into the decision-making process. Subsequently, Aragonés-Beltrán et al. [9] expanded on MCDM applications by employing both the AHP and ANP to guide a Spanish solar investment firm in prioritizing solar-thermal projects within its portfolio, facilitating informed investment decisions. In 2019, Szilágyi et al. [10] advanced this research in the context of petroleum exploration, presenting a structured framework for ranking projects that integrated industry-specific qualitative data. More recently, Lee et al. [11] introduced an approach based on the AHP as an effective decision-making tool for selecting projects within project management. Similarly, Alsanousi et al. [12] proposed a hybrid MCDM approach that combined the BWM and the TOPSIS technique for evaluating projects based on financial performance, further enhancing decision-making methodologies in project selection.

In recent years, sustainability has emerged as a central priority within project portfolio management, as financial performance alone is no longer deemed sufficient to ensure long-term success [2]. Khalili-Damghani and Tavana [13] emphasized the role of sustainability in project portfolio selection, highlighting its importance for balanced decision-making. Similarly, Siew [14] underscored the need to embed sustainability principles into construction project portfolios to foster more sustainable outcomes in the industry. RezaHoseini et al. [15] advanced this perspective by developing a model that incorporated sustainability-oriented criteria alongside time-sensitive factors, resource constraints, and budget limitations, employing a sustainability-balanced scorecard to inform project selection and scheduling. More recently, Rahimi et al. [16] reinforced the focus on sustainability in project portfolio selection, applying the Simultaneous Evaluation of Criteria and Alternatives (SECA) method to calculate sustainability scores for project alternatives, thereby strengthening sustainability's role in portfolio decision-making. In this context, Parvaneh and Hammad [17] aimed to identify a set of sustainability criteria and sub-criteria to support owners of power-generating plants in selecting the most sustainable technology for new projects, further expanding the scope of sustainability considerations in project evaluation.

Fuzzy sets have emerged as a highly valued tool in the MCDM field, owing to their exceptional capability to handle uncertainty and imprecision inherent in decision-making processes. As a result, they have found widespread application across diverse industries and domains. One of the key strengths of fuzzy sets lies in their ability to represent intricate and subjective relationships between variables. Unlike traditional binary logic, which confines variables to a strict true or false state, fuzzy sets introduce the concept of partial truth, allowing variables to exist with varying degrees of truth or falsity. This flexibility enables decision-makers to capture the complexities and subtleties of real-world problems, facilitating more nuanced and well-informed decisions.

The integration of fuzzy logic with MCDM methods has become essential in managing uncertainty, vagueness, and linguistic variables within project portfolio selection. This approach enables more refined decision-making by accommodating the imprecision and subjective judgments often in complex evaluation processes [18, 19]. Mohagheghi et al. [20] introduced a novel framework that explicitly considers vagueness and knowledge gaps, improving the selection of sustainable project portfolios under uncertain conditions. Similarly, Wu et al. [21] proposed a model for optimizing renewable energy project portfolios by utilizing a type-2 fuzzy environment, effectively addressing the uncertainties inherent in such projects. In line with these advancements, Mohagheghi and Mousavi [22] introduced the D-WASPAS method, integrating social cognition into decision-making to enhance sustainable project selection in uncertain contexts.

In 2020, Ma et al. [23] developed a multi-criteria project portfolio selection model that combines fuzzy logic with the TOPSIS method, facilitating the identification of optimal projects under uncertainty. The model's practical relevance was demonstrated through a case study in the paper manufacturing industry, underscoring its applicability in real-world scenarios. Jalilibal and Bozorgi-Amiri [24] introduced a hybrid MCDM framework for project portfolio selection, focusing on sustainability criteria. They employed a fuzzy DEMATEL technique in an uncertain environment to assess and classify the interrelationships among criteria, thereby supporting more effective project selection. Furthermore, Mohagheghi et al. [25] proposed a comprehensive framework within an interval type-2 fuzzy environment, offering a holistic solution for optimizing sustainable project portfolios under uncertainty.

Expanding on these advancements, recent studies have applied fuzzy-based MCDM techniques for financial portfolio selection. Huang [26] explored fuzzy portfolio selection to balance economic fluctuations and security returns. Lakshmi and Kumara [27] integrated a randomized weighted fuzzy AHP with TOPSIS for stock selection, while Jawad et al. [28] developed an automatic spherical fuzzy AHP algorithm for investment evaluation. In 2025, Lakshmi and Kumara [29] compared FAHP and entropy methods in the post-COVID Indian mutual fund market, highlighting their effectiveness in managing volatility. Similarly, Sharma [30] introduced a hybrid model combining Trapezoidal Bipolar Fuzzy VIKOR with Boruta-GA for improved decision-making. These studies reinforce the growing role of fuzzy MCDM in optimizing financial portfolio decisions under uncertainty.

The LBWA model, developed by Žižović and Pamucar [31], provides an efficient way to determine criteria weights in MCDM by involving experts from various fields. What sets LBWA apart from methods like the BWM and AHP is its simplicity and flexibility. It requires only minimal comparisons, making it easy to use even when dealing with a large number of criteria. Additionally, its straightforward mathematical framework ensures consistency in expert judgments while allowing decision-makers to adjust weight coefficients through an elasticity factor, offering both accuracy and adaptability in complex decision-making situations.

The LBWA method has demonstrated versatility across various fields, addressing complex decision-making challenges. In 2020, Pamucar et al. [32] applied a fuzzy LBWA-WASPAS-H model for selecting airport ground access modes, while Božanić et al. [33] used a hybrid LBWA-IR-MAIRCA model to determine the constructive elements of weapons. In 2021, Torkayesh et al. [34] introduced a BWM-LBWA-CoCoSo framework to evaluate healthcare sectors in Eastern Europe, and Jokić et al. [35] combined LBWA with fuzzy MABAC for mortar unit position selection. Pamucar and Görçün [36] evaluated European container ports with a fuzzy LBWA-CoCoSo'B approach. In 2023, Ögel et al. [37] applied Fuzzy LBWA to identify food waste causes in perishable goods, and Božanić et al. [38] developed an oil spill response strategy tool using LBWA and Z-MABAC, also Ogundoyin and Kamil [39] introduced a fuzzy-BWM, fuzzy-LBWA, and V-Fuzzy-CoCoSo-LD model for IoT gateway selection in this year. Most recently, Özekenci et al. [40] applied LBWA with TOPSIS and

GRA for personnel selection in a trading company. These studies highlight LBWA's adaptability and effectiveness in supporting decision-making across diverse domains.

The MAIRCA, developed by Pamučar et al. [41], is a powerful decision-making tool that evaluates alternatives by comparing them against both ideal and real benchmarks. Over the years, its versatility has been demonstrated across a range of applications. In 2020, Boral et al. [42] combined fuzzy AHP with MAIRCA to enhance failure modes and effects analysis, while Gul and Ak [43] applied it to flood susceptibility evaluation. Hadian et al. [44] further explored its potential in flood risk assessment, emphasizing its applicability in environmental studies. Occupational risk assessment was innovatively advanced through the integration of fuzzy BWM and MAIRCA by Gul et al. [45]. During the COVID-19 pandemic, Ecer [46] extended MAIRCA using intuitionistic fuzzy sets to aid vaccine selection. Nguyen et al. [47] utilized it alongside MARCOS and TOPSIS in manufacturing decision-making processes, while Eti and Baş [48] applied it to assess social exclusion among international students. Most recently, Tepe et al. [49] leveraged MAIRCA to evaluate psychosocial risks faced by energy sector employees, underscoring its adaptability in addressing diverse, complex decision-making challenges.

Table 1 provides an at-a-glance overview, showing how each category integrates various MCDM methods, their applications, the contributions made by the cited studies, and where further research (e.g., integrating IVFSs) is needed.

Research Gaps

Despite the demonstrated effectiveness of the LBWA and MAIRCA methods in various decision-making contexts, significant gaps persist in their development and application. The LBWA method, while proficient in clustering criteria based on relative importance, encounters challenges related to vagueness, imprecision, and incomplete knowledge—particularly in the second and third steps, where criteria are grouped into sets. A potential enhancement involves integrating Interval-Valued Fuzzy Numbers (IVFNs), which offer a more refined approach to managing uncertainty. However, this integration has yet to be explored in existing studies, leaving a critical gap in the methodological advancement of LBWA.

Similarly, while the MAIRCA method is effective in evaluating alternatives, its ability to handle uncertainty remains underdeveloped. The application of IVFSs within MAIRCA could enhance its capability to address imprecision in alternative assessments, yet this potential remains largely unexamined. Furthermore, despite the individual strengths of LBWA and MAIRCA, their combined use in sustainable project portfolio selection has not been investigated. Given the complexity of sustainability-oriented decision-making under uncertainty, the absence of such an integrated approach represents a crucial research gap that warrants further exploration.

This study aims to present a novel integrated decision-making model that introduces two enhanced versions of LBWA and MAIRCA methods, incorporating IVFSs to effectively address uncertainty and imprecision. The model utilizes performance ratings and criteria weights, expressed through linguistic terms and represented by IVFNs. By implementing this innovative approach, the study seeks to provide a practical and reliable solution for sustainable project portfolio selection. Both LBWA and MAIRCA methods are ideal for handling complex MCDM challenges, as they account for the uncertainty and vagueness inherent in real-world decision-making scenarios. This flexible framework allows decision-makers to articulate their preferences more clearly, improving the overall accuracy and comprehensiveness of decisions related to selecting sustainable project portfolios in uncertain conditions.

Table 1. Selected relevant literature review

Category	Methods/Techniques	Applications/Contexts	Key Contributions	Key Studies
Project Portfolio Selection & Sustainability (MCDM)	Single/hybrid MCDM, ANP, AHP, BWM, TOPSIS	Solar investments, petroleum exploration, general project portfolios	Develops systematic evaluation frameworks that integrate environmental and financial criteria for complex project selection	[2], [6 – 12]
Sustainability in Portfolio Selection	Sustainability-balanced scorecard, SECA, comprehensive criteria identification	Construction projects, scheduling, power-generating technology selection	Emphasizes sustainability as a core decision-making factor beyond financial performance for balanced portfolios	[13 – 17]
Fuzzy MCDM Approaches in Project Portfolio Selection	Fuzzy logic integration, type-2 fuzzy, D-WASPAS, fuzzy DEMATEL	Renewable energy, paper manufacturing, sustainable project portfolios	Addresses uncertainty and vagueness in project evaluation, enabling more nuanced decision-making	[18 – 25]
Fuzzy MCDM in Financial Portfolio Selection	Fuzzy portfolio selection, fuzzy AHP (including spherical and randomized weighted versions), hybrid fuzzy VIKOR models	Stock selection, mutual funds, investment evaluation	Optimizes financial decision-making by balancing economic fluctuations, risk, and security returns through advanced fuzzy evaluations	[26 – 30]
LBWA Method and Its Applications	LBWA and hybrid LBWA models (e.g., LBWA-IR-MAIRCA, LBWA-CoCoSo, fuzzy adaptations)	Transportation, defense, healthcare, maritime, food waste, IoT, personnel selection	Provides a simple and flexible approach to criteria weighting using minimal comparisons and adaptable frameworks across diverse fields	[31 – 37], [39, 40]
MAIRCA Method and Its Applications	MAIRCA often integrated with fuzzy AHP, fuzzy BWM, MARCOS, TOPSIS	Failure Modes and Effects Analysis (FMEA), flood risk, occupational risk, vaccine selection, social exclusion, psychosocial risks	Evaluates alternatives by benchmarking against ideal and real scenarios, offering a robust framework for complex decision-making	[41 – 49]
This Study	Proposed integration of LBWA and MAIRCA with IVFSs	Sustainable project portfolio selection	Aims to enhance uncertainty management by addressing vagueness and imprecision in current methodologies	-

Interval-Valued Fuzzy Sets

Linguistic values play a crucial role in addressing situations characterized by complexity and ambiguity, especially those resistant to precise quantification. Their primary strength is facilitating the articulation of subjective judgments and qualitative uncertainties inherent in decision-making. As highlighted by Zadeh and Zimmermann [50, 51], these values are governed by linguistic variables—terms that allow for a richer, more detailed representation of the conditions under evaluation, thereby enhancing interpretability and insight.

Several researchers contend that representing linguistic expressions solely through traditional fuzzy sets may be insufficient. Notably, Grattan-Guinness [52] and Karnik and Mendel [53] emphasized the advantages of IVFSs, which enhance the ability to handle vague or uncertain information. By accommodating a broader spectrum of potential values, this method enables decision-makers to articulate their judgments and preferences with greater flexibility and precision.

Additionally, studies have shown that IVFSs offer a clearer representation of linguistic expressions, mainly when dealing with vague or imprecise information. These sets give decision-makers a more precise way to handle uncertainty. According to Ashtiani et al. [54] and Vahdani et al. [55], IVFSs prove to be an essential tool for those making decisions in complex, ambiguous scenarios, enabling them to navigate uncertainty with greater accuracy and confidence.

This study explores fuzzy demand through the lens of IVFSs. It builds on Gorzalczany's [56] definition of \tilde{A} , an IVFS that spans the entire range from negative to positive infinity $(-\infty, \infty)$.

$$\tilde{A} = \{x, [\mu_{\tilde{A}^l}(x), \mu_{\tilde{A}^u}(x)]\}, \quad x \in (-\infty, \infty), \quad \mu_{\tilde{A}^l}, \mu_{\tilde{A}^u} : (-\infty, \infty) \rightarrow [0, 1], \quad (1)$$

$$\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}^l}(x), \mu_{\tilde{A}^u}(x)], \quad \mu_{\tilde{A}^l}(x) \leq \mu_{\tilde{A}^u}(x), \quad \forall x \in (-\infty, \infty), \quad (2)$$

where, $\mu_{\tilde{A}^l}(x)$ and $\mu_{\tilde{A}^u}(x)$ correspond to the lower and upper limits of the membership degree, respectively.

Additionally, Yao and Lin's [57] definition of triangular IVFNs, as shown in Fig. 1, describes a triangular IVFN as $\tilde{A} = [\tilde{A}_x^l, \tilde{A}_x^u] = [(a_1^l, a_2^l, a_3^l; \hat{h}_A^l), (a_1^u, a_2^u, a_3^u; \hat{h}_A^u)]$. This representation captures both the lower and upper bounds of the triangular fuzzy numbers along with their associated membership values.

In this context, \tilde{A}^l and \tilde{A}^u represent the lower and upper triangular IVFNs, respectively, with $\tilde{A}^l \subset \tilde{A}^u$. The membership function $\mu_{\tilde{A}}(x)$ indicates the degree to which an event x belongs to \tilde{A} . Specifically, $\mu_{\tilde{A}^l}(x) = \hat{h}_A^l$ and $\mu_{\tilde{A}^u}(x) = \hat{h}_A^u$ represent the lower and upper membership functions, respectively. Based on Fig. 1, the following relationships can be derived:

1. If \tilde{A}^l equals \tilde{A}^u , the triangular IVFN \tilde{A} can be considered a generalized triangular fuzzy number.
2. If the conditions $a_1^l = a_1^u, a_2^l = a_2^u, a_3^l = a_3^u$ and $\hat{h}_A^l = \hat{h}_A^u$ hold, the triangular IVFN \tilde{A} represents a crisp value.
3. If $\hat{h}_A^l = \hat{h}_A^u = 1$ and $a_2^l = a_2^u$, the triangular IVFN \tilde{A} can be expressed as $\tilde{A} = [\tilde{A}_x^l, \tilde{A}_x^u] = [(a_1^u, a_1^l), (a_2^l = a_2^u), (a_3^l, a_3^u)]$.

Using the third relation described above, two triangular IVFNs can be represented as $\tilde{A} = [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)]$ and $\tilde{B} = [(b_1^u, b_1^l), b_2, (b_3^l, b_3^u)]$. Following this, the studies of Chen, Hong and Lee, Chen and Chen, and Vahdani et al. [58-60], [55] proposed operations for addition, subtraction, multiplication, and generalized division between \tilde{A} and \tilde{B} , which are outlined below:

1. Addition of IVFNs \oplus :

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)] \oplus [(b_1^u, b_1^l), b_2, (b_3^l, b_3^u)] \\ &= [(a_1^u + b_1^u, a_1^l + b_1^l), a_2 + b_2, (a_3^l + b_3^l, a_3^u + b_3^u)] \end{aligned} \quad (3)$$

2. Subtraction of IVFNs \ominus :

$$\begin{aligned}\tilde{A} \ominus \tilde{B} &= [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)] \ominus [(b_1^u, b_1^l), b_2, (b_3^l, b_3^u)] \\ &= [(a_1^u - b_1^u, a_1^l - b_1^l), a_2 - b_2, (a_3^l - b_3^l, a_3^u - b_3^u)]\end{aligned}\quad (4)$$

3. Multiplication of IVFNs \otimes :

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)] \otimes [(b_1^u, b_1^l), b_2, (b_3^l, b_3^u)] \\ &= [(a_1^u \times b_1^u, a_1^l \times b_1^l), a_2 \times b_2, (a_3^l \times b_3^l, a_3^u \times b_3^u)]\end{aligned}\quad (5)$$

4. Generalized division of IVFNs \oslash :

$$\begin{aligned}\tilde{A} \oslash \tilde{B} &= [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)] \oslash [(b_1^u, b_1^l), b_2, (b_3^l, b_3^u)] \\ &= [(a_1^u \div b_3^u, a_1^l \div b_3^l), a_2 \div b_2, (a_3^l \div b_1^l, a_3^u \div b_1^u)]\end{aligned}\quad (6)$$

5. Scalar multiplication of the IVFN by k :

$$k \times \tilde{A} = k \times [(a_1^u, a_1^l), a_2, (a_3^l, a_3^u)] = [(ka_1^u, ka_1^l), ka_2, (ka_3^l, ka_3^u)]\quad (7)$$

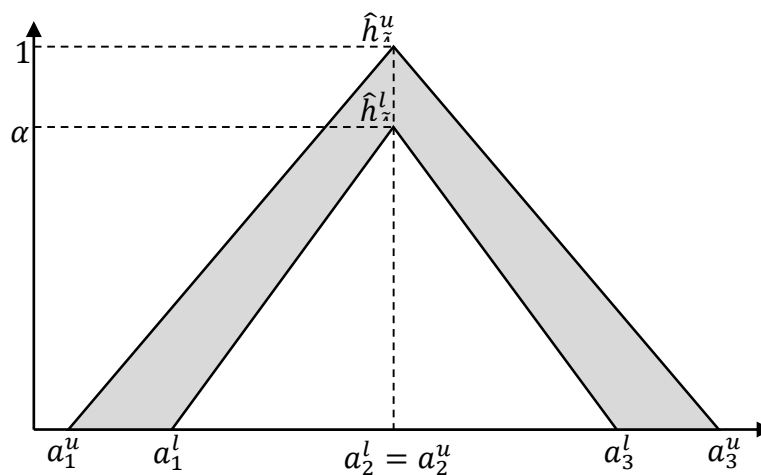


Figure 1. An interval-valued triangular fuzzy number

Proposed Methodology

This study introduces a novel approach that presents two new developed decision-making methods under uncertainty, IVF-LBWA and IVF-MAIRCA, to effectively address MCDM challenges. The proposed methodology is specifically designed to manage uncertainty, ambiguity, and imprecision by incorporating linguistic variables and addressing the inherent subjectivity in decision-making. The approach unfolds in two distinct phases: in the first phase, IVF-LBWA is presented to derive criteria weights, effectively capturing the relative importance of each criterion. In the second phase, IVF-MAIRCA is developed to evaluate and rank alternatives based on the weighted criteria. To ensure accurate and flexible representation of preferences, the approach leverages IVFNs for both criteria weights and performance ratings, with linguistic variables converted into IVFNs as detailed in Table 2.

Managing the uncertainty and ambiguity inherent in real-world decision-making is effectively achieved through the IVF-LBWA and IVF-MAIRCA methods. By leveraging linguistic expressions, these methods enable a more detailed and meaningful articulation of subjective judgments. This enriched representation ensures that decision problems are modeled comprehensively, capturing the complexities and nuances of real-world scenarios. The process emphasizes aligning both criteria weighting and alternative evaluations with the strategic priorities of decision-makers. As a result, the approach facilitates informed, context-sensitive choices that enhance decision quality and ensure consistency with long-term organizational

goals.

The proposed method presents considerable practical advantages, particularly in the realm of sustainable project portfolio selection. Given the complexity of this process, which involves balancing multiple criteria and factors, the approach provides a structured and systematic framework to guide decision-makers. It enables the comprehensive assessment and prioritization of projects based on a wide array of sustainability-related criteria. By facilitating this in-depth evaluation, the method ensures that project portfolios are not only selected according to strategic objectives but also aligned with long-term sustainability goals, ultimately supporting more informed, effective, and impactful decision-making.

Table 2. IVFNs corresponding to linguistic variables

Linguistic variable	Abbreviation	IVFN
Extremely Unimportant / Extremely Bad	EI / EB	[(0, 0.25), 0.75, (1.5, 2)]
Very Unimportant / Very Bad	VI / VB	[(1, 1.25), 1.75, (2.5, 3)]
Unimportant / Bad	I / B	[(2, 2.25), 2.75, (3.5, 4)]
Moderately Unimportant / Moderately Bad	MI / MB	[(3, 3.25), 3.75, (4.5, 5)]
Fair	F	[(4, 4.25), 4.75, (5.5, 6)]
Moderately Important / Moderately Good	MU / MG	[(5, 5.25), 5.75, (6.5, 7)]
Important / Good	U / G	[(6, 6.25), 6.75, (7.5, 8)]
Very Important / Very Good	VU / VG	[(7, 7.25), 7.75, (8.5, 9)]
Extremely Important / Excellent	EU / E	[(8, 8.25), 8.75, (9.5, 10)]
The Most Important	TMI	[(10, 10), 10, (10, 10)]

Introduced IVF-LBWA Method

This section presents the IVF enhancement of the LBWA method. Within an MCDM framework, where the criteria set is defined as $S = \{C_1, C_2, \dots, C_n\}$, the procedure for deriving criteria weight coefficients using the IVF-LBWA model is outlined below:

Step 1: Decision makers D_1 to D_v share their perspectives on the importance of each criterion using linguistic terms. Feedback can be gathered through various means, such as interviews, questionnaires, or collaborative brainstorming sessions. While identifying a single criterion as the most important can enhance the decision-making process, this is not strictly required. A linguistic evaluation of the importance of each criterion by each decision-maker is sufficient to proceed.

Step 2: The linguistic expressions provided by each decision-maker for each criterion are converted into their corresponding IVFNs, as outlined in Table 2. Now, for each of n criteria, an IVFN $\tilde{C}_{j(k)} = \left[\left(c_{j(k)_1}^u, c_{j(k)_1}^l \right), c_{j(k)_2}, \left(c_{j(k)_3}^l, c_{j(k)_3}^u \right) \right]$ is assigned according to the opinion of v decision-makers. The criteria leveling process is then carried out using the following equations:

Step 2.1: Calculate the Maximum Values $\tilde{M}_{(j)}$ for based on each decision maker's opinion:

$$\tilde{M}_{(k)} = \left[\left(m_{(k)_1}^u, m_{(k)_1}^l \right), m_{(k)_2}, \left(m_{(k)_3}^l, m_{(k)_3}^u \right) \right]. \tag{8}$$

where, $m_{(k)_1}^u = \max_j c_{j(k)_1}^u$, $m_{(k)_1}^l = \max_j c_{j(k)_1}^l$, $m_{(k)_2} = \max_j c_{j(k)_2}$, $m_{(k)_3}^l = \max_j c_{j(k)_3}^l$, $m_{(k)_3}^u = \max_j c_{j(k)_3}^u$, $j = \{1, 2, \dots, n\}$, and $k = \{1, 2, \dots, v\}$.

Step 2.2: Compute the difference from maximum value for each criterion based on each decision maker's opinion:

$$\tilde{D}_{j(k)} = \tilde{M}_{(k)} \ominus \tilde{C}_{j(k)} \tag{9}$$

$\tilde{d}_{i(j)}$ is an IVFN and can be represented as $\left[\left(d_{i(j)_1}^u, d_{i(j)_1}^l \right), d_{i(j)_2}, \left(d_{i(j)_3}^l, d_{i(j)_3}^u \right) \right]$.

Step 2.3: Assigning levels of criteria based on each decision maker's opinion:

$$Levels_{(k)_1}^u = \begin{cases} c_{j(k)_1}^u \in Level S_{11}^u & \text{if } 1 \leq d_{j(k)_1}^u < 2 \\ c_{j(k)_1}^u \in Level S_{21}^u & \text{if } 2 \leq d_{j(k)_1}^u < 3 \\ \vdots \\ c_{j(k)_1}^u \in Level S_{\lfloor m_{(k)_1}^u \rfloor}^u & \text{if } \lfloor m_{(k)_1}^u \rfloor \leq d_{j(k)_1}^u < \lfloor m_{(k)_1}^u \rfloor + 1 \end{cases} \quad (10)$$

$$Levels_{(k)_1}^l = \begin{cases} c_{j(k)_1}^l \in Level S_{11}^l & \text{if } 1 \leq d_{j(k)_1}^l < 2 \\ c_{j(k)_1}^l \in Level S_{21}^l & \text{if } 2 \leq d_{j(k)_1}^l < 3 \\ \vdots \\ c_{j(k)_1}^l \in Level S_{\lfloor m_{(k)_1}^l \rfloor}^l & \text{if } \lfloor m_{(k)_1}^l \rfloor \leq d_{j(k)_1}^l < \lfloor m_{(k)_1}^l \rfloor + 1 \end{cases} \quad (11)$$

$$Levels_{(k)_2} = \begin{cases} c_{j(k)_2} \in Level S_{12} & \text{if } 1 \leq d_{j(k)_2} < 2 \\ c_{j(k)_2} \in Level S_{22} & \text{if } 2 \leq d_{j(k)_2} < 3 \\ \vdots \\ c_{j(k)_2} \in Level S_{\lfloor m_{(k)_2} \rfloor} & \text{if } \lfloor m_{(k)_2} \rfloor \leq d_{j(k)_2} < \lfloor m_{(k)_2} \rfloor + 1 \end{cases} \quad (12)$$

$$Levels_{(k)_3}^l = \begin{cases} c_{j(k)_3}^l \in Level S_{13}^l & \text{if } 1 \leq d_{j(k)_3}^l < 2 \\ c_{j(k)_3}^l \in Level S_{23}^l & \text{if } 2 \leq d_{j(k)_3}^l < 3 \\ \vdots \\ c_{j(k)_3}^l \in Level S_{\lfloor m_{(k)_3}^l \rfloor}^l & \text{if } \lfloor m_{(k)_3}^l \rfloor \leq d_{j(k)_3}^l < \lfloor m_{(k)_3}^l \rfloor + 1 \end{cases} \quad (13)$$

$$Levels_{(k)_3}^u = \begin{cases} c_{j(k)_3}^u \in Level S_{13}^u & \text{if } 1 \leq d_{j(k)_3}^u < 2 \\ c_{j(k)_3}^u \in Level S_{23}^u & \text{if } 2 \leq d_{j(k)_3}^u < 3 \\ \vdots \\ c_{j(k)_3}^u \in Level S_{\lfloor m_{(k)_3}^u \rfloor}^u & \text{if } \lfloor m_{(k)_3}^u \rfloor \leq d_{j(k)_3}^u < \lfloor m_{(k)_3}^u \rfloor + 1 \end{cases} \quad (14)$$

Step 3: By following the outlined rules, the decision makers perform an initial classification of the criteria, organizing them into groups based on their corresponding IVFNs. If the significance of criterion C_j , based on decision maker k 's opinion and the upper bound's first character, is denoted by $s_{(k)_1}^u(C_j)$, where $j \in \{1, 2, \dots, n\}$ and $k \in \{1, 2, \dots, v\}$, then we have $S_{(k)_1}^u = S_{11}^u \cup S_{21}^u \cup \dots \cup S_{\lfloor m_{(k)_1}^u \rfloor}^u$, where for every level $e = \{1, 2, \dots, \lfloor m_{(k)_1}^u \rfloor\}$, the following applies:

$$S_{e(k)_1}^u = \{C_{e_{11}}^u, C_{e_{21}}^u, \dots, C_{e_{s_1}}^u\} = \{C_j \in S_{(k)_1}^u: e \leq s_{(k)_1}^u(C_j) < e + 1\} \quad (15)$$

Additionally, for each $p, q \in \{1, 2, \dots, \lfloor m_{(k)_1}^u \rfloor\}$ where $p \neq q$, the condition $S_{p(k)_1}^u \cap S_{q(k)_1}^u = \emptyset$ holds. Thus, this approach establishes a well-defined partition of the criteria set $S_{(k)_1}^u$. Within the formed subsets (levels) of criterion influence, criteria are compared based on their significance. Each criterion $C_{ep_1}^u \in S_{e(k)_1}^u$ within the subset $S_{e(k)_1}^u = \{C_{e_{11}}^u, C_{e_{21}}^u, \dots, C_{e_{s_1}}^u\}$ is assigned an integer $I_{ep_1}^u \in \{0, 1, \dots, r_{e(k)_1}^u\}$. The most important criterion C_{11}^u is assigned $I_{11}^u = 0$. If criterion $C_{ep_1}^u$ is more significant than $C_{eq_1}^u$, then $I_{ep_1}^u <$

$I_{e_{q_1}}^u$; if they are equivalent, $I_{e_{p_1}}^u = I_{e_{q_1}}^u$. The maximum value for the comparison scale is determined using the following expression:

$$r_{e(k)_1}^u = \max \left\{ |S_{1_1}^u|, |S_{2_1}^u|, \dots, |S_{[m(k)_1]_1}^u| \right\}. \tag{16}$$

Step 4: The maximum value $r_{e(k)_1}^u$, determined by Eq. 16 for the criteria comparison scale, is used to define an elasticity coefficient $r'_{e(k)_1}^u \in \mathbb{N}$ (where \mathbb{N} represents the set of real numbers). This coefficient must satisfy the condition $r'_{e(k)_1}^u > r_{e(k)_1}^u$.

Step 5: Computation of the criteria's influence function.

For each criterion $C_{ep_1}^u \in S_{e(k)_1}^u$, the influence function for that criterion can be calculated as follows:

$$f_{(k)_1}^u(C_{ep_1}^u) = \frac{r'_{e(k)_1}^u}{e.r'_{e(k)_1}^u + I_{ep_1}^u}. \tag{17}$$

The concept applied to the first character of the upper bound can also be extended to include the first character of the lower bound, the middle character, the third character of the lower bound, and the third character of the upper bound. As a result, the following relationships emerge:

$$f_{(k)_1}^l(C_{ep_1}^l) = \frac{r'_{e(k)_1}^l}{e.r'_{e(k)_1}^l + I_{ep_1}^l}, \tag{18}$$

$$f_{(k)_2}(C_{ep_2}) = \frac{r'_{e(k)_2}}{e.r'_{e(k)_2} + I_{ep_2}}, \tag{19}$$

$$f_{(k)_3}^l(C_{ep_3}^l) = \frac{r'_{e(k)_3}^l}{e.r'_{e(k)_3}^l + I_{ep_3}^l}, \tag{20}$$

$$f_{(k)_3}^u(C_{ep_3}^u) = \frac{r'_{e(k)_3}^u}{e.r'_{e(k)_3}^u + I_{ep_3}^u}. \tag{21}$$

$\tilde{F}_{(k)}(C_j)$ is an IVFN and can be represented as $\left[\left(f_{(k)_1}^u(C_{ep_1}^u), f_{(k)_1}^l(C_{ep_1}^l) \right), f_{(k)_2}(C_{ep_2}), \left(f_{(k)_3}^l(C_{ep_3}^l), f_{(k)_3}^u(C_{ep_3}^u) \right) \right]$.

Step 6: In this step, the criterion weights are calculated according to the opinion of decision maker k :

$$\tilde{W}_{j(k)} = \tilde{F}_{(k)}(C_j) \odot \sum_{j=1}^n \tilde{F}_{(k)}(C_j). \tag{22}$$

where, $\tilde{W}_{j(k)}$ is an IVFN and can be represented as $\left[\left(w_{j(k)_1}^u, w_{j(k)_1}^l \right), w_{j(k)_2}, \left(w_{j(k)_3}^l, w_{j(k)_3}^u \right) \right]$.

Step 7: In this step, the final weight of each criterion is obtained by computing the average of the individual weights assigned to each criterion, which were derived from the assessments of all decision makers in the previous step.

$$\tilde{W}_j = \frac{\sum_{k=1}^v \tilde{W}_{j(k)}}{v}. \tag{23}$$

where, \tilde{W}_j is an IVFN and can be represented as $\left[\left(w_{j_1}^u, w_{j_1}^l \right), w_{j_2}, \left(w_{j_3}^l, w_{j_3}^u \right) \right]$.

Introduced IVF-MAIRCA Method

The suggested approach employs IVFN to capture performance evaluations using linguistic variables, facilitating a more precise and inclusive expression of preferences. As a result, the IVF-MAIRCA framework is organized into the following stages:

Step 1: An initial linguistic decision matrix D_L is constructed based on the linguistic assessments of alternatives against the selected criteria. For generalization purposes, assume that v decision makers evaluate m alternatives across n criteria. The resulting decision matrix is formulated as shown in Eq. 24.

$$D_L = \begin{pmatrix} l_{11}^1, l_{12}^1, \dots, l_{1n}^1 & l_{11}^2, l_{12}^2, \dots, l_{1n}^2 & \dots & l_{11}^v, l_{12}^v, \dots, l_{1n}^v \\ l_{21}^1, l_{22}^1, \dots, l_{2n}^1 & l_{21}^2, l_{22}^2, \dots, l_{2n}^2 & \dots & l_{21}^v, l_{22}^v, \dots, l_{2n}^v \\ \vdots & \vdots & \ddots & \vdots \\ l_{m1}^1, l_{m2}^1, \dots, l_{mn}^1 & l_{m1}^2, l_{m2}^2, \dots, l_{mn}^2 & \dots & l_{m1}^v, l_{m2}^v, \dots, l_{mn}^v \end{pmatrix} \tag{24}$$

where, l_{ij}^k signifies the linguistic assessment provided by the k^{th} expert for the i^{th} alternative relative to the j^{th} criterion. Where $i = \{1, 2, \dots, m\}, j = \{1, 2, \dots, n\},$ and $k = \{1, 2, \dots, v\}.$

Step 2: Using Table 2, the linguistic expressions by each decision maker are transformed into corresponding IVFNs.

$$\tilde{D}^1 = \begin{pmatrix} \tilde{d}_{11}^1 & \tilde{d}_{12}^1 & \dots & \tilde{d}_{1n}^1 \\ \tilde{d}_{21}^1 & \tilde{d}_{22}^1 & \dots & \tilde{d}_{2n}^1 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{d}_{m1}^1 & \tilde{d}_{m2}^1 & \dots & \tilde{d}_{mn}^1 \end{pmatrix}, \tilde{D}^2 = \begin{pmatrix} \tilde{d}_{11}^2 & \tilde{d}_{12}^2 & \dots & \tilde{d}_{1n}^2 \\ \tilde{d}_{21}^2 & \tilde{d}_{22}^2 & \dots & \tilde{d}_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{d}_{m1}^2 & \tilde{d}_{m2}^2 & \dots & \tilde{d}_{mn}^2 \end{pmatrix}, \dots, \tilde{D}^v = \begin{pmatrix} \tilde{d}_{11}^v & \tilde{d}_{12}^v & \dots & \tilde{d}_{1n}^v \\ \tilde{d}_{21}^v & \tilde{d}_{22}^v & \dots & \tilde{d}_{2n}^v \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{d}_{m1}^v & \tilde{d}_{m2}^v & \dots & \tilde{d}_{mn}^v \end{pmatrix} \tag{25}$$

where, \tilde{d}_{ij}^k is an IVFN representing the performance of alternative i on criterion j as evaluated by decision-maker $k.$

Step 3: The IVF aggregated decision matrix is constructed by consolidating the individual evaluations provided by all decision-makers.

$$\tilde{A} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{pmatrix} \tag{26}$$

where, $\tilde{a}_{ij} = \left[\left(a_{ij_1}^u, a_{ij_1}^l \right), a_{ij_2}, \left(a_{ij_3}^l, a_{ij_3}^u \right) \right]$ is IVFN representing the aggregated performance of alternative i on criterion $j,$ calculated as follows:

$$\tilde{a}_{ij} = \frac{\tilde{d}_{ij}^1 + \tilde{d}_{ij}^2 + \dots + \tilde{d}_{ij}^v}{v} \tag{27}$$

Step 4: At this stage, the decision-maker is assumed to maintain impartiality in the selection process. Given that all alternatives are considered equally likely to be chosen, their preferences are uniformly distributed and can be represented as:

$$P_{A_i} = \frac{1}{m}; \sum_{i=1}^m P_{A_i} = 1. \tag{28}$$

Step 5: The IVF theoretical evaluation matrix \tilde{T}_{P_A} is generated by multiplying the IVF criteria weights, derived through the IVF-LBWA method, with the equal preference values assigned to each alternative (P_{A_i}).

$$\tilde{T}_{P_A} = \begin{pmatrix} \frac{1}{m}\tilde{w}_1 & \frac{1}{m}\tilde{w}_2 & \dots & \frac{1}{m}\tilde{w}_n \\ \frac{1}{m}\tilde{w}_1 & \frac{1}{m}\tilde{w}_2 & \dots & \frac{1}{m}\tilde{w}_n \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{m}\tilde{w}_1 & \frac{1}{m}\tilde{w}_2 & \dots & \frac{1}{m}\tilde{w}_n \end{pmatrix} = \begin{pmatrix} \tilde{t}_{p11} & \tilde{t}_{p12} & \dots & \tilde{t}_{p1n} \\ \tilde{t}_{p21} & \tilde{t}_{p22} & \dots & \tilde{t}_{p2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{pm1} & \tilde{t}_{pm2} & \dots & \tilde{t}_{pmn} \end{pmatrix}. \tag{29}$$

where, $\tilde{t}_{pij} = \left[\left(t_{pij_1^u}, t_{pij_1^l} \right), t_{pij_2}, \left(t_{pij_3^l}, t_{pij_3^u} \right) \right]$ is IVFN.

Step 6: This step involves transforming the IVF aggregated decision matrix, obtained in Step 3, into the IVF normalized decision matrix \tilde{N} through a normalization process. Eqs. 30-34 in the IVF-MAIRCA method is applied to streamline computational complexity while enhancing numerical precision. Additionally, this approach eliminates the need for the decision-maker to account for the type of criteria—whether benefit-oriented or cost-oriented. This feature is particularly advantageous in scenarios where decision-making involves numerous conflicting criteria, a common challenge in complex evaluations.

$$n_{ij_1^u} = \frac{a_{ij_1^u}}{\sqrt{\sum_{i=1}^m \left[(a_{ij_1^u})^2 + (a_{ij_1^l})^2 + (a_{ij_2})^2 + (a_{ij_3^l})^2 + (a_{ij_3^u})^2 \right]}}, \tag{30}$$

$$n_{ij_1^l} = \frac{a_{ij_1^l}}{\sqrt{\sum_{i=1}^m \left[(a_{ij_1^u})^2 + (a_{ij_1^l})^2 + (a_{ij_2})^2 + (a_{ij_3^l})^2 + (a_{ij_3^u})^2 \right]}}, \tag{31}$$

$$n_{ij_2} = \frac{a_{ij_2}}{\sqrt{\sum_{i=1}^m \left[(a_{ij_1^u})^2 + (a_{ij_1^l})^2 + (a_{ij_2})^2 + (a_{ij_3^l})^2 + (a_{ij_3^u})^2 \right]}}, \tag{32}$$

$$n_{ij_3^l} = \frac{a_{ij_3^l}}{\sqrt{\sum_{i=1}^m \left[(a_{ij_1^u})^2 + (a_{ij_1^l})^2 + (a_{ij_2})^2 + (a_{ij_3^l})^2 + (a_{ij_3^u})^2 \right]}}, \tag{33}$$

$$n_{ij_3^u} = \frac{a_{ij_3^u}}{\sqrt{\sum_{i=1}^m \left[(a_{ij_1^u})^2 + (a_{ij_1^l})^2 + (a_{ij_2})^2 + (a_{ij_3^l})^2 + (a_{ij_3^u})^2 \right]}}. \tag{34}$$

where, \tilde{n}_{ij} represents an element of the IVF normalized decision matrix \tilde{N} , and is defined as $\left[\left(n_{ij_1^u}, n_{ij_1^l} \right), n_{ij_2}, \left(n_{ij_3^l}, n_{ij_3^u} \right) \right]$.

Step 7: In this phase, the IVF elements of the actual weight matrix \tilde{T}_{r_A} are determined. This is achieved by performing element-wise multiplication of the normalized decision matrix with the corresponding elements of the theoretical evaluation matrix.

$$\tilde{T}_{r_A} = \begin{pmatrix} \tilde{n}_{11} \times \tilde{t}_{p11} & \tilde{n}_{12} \times \tilde{t}_{p12} & \dots & \tilde{n}_{1n} \times \tilde{t}_{p1n} \\ \tilde{n}_{21} \times \tilde{t}_{p21} & \tilde{n}_{22} \times \tilde{t}_{p22} & \dots & \tilde{n}_{2n} \times \tilde{t}_{p2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{n}_{m1} \times \tilde{t}_{pm1} & \tilde{n}_{m2} \times \tilde{t}_{pm2} & \dots & \tilde{n}_{mn} \times \tilde{t}_{pmn} \end{pmatrix} = \begin{pmatrix} \tilde{t}_{r11} & \tilde{t}_{r12} & \dots & \tilde{t}_{r1n} \\ \tilde{t}_{r21} & \tilde{t}_{r22} & \dots & \tilde{t}_{r2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{rm1} & \tilde{t}_{rm2} & \dots & \tilde{t}_{rmn} \end{pmatrix}. \tag{35}$$

Here, $\tilde{t}_{rij} = \left[\left(t_{rij_1^u}, t_{rij_1^l} \right), t_{rij_2}, \left(t_{rij_3^l}, t_{rij_3^u} \right) \right]$ is IVFN.

Step 8: This step focuses on quantifying the difference between the theoretical and actual evaluations of each alternative across all criteria. To simplify the process and directly obtain the gap values, the calculation follows a streamlined approach. The distance measurement

formula for IVFNs, as introduced by Ashtiani et al. [61], is utilized. Accordingly, the total gap matrix elements are computed using the Eq. 36:

$$g_{ij} = \sqrt{\frac{1}{6} \left[(t_{pij_1^u} - t_{rij_1^u})^2 + (t_{pij_1^l} - t_{rij_1^l})^2 + 2(t_{pij_2} - t_{rij_2})^2 + (t_{pij_3^l} - t_{rij_3^l})^2 + (t_{pij_3^u} - t_{rij_3^u})^2 \right]} \quad (36)$$

Step 9: In this step, the gap values for each alternative across all criteria are aggregated to compute the final values of the criteria functions, following Eq. 37. These computed values are then sorted in ascending order to establish the ranking of alternatives.

$$Q_i = \sum_{j=1}^n g_{ij}, \text{ where } i = 1, 2, \dots, m. \quad (37)$$

The research workflow for this study is visually represented in Fig. 2.

Numerical Example

A project-oriented organization is engaged in the task of selecting initiatives for its sustainable project portfolio. This process involves four decision makers who have shortlisted six potential projects based on their relevance to strategic objectives. Research in project portfolio management highlights the importance of well-defined evaluation criteria. RezaHoseini et al. [15] emphasize six key dimensions for project assessment: social impact, customer-oriented outcomes, efficiency in internal processes, opportunities for growth and learning, environmental sustainability, and financial feasibility. Guided by these factors, the organization aims to determine the most suitable projects from the identified candidates.

In the initial phase, the weights of each criterion are determined using the methodology detailed in Section 4.1. This process begins with collecting the linguistic evaluations provided by the decision makers for each criterion, which are summarized in Table 3. These assessments serve as the foundation for deriving the relative importance of the criteria.

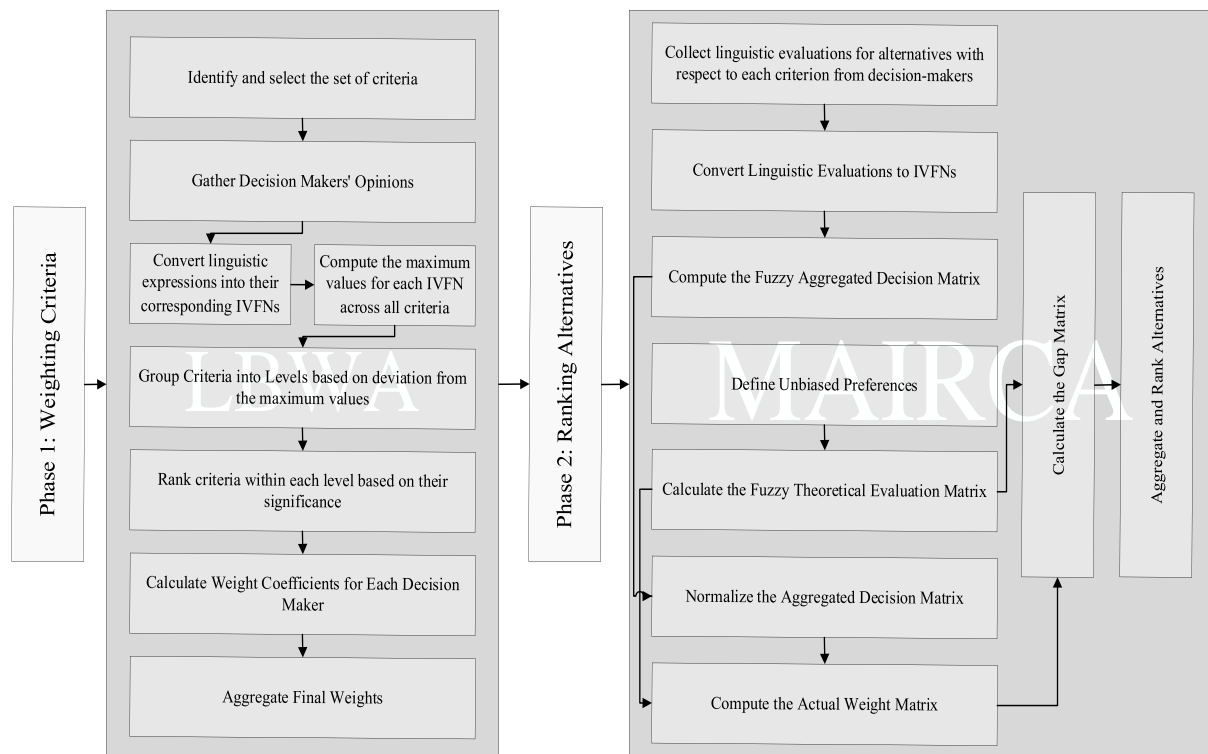


Figure 2. The proposed integrated IVF-LBWA and IVF-MAIRCA approach

The process continues by converting the linguistic variables for each criterion into equivalent IVFNs. This conversion relies on the reference values outlined in Table 2, and the resulting IVFNs are summarized in Table 4.

Table 3. Linguistic variable of the importance of criterion

Criteria	DM1	DM2	DM3	DM4
Social (C1)	F	MU	MU	MI
Customer (C2)	EU	U	VU	F
Internal processes (C3)	U	U	VU	U
Growth and learning (C4)	MI	MI	MU	U
Environmental (C5)	TMI	VI	I	TMI
Financial (C6)	EI	TMI	TMI	VI

Next, the differences between each criterion's value and the maximum value, based on the opinions of the decision-makers, are computed using Eqs. 8 and 9. Then, using the calculated values and Eqs. 10-14, the criteria are categorized into different levels, as can be seen in Table 5.

In the next step, Eqs. 15 and 16 were applied to evaluate the relative importance of each criterion within its respective level, as compared to others in the same level. These evaluations, based on the opinions of individual decision-makers, are summarized in Table 6.

In the following step, Eqs. 17-21 were employed to calculate the influence function of each criterion for every decision-maker. The results of these calculations are comprehensively presented in Table 7.

Table 4. Assigned IVFN to each criterion

Criteria	DM1	DM2
C1	[(4, 4.25), 4.75, (5.5, 6)]	[(3, 3.25), 3.75, (4.5, 5)]
C2	[(0, 0.25), 0.75, (1.5, 2)]	[(2, 2.25), 2.75, (3.5, 4)]
C3	[(2, 2.25), 2.75, (3.5, 4)]	[(2, 2.25), 2.75, (3.5, 4)]
C4	[(5, 5.25), 5.75, (6.5, 7)]	[(5, 5.25), 5.75, (6.5, 7)]
C5	[(10, 10), 10, (10, 10)]	[(7, 7.25), 7.75, (8.5, 9)]
C6	[(8, 8.25), 8.75, (9.5, 10)]	[(10, 10), 10, (10, 10)]
Criteria	DM3	DM4
C1	[(3, 3.25), 3.75, (4.5, 5)]	[(5, 5.25), 5.75, (6.5, 7)]
C2	[(1, 1.25), 1.75, (2.5, 3)]	[(4, 4.25), 4.75, (5.5, 6)]
C3	[(1, 1.25), 1.75, (2.5, 3)]	[(2, 2.25), 2.75, (3.5, 4)]
C4	[(3, 3.25), 3.75, (4.5, 5)]	[(2, 2.25), 2.75, (3.5, 4)]
C5	[(6, 6.25), 6.75, (7.5, 8)]	[(10, 10), 10, (10, 10)]
C6	[(10, 10), 10, (10, 10)]	[(7, 7.25), 7.75, (8.5, 9)]

Table 5. Criteria classification into levels based on the opinions of decision makers

S_1^u	DM1	DM2	DM3	DM4	S_1^l	DM1	DM2	DM3	DM4	S_2	DM1	DM2	DM3	DM4
1	C5, C6	C6, C5	C6	C5, C6	1	C5, C6	C6, C5	C6	C5, C6	1	C5, C6	C6, C5	C6	C5, C6
2			C5		2			C5		2			C5	
3	C4	C4		C1	3	C4	C4		C1	3	C4	C4		C1
4	C1			C2	4	C1			C2	4	C1			C2
5		C1	C1, C4		5		C1	C1, C4		5		C1	C1, C4	
6	C3	C2, C3		C3, C4	6	C3	C2, C3		C3, C4	6	C3	C2, C3		C3, C4
7			C2, C3		7			C2, C3		7			C2, C3	
8	C2				8	C2				8	C2			
9					9					9				
10					10					10				

S_3^l	DM1	DM2	DM3	DM4	S_3^u	DM1	DM2	DM3	DM4					
1	C5, C6	C6	C6	C5	1	C5	C6	C6	C5					
2		C5		C6	2	C6								
3			C5		3		C5		C6					
4	C4	C4		C1	4			C5						
5	C1			C2	5	C4	C4		C1					
6		C1	C1, C4		6	C1			C2					
7	C3	C2, C3		C3, C4	7		C1	C1, C4						
8			C2, C3		8	C3	C2, C3		C3, C4					
9	C2				9			C2, C3						
10					10	C2								

Table 6. Sorting criteria within each level based on their significance

S_1^u	DM 1	DM 2	DM 3	DM 4	S_1^l	DM 1	DM 2	DM 3	DM 4	S_2	DM 1	DM 2	DM 3	DM 4
r	2	2	2	2	r	2	2	2	2	r	2	2	2	2
r'	3	3	3	3	r'	3	3	3	3	r'	3	3	3	3
l_{C_1}	0	0	0	0	l_{C_1}	0	0	0	0	l_{C_1}	0	0	0	0
l_{C_2}	0	0	0	0	l_{C_2}	0	0	0	0	l_{C_2}	0	0	0	0
l_{C_3}	0	1	1	0	l_{C_3}	0	1	1	0	l_{C_3}	0	1	1	0
l_{C_4}	0	0	1	1	l_{C_4}	0	0	1	1	l_{C_4}	0	0	1	1
l_{C_5}	0	1	0	0	l_{C_5}	0	1	0	0	l_{C_5}	0	1	0	0
l_{C_6}	1	0	0	1	l_{C_6}	1	0	0	1	l_{C_6}	1	0	0	1
S_3^l	DM 1	DM 2	DM 3	DM 4	S_3^u	DM 1	DM 2	DM 3	DM 4					
r	2	2	2	2	r	1	2	2	2					
r'	3	3	3	3	r'	2	3	3	3					
l_{C_1}	0	0	0	0	l_{C_1}	0	0	0	0					
l_{C_2}	0	0	0	0	l_{C_2}	0	0	0	0					
l_{C_3}	0	2	2	0	l_{C_3}	0	2	2	0					
l_{C_4}	0	0	2	2	l_{C_4}	0	0	2	2					
l_{C_5}	0	0	0	0	l_{C_5}	0	0	0	0					
l_{C_6}	2	0	0	0	l_{C_6}	0	0	0	0					

Table 7. Influence function of each criterion for every decision maker

	DM1	DM2	DM3	DM4
$\tilde{F}(C_1)$	[(0.17, 0.2), 0.25, (0.25, 0.25)]	[(0.14, 0.17), 0.2, (0.2, 0.2)]	[(0.14, 0.17), 0.2, (0.2, 0.2)]	[(0.2, 0.25), 0.33, (0.33, 0.33)]
$\tilde{F}(C_2)$	[(0.1, 0.11), 0.13, (0.13, 0.13)]	[(0.13, 0.14), 0.17, (0.17, 0.17)]	[(0.11, 0.13), 0.14, (0.14, 0.14)]	[(0.17, 0.2), 0.25, (0.25, 0.25)]
$\tilde{F}(C_3)$	[(0.13, 0.14), 0.17, (0.17, 0.17)]	[(0.12, 0.13), 0.16, (0.16, 0.16)]	[(0.1, 0.12), 0.14, (0.14, 0.14)]	[(0.13, 0.14), 0.17, (0.17, 0.17)]
$\tilde{F}(C_4)$	[(0.2, 0.25), 0.33, (0.33, 0.33)]	[(0.2, 0.25), 0.33, (0.33, 0.33)]	[(0.13, 0.15), 0.19, (0.19, 0.19)]	[(0.12, 0.13), 0.16, (0.16, 0.16)]
$\tilde{F}(C_5)$	[(1, 1), 1, (1, 1)]	[(0.33, 0.5), 0.75, (0.75, 0.75)]	[(0.25, 0.33), 0.5, (0.5, 0.5)]	[(1, 1), 1, (1, 1)]
$\tilde{F}(C_6)$	[(0.5, 0.6), 0.75, (0.75, 0.75)]	[(1, 1), 1, (1, 1)]	[(1, 1), 1, (1, 1)]	[(0.33, 0.5), 0.75, (0.75, 0.75)]

In the final step, Eq. 22 was applied to calculate the IVF weight of each criterion based on the opinions of each decision maker. Following this, the aggregate IVF weights of the criteria

were determined using Eq. 23. The results are summarized in Tables 8 and 9.

In the second phase, project evaluation is conducted using the method outlined in Section 4.2, which assesses the alignment of each project with the organization’s objectives. This process culminates in the selection of the project that best supports the organization’s long-term goals and sustainability initiatives. The first step involves compiling the decision makers' linguistic evaluations of each project's performance across all criteria, as shown in Table 10, providing a comprehensive overview of the assessment process.

Table 8. IVF weight of each criterion based on the opinions of each decision maker

Weig hts	DM1	DM2	DM3	DM4
\bar{w}_1	[(0.06, 0.08), 0.1, (0.11, 0.12)]	[(0.05, 0.06), 0.08, (0.09, 0.1)]	[(0.07, 0.08), 0.09, (0.11, 0.12)]	[(0.08, 0.09), 0.13, (0.15, 0.17)]
\bar{w}_2	[(0.04, 0.04), 0.05, (0.05, 0.06)]	[(0.05, 0.05), 0.06, (0.08, 0.09)]	[(0.05, 0.06), 0.07, (0.08, 0.08)]	[(0.06, 0.08), 0.09, (0.11, 0.13)]
\bar{w}_3	[(0.05, 0.05), 0.06, (0.07, 0.08)]	[(0.04, 0.05), 0.06, (0.07, 0.08)]	[(0.05, 0.05), 0.06, (0.07, 0.08)]	[(0.05, 0.05), 0.06, (0.07, 0.09)]
\bar{w}_4	[(0.08, 0.1), 0.13, (0.14, 0.16)]	[(0.08, 0.1), 0.13, (0.15, 0.17)]	[(0.06, 0.07), 0.09, (0.1, 0.11)]	[(0.04, 0.05), 0.06, (0.07, 0.08)]
\bar{w}_5	[(0.38, 0.38), 0.38, (0.43, 0.48)]	[(0.13, 0.19), 0.29, (0.34, 0.39)]	[(0.12, 0.15), 0.23, (0.26, 0.29)]	[(0.38, 0.38), 0.38, (0.45, 0.52)]
\bar{w}_6	[(0.19, 0.23), 0.29, (0.33, 0.36)]	[(0.38, 0.38), 0.38, (0.46, 0.52)]	[(0.46, 0.46), 0.46, (0.53, 0.58)]	[(0.13, 0.19), 0.28, (0.34, 0.39)]

Table 9. Aggregate IVF weights of the criteria

Final Weights	
\bar{w}_1	[(0.06, 0.08), 0.1, (0.11, 0.13)]
\bar{w}_2	[(0.05, 0.06), 0.07, (0.08, 0.09)]
\bar{w}_3	[(0.05, 0.05), 0.06, (0.07, 0.08)]
\bar{w}_4	[(0.06, 0.08), 0.1, (0.12, 0.13)]
\bar{w}_5	[(0.25, 0.28), 0.32, (0.37, 0.42)]
\bar{w}_6	[(0.29, 0.32), 0.35, (0.41, 0.46)]

In the subsequent step, the linguistic evaluations provided by each decision-maker are systematically translated into their corresponding IVFNs using the conversion process outlined in Table 2. The translated results, reflecting the decision makers' assessments, are compiled and presented in Table 11.

Then, the IVF aggregated decision matrix is constructed by applying Eqs. 26 and 27, which combine the individual decision makers' evaluations into a unified matrix. The resulting IVF aggregated decision matrix is presented in Table 12.

In the next step, the IVF theoretical evaluation matrix is constructed by applying Eqs. 28 and 27. Subsequently, the IVF normalized decision matrix is formed using Eqs. 30-34, ensuring that the data is adjusted for consistency and comparability across criteria. Finally, the IVF elements of the actual weight matrix are determined through element-wise multiplication of the normalized decision matrix with the corresponding elements of the theoretical evaluation matrix in Tables 13 and 14.

In the next step, the elements of the total gap matrix are calculated using Eq. 36, and the results are presented in Table 15.

In the final step, the gap values for each alternative across all criteria are aggregated using Eq. 37 to calculate the final criteria function values. These values are then sorted in ascending order to establish the ranking of alternatives, with the results summarized in Table 16.

Based on the evaluation process, the projects are ranked in descending order of preference as follows: Project A4 is ranked the highest, followed by Project A2 in second place, Project

A3 in third, Project A1 in fourth, and finally, Project A5 in fifth place.

Table 10. Linguistic evaluations of each project's performance across all criteria

DM1						
	C1	C2	C3	C4	C5	C6
P1	G	VG	MG	F	B	MB
P2	VG	F	B	MG	E	MB
P3	F	F	MB	VG	G	EB
P4	B	E	B	G	B	E
P5	F	G	VG	VB	G	EB
DM2						
	C1	C2	C3	C4	C5	C6
P1	MG	VG	F	G	B	B
P2	G	B	MB	F	G	F
P3	F	G	B	G	MG	B
P4	VB	G	B	VG	MB	E
P5	MG	MG	G	B	MG	EB
DM3						
	C1	C2	C3	C4	C5	C6
P1	G	E	G	F	B	B
P2	E	B	B	MG	G	MB
P3	B	G	B	G	F	B
P4	VB	VG	EB	G	MB	G
P5	MG	MG	G	VB	MG	VB
DM4						
	C1	C2	C3	C4	C5	C6
P1	MG	G	G	F	B	B
P2	G	F	F	G	G	F
P3	F	MG	F	E	MG	B
P4	VB	G	VB	E	F	VG
P5	MG	G	E	EB	F	VB

Table 11. Corresponding IVFN of each project's performance across all criteria

	C1	C2	C3	C4	C5	C6
A1	[(6, 6.25), 6.75, (7.5, 8)]	[(7, 7.25), 7.75, (8.5, 9)]	[(5, 5.25), 5.75, (6.5, 7)]	[(4, 4.25), 4.75, (5.5, 6)]	[(2, 2.25), 2.75, (3.5, 4)]	[(3, 3.25), 3.75, (4.5, 5)]
A2	[(7, 7.25), 7.75, (8.5, 9)]	[(4, 4.25), 4.75, (5.5, 6)]	[(2, 2.25), 2.75, (3.5, 4)]	[(5, 5.25), 5.75, (6.5, 7)]	[(8, 8.25), 8.75, (9.5, 10)]	[(3, 3.25), 3.75, (4.5, 5)]
A3	[(4, 4.25), 4.75, (5.5, 6)]	[(4, 4.25), 4.75, (5.5, 6)]	[(3, 3.25), 3.75, (4.5, 5)]	[(7, 7.25), 7.75, (8.5, 9)]	[(6, 6.25), 6.75, (7.5, 8)]	[(0, 0.25), 0.75, (1.5, 2)]
A4	[(2, 2.25), 2.75, (3.5, 4)]	[(8, 8.25), 8.75, (9.5, 10)]	[(2, 2.25), 2.75, (3.5, 4)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(8, 8.25), 8.75, (9.5, 10)]
A5	[(4, 4.25), 4.75, (5.5, 6)]	[(6, 6.25), 6.75, (7.5, 8)]	[(7, 7.25), 7.75, (8.5, 9)]	[(1, 1.25), 1.75, (2.5, 3)]	[(6, 6.25), 6.75, (7.5, 8)]	[(0, 0.25), 0.75, (1.5, 2)]
DM2						
	C1	C2	C3	C4	C5	C6
A1	[(5, 5.25), 5.75, (6.5, 7)]	[(7, 7.25), 7.75, (8.5, 9)]	[(4, 4.25), 4.75, (5.5, 6)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(2, 2.25), 2.75, (3.5, 4)]
A2	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(3, 3.25), 3.75, (4.5, 5)]	[(4, 4.25), 4.75, (5.5, 6)]	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]
A3	[(4, 4.25), 4.75, (5.5, 6)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(6, 6.25), 6.75, (7.5, 8)]	[(5, 5.25), 5.75, (6.5, 7)]	[(2, 2.25), 2.75, (3.5, 4)]
A4	[(1, 1.25), 1.75, (2.5, 3)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(7, 7.25), 7.75, (8.5, 9)]	[(3, 3.25), 3.75, (4.5, 5)]	[(8, 8.25), 8.75, (9.5, 10)]
A5	[(5, 5.25), 5.75, (6.5, 7)]	[(5, 5.25), 5.75, (6.5, 7)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(5, 5.25), 5.75, (6.5, 7)]	[(0, 0.25), 0.75, (1.5, 2)]

DM3						
	C1	C2	C3	C4	C5	C6
A1	[(6, 6.25), 6.75, (7.5, 8)]	[(8, 8.25), 8.75, (9.5, 10)]	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]	[(2, 2.25), 2.75, (3.5, 4)]	[(2, 2.25), 2.75, (3.5, 4)]
A2	[(8, 8.25), 8.75, (9.5, 10)]	[(2, 2.25), 2.75, (3.5, 4)]	[(2, 2.25), 2.75, (3.5, 4)]	[(5, 5.25), 5.75, (6.5, 7)]	[(6, 6.25), 6.75, (7.5, 8)]	[(3, 3.25), 3.75, (4.5, 5)]
A3	[(2, 2.25), 2.75, (3.5, 4)]	[(6, 6.25), 6.75, (7.5, 8)]	[(2, 2.25), 2.75, (3.5, 4)]	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]	[(2, 2.25), 2.75, (3.5, 4)]
A4	[(1, 1.25), 1.75, (2.5, 3)]	[(7, 7.25), 7.75, (8.5, 9)]	[(0, 0.25), 0.75, (1.5, 2)]	[(6, 6.25), 6.75, (7.5, 8)]	[(3, 3.25), 3.75, (4.5, 5)]	[(6, 6.25), 6.75, (7.5, 8)]
A5	[(5, 5.25), 5.75, (6.5, 7)]	[(5, 5.25), 5.75, (6.5, 7)]	[(6, 6.25), 6.75, (7.5, 8)]	[(1, 1.25), 1.75, (2.5, 3)]	[(5, 5.25), 5.75, (6.5, 7)]	[(1, 1.25), 1.75, (2.5, 3)]
DM4						
	C1	C2	C3	C4	C5	C6
A1	[(5, 5.25), 5.75, (6.5, 7)]	[(6, 6.25), 6.75, (7.5, 8)]	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]	[(2, 2.25), 2.75, (3.5, 4)]	[(2, 2.25), 2.75, (3.5, 4)]
A2	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]	[(4, 4.25), 4.75, (5.5, 6)]	[(6, 6.25), 6.75, (7.5, 8)]	[(6, 6.25), 6.75, (7.5, 8)]	[(4, 4.25), 4.75, (5.5, 6)]
A3	[(4, 4.25), 4.75, (5.5, 6)]	[(5, 5.25), 5.75, (6.5, 7)]	[(4, 4.25), 4.75, (5.5, 6)]	[(8, 8.25), 8.75, (9.5, 10)]	[(5, 5.25), 5.75, (6.5, 7)]	[(2, 2.25), 2.75, (3.5, 4)]
A4	[(1, 1.25), 1.75, (2.5, 3)]	[(6, 6.25), 6.75, (7.5, 8)]	[(1, 1.25), 1.75, (2.5, 3)]	[(8, 8.25), 8.75, (9.5, 10)]	[(4, 4.25), 4.75, (5.5, 6)]	[(7, 7.25), 7.75, (8.5, 9)]
A5	[(5, 5.25), 5.75, (6.5, 7)]	[(6, 6.25), 6.75, (7.5, 8)]	[(8, 8.25), 8.75, (9.5, 10)]	[(0, 0.25), 0.75, (1.5, 2)]	[(4, 4.25), 4.75, (5.5, 6)]	[(1, 1.25), 1.75, (2.5, 3)]

Table 12. The IVF aggregated decision matrix

Aggregated decision matrix						
	C1	C2	C3	C4	C5	C6
A1	[(5.5, 5.75), 6.25, (7, 7.5)]	[(7, 7.25), 7.75, (8.5, 9)]	[(5.25, 5.5), 6, (6.75, 7.25)]	[(4.5, 4.75), 5.25, (6, 6.5)]	[(2, 2.25), 2.75, (3.5, 4)]	[(2.25, 2.5), 3, (3.75, 4.25)]
A2	[(6.75, 7), 7.5, (8.25, 8.75)]	[(3, 3.25), 3.75, (4.5, 5)]	[(2.75, 3), 3.5, (4.25, 4.75)]	[(5, 5.25), 5.75, (6.5, 7)]	[(6.5, 6.75), 7.25, (8, 8.5)]	[(3.5, 3.75), 4.25, (5, 5.5)]
A3	[(3.5, 3.75), 4.25, (5, 5.5)]	[(5.25, 5.5), 6, (6.75, 7.25)]	[(2.75, 3), 3.5, (4.25, 4.75)]	[(6.75, 7), 7.5, (8.25, 8.75)]	[(5, 5.25), 5.75, (6.5, 7)]	[(1.5, 1.75), 2.25, (3, 3.5)]
A4	[(1.25, 1.5), 2, (2.75, 3.25)]	[(6.75, 7), 7.5, (8.25, 8.75)]	[(1.25, 1.5), 2, (2.75, 3.25)]	[(6.75, 7), 7.5, (8.25, 8.75)]	[(3, 3.25), 3.75, (4.5, 5)]	[(7.25, 7.5), 8, (8.75, 9.25)]
A5	[(4.75, 5), 5.5, (6.25, 6.75)]	[(5.5, 5.75), 6.25, (7, 7.5)]	[(6.75, 7), 7.5, (8.25, 8.75)]	[(1, 1.25), 1.75, (2.5, 3)]	[(5, 5.25), 5.75, (6.5, 7)]	[(0.5, 0.75), 1.25, (2, 2.5)]

Table 13. The IVF Normalized decision matrix

Normalized decision matrix						
	C1	C2	C3	C4	C5	C6
A1	[(0.196, 0.204), 0.222, (0.249, 0.267)]	[(0.212, 0.22), 0.235, (0.258, 0.273)]	[(0.206, 0.215), 0.235, (0.264, 0.284)]	[(0.147, 0.155), 0.171, (0.196, 0.212)]	[(0.073, 0.082), 0.1, (0.127, 0.146)]	[(0.098, 0.108), 0.13, (0.163, 0.184)]
A2	[(0.24, 0.249), 0.267, (0.293, 0.311)]	[(0.091, 0.098), 0.114, (0.136, 0.152)]	[(0.108, 0.117), 0.137, (0.166, 0.186)]	[(0.163, 0.171), 0.188, (0.212, 0.229)]	[(0.237, 0.246), 0.264, (0.291, 0.31)]	[(0.152, 0.163), 0.184, (0.217, 0.239)]
A3	[(0.124, 0.133), 0.151, (0.178, 0.196)]	[(0.159, 0.167), 0.182, (0.205, 0.22)]	[(0.108, 0.117), 0.137, (0.166, 0.186)]	[(0.22, 0.229), 0.245, (0.269, 0.286)]	[(0.182, 0.191), 0.209, (0.237, 0.255)]	[(0.065, 0.076), 0.098, (0.13, 0.152)]
A4	[(0.044, 0.053), 0.071, (0.098, 0.116)]	[(0.205, 0.212), 0.227, (0.25, 0.265)]	[(0.049, 0.059), 0.078, (0.108, 0.127)]	[(0.22, 0.229), 0.245, (0.269, 0.286)]	[(0.109, 0.118), 0.137, (0.164, 0.182)]	[(0.315, 0.325), 0.347, (0.38, 0.401)]
A5	[(0.169, 0.178), 0.196, (0.222, 0.24)]	[(0.167, 0.174), 0.189, (0.212, 0.227)]	[(0.264, 0.274), 0.294, (0.323, 0.343)]	[(0.033, 0.041), 0.057, (0.082, 0.098)]	[(0.182, 0.191), 0.209, (0.237, 0.255)]	[(0.022, 0.033), 0.054, (0.087, 0.108)]

Table 14. The IVF actual weight decision matrix

Actual weight decision matrix						
	C1	C2	C3	C4	C5	C6
A1	[(0.013, 0.016), 0.022, (0.028, 0.034)]	[(0.011, 0.013), 0.016, (0.021, 0.024)]	[(0.01, 0.011), 0.015, (0.019, 0.023)]	[(0.009, 0.012), 0.017, (0.023, 0.028)]	[(0.018, 0.023), 0.032, (0.048, 0.061)]	[(0.028, 0.034), 0.046, (0.067, 0.085)]
A2	[(0.016, 0.019), 0.026, (0.033, 0.04)]	[(0.005, 0.006), 0.008, (0.011, 0.014)]	[(0.005, 0.006), 0.009, (0.012, 0.015)]	[(0.01, 0.013), 0.019, (0.025, 0.03)]	[(0.059, 0.068), 0.084, (0.109, 0.129)]	[(0.044, 0.051), 0.065, (0.089, 0.11)]
A3	[(0.008, 0.01), 0.015, (0.02, 0.025)]	[(0.008, 0.01), 0.012, (0.016, 0.02)]	[(0.005, 0.006), 0.009, (0.012, 0.015)]	[(0.014, 0.018), 0.025, (0.031, 0.037)]	[(0.046, 0.053), 0.067, (0.088, 0.107)]	[(0.019, 0.024), 0.034, (0.054, 0.07)]
A4	[(0.003, 0.004), 0.007, (0.011, 0.015)]	[(0.01, 0.012), 0.015, (0.02, 0.024)]	[(0.002, 0.003), 0.005, (0.008, 0.01)]	[(0.014, 0.018), 0.025, (0.031, 0.037)]	[(0.027, 0.033), 0.044, (0.061, 0.076)]	[(0.091, 0.103), 0.123, (0.156, 0.185)]
A5	[(0.011, 0.014), 0.019, (0.025, 0.031)]	[(0.008, 0.01), 0.013, (0.017, 0.02)]	[(0.012, 0.014), 0.018, (0.024, 0.028)]	[(0.002, 0.003), 0.006, (0.01, 0.013)]	[(0.046, 0.053), 0.067, (0.088, 0.107)]	[(0.006, 0.01), 0.019, (0.036, 0.05)]

Table 15. The total gap matrix

Total gap matrix						
	C1	C2	C3	C4	C5	C6
A1	0.075407	0.052731	0.04821	0.082103	0.29322	0.315341
A2	0.071021	0.061208	0.054492	0.080459	0.239078	0.295364
A3	0.082426	0.056439	0.054492	0.074703	0.257124	0.327328
A4	0.090322	0.05326	0.058262	0.074703	0.281188	0.235448
A5	0.078039	0.055909	0.044442	0.093615	0.257124	0.343311

Table 16. The final values of the criteria functions

Alternatives	Final Criteria function value	Ranks
A1	0.867012555	4
A2	0.80162179	2
A3	0.85251286	3
A4	0.793184021	1
A5	0.872440225	5

Interpretation and Implications of Results

Result Discussion

The final ranking approach reveals that Project 4 delivered the best performance, achieving the lowest overall criteria function value of 0.793184, which reflects its strong alignment with the organization's strategic objectives (Fig. 3). The weight assigned to criteria played a crucial role in the project evaluation, with financial and environmental factors being the primary determinants in the assessment framework. Project 2 ranked second, particularly in environmental and financial aspects, with a criteria function value of 0.80162179. On the other hand, Project 5 had the highest criteria function value of 0.872440225, indicating the most significant performance discrepancies across the assessed dimensions. A detailed analysis of the total gap matrix shows distinct performance differences: Project 2 stood out for its exceptional environmental performance (0.239078), far surpassing the other projects, while Project 4 excelled in financial performance with the smallest financial gap of 0.235448 (Fig. 4).

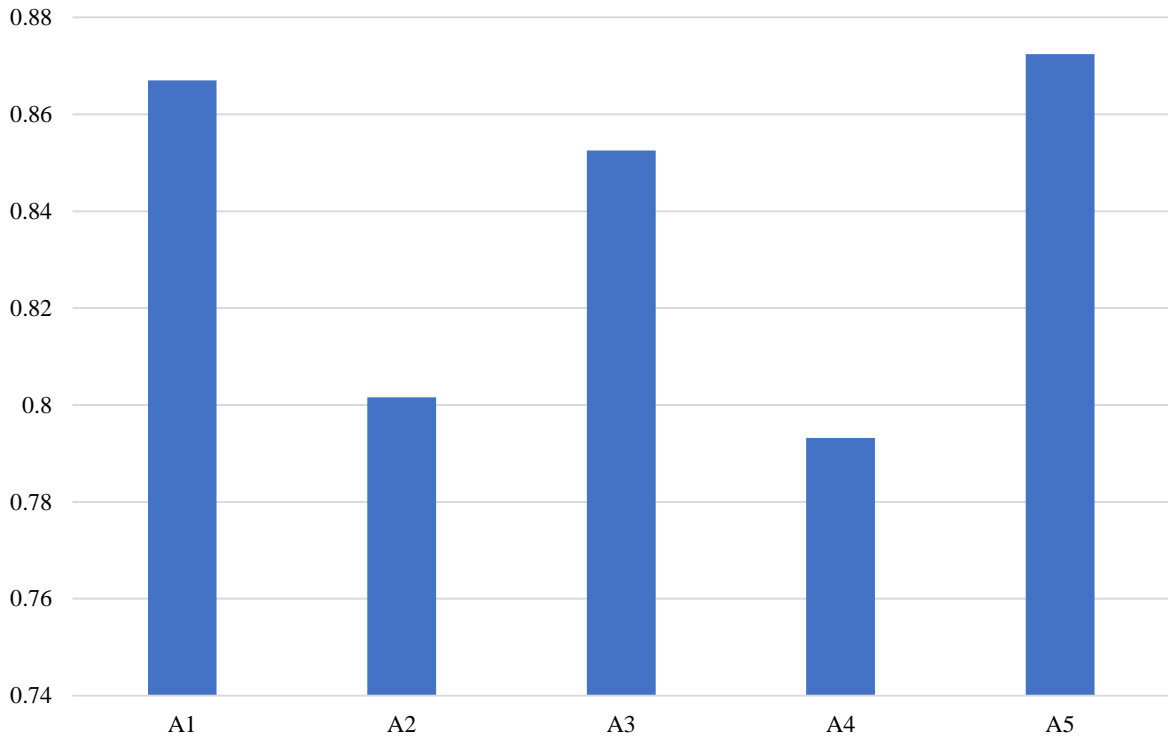


Figure 3. Final function value

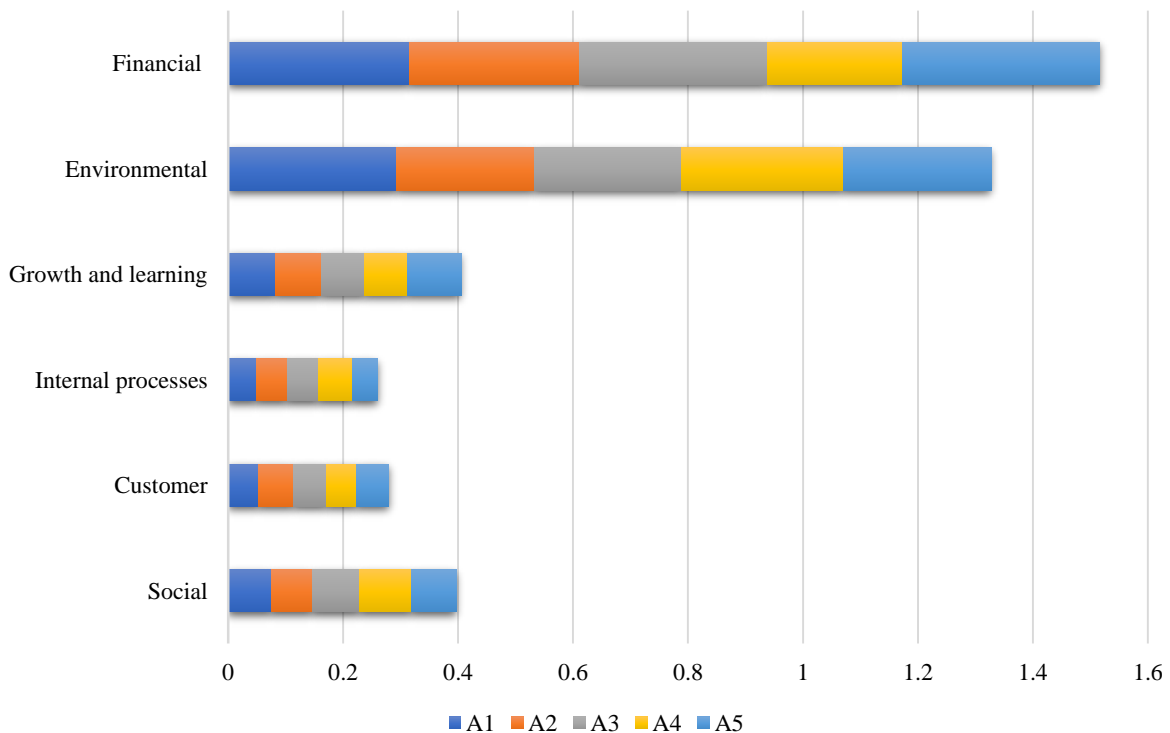


Figure 4. Total gaps of projects from each criterion

When analyzing the differences between project performance, the variation in scores becomes particularly noteworthy. While Project 2 achieved the second-highest ranking with a score of 0.80162179, the difference between its performance and Project 4's top-ranked score is relatively small at approximately 0.008. However, the gap between Project 2 and Project 5 (the lowest-ranked project) is substantially larger, at about 0.071, indicating more significant

performance discrepancies.

An examination of the detailed gap matrix presented in Table 15 reveals distinct patterns in project performance across various strategic dimensions. While some projects exhibit consistency across criteria, others demonstrate more pronounced variations. Notably, Project 4 maintains relatively low gaps across most criteria, particularly in C2, C3, and C4, with values of 0.05326, 0.058262, and 0.074703, respectively. These consistently lower gaps contribute to its top-ranking position, reflected in its final criteria function value of 0.793184, as indicated in Table 16. In contrast, Project 5 displays more significant performance fluctuations, particularly in C6, where the gap reaches 0.343311, the highest among the projects. This variability, coupled with a higher overall criteria function value of 0.872440225, explains its lower rank at position 5. These performance discrepancies across the criteria provide a clear rationale for the rankings, emphasizing the relative strengths and weaknesses of each project.

In the context of project portfolio selection, a critical aspect to consider is the effect of outliers or extreme data points. When certain projects display performance scores that significantly deviate from the others, these anomalies can skew the overall average and potentially alter the final rankings. To ensure the integrity of the results, it is imperative to carefully examine the data for any such outliers and address them appropriately, preventing them from distorting the selection process.

Moreover, the results of the portfolio selection must be interpreted with attention to the various contextual factors that may influence the evaluation. A key consideration is whether all projects were evaluated under uniform conditions in terms of scope, resources, and timelines. Discrepancies in these areas could create unfair comparisons between projects, leading to invalid conclusions. Ensuring a consistent baseline for all projects is essential for an accurate and equitable assessment.

Additionally, the selection criteria used to evaluate the projects should be scrutinized for their adequacy and relevance. If the criteria do not comprehensively capture the full spectrum of a project's alignment with organizational objectives or miss critical success factors, the resulting evaluation may be incomplete or biased. It is vital that the criteria encompass all pertinent dimensions to ensure a well-rounded and objective assessment.

In summary, while average scores and rankings provide valuable insights into the relative performance of projects, a more nuanced approach is required. By factoring in potential outliers, ensuring consistent evaluation conditions, and using comprehensive criteria, the final selection process can more accurately reflect the true value of each project within the portfolio.

Sensitivity Analysis

To evaluate the reliability of the decision-making process, a sensitivity analysis was conducted. This method is critical for understanding how variations in the model's key components can impact the overall outcomes, ensuring that decisions are based on sound and stable foundations. In this study, the sensitivity analysis was performed using a randomly generated weight replacement strategy, consisting of 40 tests where the weights of the criteria were altered. The primary objective was to observe how these changes influenced the rankings of the alternatives, allowing for an assessment of both the accuracy and variability of the decision results. Ultimately, the analysis validated the outcomes, confirming their consistency and ensuring the reliability of the decision-making approach.

As depicted in Fig. 5, the analysis indicates that Project 2 ranked first in 53% of the tests. This suggests that project 2 consistently outperforms the other alternatives across most criteria. Despite variations in the weights assigned to the six criteria, Project 2 was selected as the top-ranked options in 68% of the tests. These findings provide decision-makers with a high degree of confidence that Project 2 is the most aligned with the organization's strategic objectives for the task at hand.

The ranking of Project 4 demonstrates significant variability depending on the prioritization of the evaluation criteria. Its position is highly influenced by the relative importance assigned to each criterion. For instance, when the financial criterion is assigned the lowest priority, Project 4 consistently ranks at the bottom. Nevertheless, Project 4 achieves a position within the top two ranks in 63% of the conducted tests. However, it is also susceptible to being ranked last when the priority of the criteria is altered, underscoring the sensitivity of its performance to changes in the weighting structure.

Similarly, while Project 1 achieved the top rank in just 15% of the tests, it was ranked last only once and demonstrated strong performance by securing a position among the top three projects in 68% of the tests. In contrast, Project 5 has consistently failed to achieve the top rank, irrespective of the prioritization or importance assigned to the criteria. Moreover, it has been ranked last in over half of the conducted tests, highlighting its relatively weaker performance compared to the other projects.

To conclude, this study's sensitivity analysis serves as a vital tool for enhancing decision-making by providing a deeper understanding of the variability in n project portfolio selection. By exploring alternative weight configurations, the analysis evaluates the consistency and dependability of the decision-making framework. This process equips decision-makers with the confidence to make informed choices, ensuring their decisions are based on a reliable and well-structured evaluation.

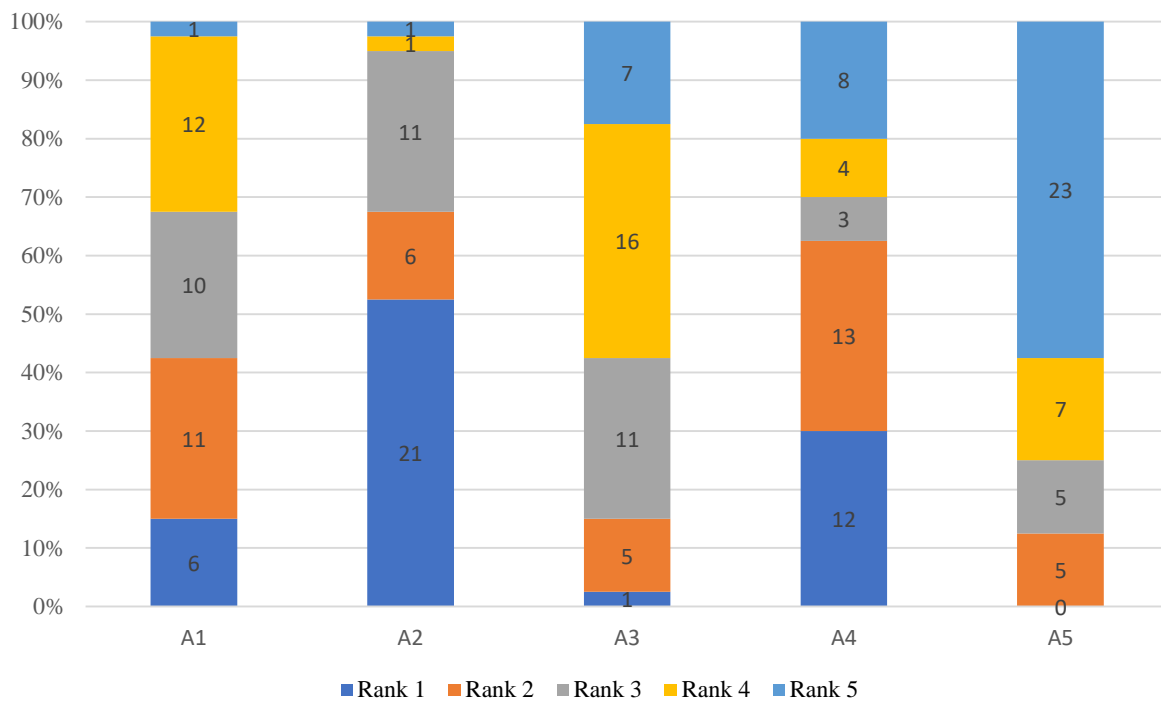


Figure 5. Distribution of ranks for each project across 40 tests

Managerial Insights

The evaluation process for selecting projects in the sustainable portfolio highlights several critical insights that can guide strategic decision-making. First and foremost, the importance of criteria weighting emerges as a central theme. The analysis shows that environmental sustainability and financial feasibility play pivotal roles in determining project outcomes. This finding underscores the need for managers to periodically review and validate the weighting scheme to ensure it accurately reflects the organization's evolving strategic priorities.

Adjustments in these weights, if necessary, should be carefully managed to maintain alignment with long-term sustainability and financial objectives.

A combined analysis of project rankings and strategic alignment reveals that, while the top projects exhibit close overall performance, their detailed assessments vary notably across key dimensions. Sensitivity analysis confirms that even slight adjustments in the criteria weights can influence the rankings, underscoring the need to balance quantitative scores with qualitative considerations such as strategic fit and long-term benefits.

Sensitivity testing further indicates that project performance is somewhat volatile under different weight configurations. For example, one project consistently achieves top rankings in many tests, while another shows significant variability—at times excelling and at other times lagging behind. These findings enable decision-makers to gauge the stability and reliability of the evaluation framework, identify potential risks, and appreciate the nuanced differences among the projects.

Lastly, the iterative and adaptive nature of the decision-making process is essential in a dynamic environment. The sensitivity analysis reveals that variations in criteria weights can lead to shifts in project rankings. Therefore, an iterative evaluation approach, which includes regular re-assessment of project performance under different scenarios, is crucial. This adaptability allows organizations to respond to changing market conditions and evolving internal priorities, ensuring that the selected project remains the most viable and strategically aligned option.

Conclusion

To conclude, sustainable project portfolio selection is a significant challenge for project-based organizations, as it is closely aligned with their long-term strategic goals. This study presents the new integrated group decision approach as comprehensive framework specifically designed to address the complexities of MCDM in this context; in this regard, two new IVF-LBWA and IVF-MAIRCA are developed. Through the application of these methods to a practical case, their effectiveness in evaluating and prioritizing projects has been clearly demonstrated. The findings emphasize the potential of these methods to support project-based organization managers in making well-informed, strategic decisions that are in line with both sustainability objectives and organizational goals. Additionally, the sensitivity analysis conducted in this study has provided valuable insights into how variations in criteria weights affect the final project rankings. It highlights the flexibility and resilience of the proposed methods, empowering decision-makers to adjust their criteria preferences and assess the resulting impact on the sustainable project portfolio selection process. In summary, the integrated IVF-LBWA and IVF-MAIRCA approach introduced in this study offers a comprehensive and methodical framework for sustainable project portfolio selection. The application of these methods, along with the outcomes and insights gained from the sensitivity analysis, demonstrate their reliability and practical utility. By utilizing these approaches, managers in project-based organizations can make informed decisions, thereby increasing the likelihood of achieving organizational success.

Looking ahead, several promising avenues for future research emerge to further refine sustainable project portfolio selection. Expanding the range of evaluation criteria can significantly enhance the framework's comprehensiveness and practical relevance, ensuring a more holistic assessment of project sustainability. In addition, integrating advanced fuzzy set methodologies, particularly interval type-2 fuzzy numbers (IT2FNs), offers a powerful means to better capture and manage the inherent uncertainties in decision-making (e.g., [62-64]). By incorporating IT2FNs, researchers can achieve a more nuanced representation of imprecise and vague data, leading to more robust and precise project evaluations. This refined approach is expected to foster a deeper understanding of project performance across diverse criteria,

ultimately supporting more resilient and informed decision-making in dynamic environments.

Acknowledgments

The authors express their gratitude to reviewers for their comments on the study.

References

- [1] Schoper, Y.-G., Wald, A., Ingason, H. T., & Fridgeirsson, T. V. (2018). Projectification in Western economies: A comparative study of Germany, Norway and Iceland. *International Journal of Project Management*, 36(1), 71–82.
- [2] Zahmatkesh, M. and Shakhshi-Niaei, M. (2023). Sustainable Project Portfolio Selection Considering Combined Rankings Under Uncertainty: A Case Study. *Advances in Industrial Engineering*, 57(2), 187-201.
- [3] Hatami-Marbini, A., Saati, S., & Sajadi, S. M. (2018). Efficiency analysis in two-stage structures using fuzzy data envelopment analysis. *Central European Journal of Operations Research*, 26(4), 909–932.
- [4] Lorincová, S., Štarchoň, P., Weberová, D., Hitka, M., & Lipoldová, M. (2019). Employee motivation as a tool to achieve sustainability of business processes. *Sustainability*, 11(13), 3509.
- [5] Kandakoglu, M., Walther, G., & Ben Amor, S. (2024). The use of multi-criteria decision-making methods in project portfolio selection: a literature review and future research directions. *Annals of Operations Research*, 332(1–3), 807–830.
- [6] Bouyari, F., Moussavi, S. M. and Vahdani, B. (2018). A Mathematical Model for Project Portfolio Selection with Project Interdependency, Project Divisibility, Reinvestment and Resourcing in Different Ways. *Advances in Industrial Engineering*, 52(2), 163-177.
- [7] Touni, Z., Makui, A. and Mohammadi, E. (2020). A More Human-Like Portfolio Optimization Approach: Using Utility Function to Find an Individualized Portfolio. *Advances in Industrial Engineering*, 54(3), 293-310.
- [8] Zhu, Q., Dou, Y., & Sarkis, J. (2010). A portfolio-based analysis for green supplier management using the analytical network process. *Supply Chain Management: An International Journal*, 15(4), 306–319.
- [9] Aragonés-Beltrán, P., Chaparro-González, F., Pastor-Ferrando, J.-P., & Pla-Rubio, A. (2014). An AHP (Analytic Hierarchy Process)/ANP (Analytic Network Process)-based multi-criteria decision approach for the selection of solar-thermal power plant investment projects. *Energy (Oxford, England)*, 66, 222–238.
- [10] Szilágyi, I., Sebestyén, Z., & Tóth, T. (2019). Project Ranking in Petroleum Exploration. *The Engineering Economist*, 65(1), 66–87.
- [11] Lee, H. Y., Heng, Y. P., Selvanathan, K., Chandrahasan, P., & Chemmangattuvalappil, N. G. (2024). Multi-criteria decision-making tools for project selection by international conglomerates. *Process Integration and Optimization for Sustainability*, 8(2), 375–393.
- [12] Alsanousi, A. T., Alqahtani, A. Y., Makki, A. A., & Baghdadi, M. A. (2024). A Hybrid MCDM Approach Using the BWM and the TOPSIS for a Financial Performance-Based Evaluation of Saudi Stocks. *Information*, 15(5), 258.
- [13] Khalili-Damghani, K., & Tavana, M. (2014). A comprehensive framework for sustainable project portfolio selection based on structural equation modeling. *Project Management Journal*, 45(2), 83–97.
- [14] Siew, R. Y. J. (2016). Integrating sustainability into construction project portfolio management. *KSCE Journal of Civil Engineering*, 20(1), 101–108.
- [15] RezaHoseini, A., Ghannadpour, S. F., & Hemmati, M. (2020). A comprehensive mathematical model for resource-constrained multi-objective project portfolio selection and scheduling considering sustainability and projects splitting. *Journal of Cleaner Production*, 269(122073), 122073.
- [16] Rahimi, F., Davari-Ardakani, H., Ameli, M., & Kabiri Beheshtkhal, M. (2024). Sustainable project selection and scheduling using scenario-based stochastic programming: a case study of industrial projects. *Stochastic Environmental Research and Risk Assessment: Research Journal*, 38(2), 593–619.
- [17] Parvaneh, F., & Hammad, A. (2024). Application of Multi-Criteria Decision-Making (MCDM) to Select the Most Sustainable Power-Generating Technology. *Sustainability*, 16(8), 3287.
- [18] Askari Lasaki, A., Adlparvar, M. R. and Shahbazi, M. S. (2021). An Integrated Soft Computing Method Based on Intuitionistic Fuzzy Environment to Appraise the Urban Bridge Maintenance Models. *Advances in Industrial Engineering*, 55(3), 307-322.
- [19] Salimian, S., & Mousavi, S. M. (2021). Healthcare waste disposal location selection by a multi-criteria decision-making method with intuitionistic fuzzy sets. *Journal of Quality Engineering and Production Optimization*, 6(2), 143-156.
- [20] Mohagheghi, V., Mousavi, S. M., & Siadat, A. (2015). A new approach in considering vagueness and lack of knowledge for selecting sustainable portfolio of production projects. *2015 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, 1732–1736.

- [21] Wu, Y., Xu, C., Ke, Y., Tao, Y., & Li, X. (2019). Portfolio optimization of renewable energy projects under type-2 fuzzy environment with sustainability perspective. *Computers & Industrial Engineering*, 133, 69–82.
- [22] Mohagheghi, V., & Mousavi, S. M. (2020). D-WASPAS: Addressing social cognition in uncertain decision-making with an application to a sustainable project portfolio problem. *Cognitive Computation*, 12(3), 619–641.
- [23] Ma, J., Harstvedt, J. D., Jaradat, R., & Smith, B. (2020). Sustainability driven multi-criteria project portfolio selection under uncertain decision-making environment. *Computers & Industrial Engineering*, 140(106236), 106236.
- [24] Jalilibal, Z., & Bozorgi-Amiri, A. (2022). A Hybrid Grounded Theory, Fuzzy DEMATEL and ISM Method for Assessment of Sustainability Criteria for Project Portfolio Selection Problems. *Interdisciplinary Journal of Management Studies (Formerly known as Iranian Journal of Management Studies)*, 15(3), 425–442.
- [25] Mohagheghi, V., Mousavi, S. M., & Shahabi-Shahmiri, R. (2022). Sustainable project portfolio selection and optimization with considerations of outsourcing decisions, financing options and staff assignment under interval type-2 fuzzy uncertainty. *Neural Computing & Applications*, 34(17), 14577–14598.
- [26] Huang, Y. Y., Pan, J., Tsaur, R. C., & Lin, N. C. (2024). Fuzzy portfolio selection in the trade-off between different economy trends and security returns. *Applied Mathematics in Science and Engineering*, 33(1).
- [27] Vasantha Lakshmi, K., & Udaya Kumara, K. N. (2024). A novel randomized weighted fuzzy AHP by using modified normalization with the TOPSIS for optimal stock portfolio selection model integrated with an effective sensitive analysis. *Expert Systems with Applications*, 243(122770), 122770.
- [28] Jawad, M., Naz, M., & Muqaddus, H. (2024). A multi-criteria decision-making approach for portfolio selection by using an automatic spherical fuzzy AHP algorithm. *The Journal of the Operational Research Society*, 75(1), 85–98.
- [29] Vasantha Lakshmi, K., & Udaya Kumara, K. N. (2025). Fuzzy MCDM techniques for portfolio selection in the post-COVID Indian mutual fund market: a comparative study of FAHP and entropy methods. *Journal of Ambient Intelligence and Humanized Computing*, 16(1), 97–107.
- [30] Sharma, S. K. (2025). Enhancing Stock Portfolio Selection with Trapezoidal Bipolar Fuzzy VIKOR Technique with Boruta-GA Hybrid Optimization Model: A Multicriteria Decision-Making Approach. *International Journal of Computational Intelligence Systems*, 18(1), 17.
- [31] Žižović, M., Faculty of Technical Sciences in Čačak, University of Kragujevac, Serbia, Pamučar, D., & Department of logistics, Military academy, University of defence, Serbia. (2019). New model for determining criteria weights: Level Based Weight Assessment (LBWA) model. *Decision Making Applications in Management and Engineering*, 2(2).
- [32] Pamucar, D., Deveci, M., Canitez, F., & Lukovac, V. (2020). Selecting an airport ground access mode using novel fuzzy LBWA-WASPAS-H decision making model. *Engineering Applications of Artificial Intelligence*, 93(103703), 103703.
- [33] Božanić, D., Randelović, A., Radovanović, M., & Tešić, D. (2020). A hybrid lbwa - Ir-mairca multi-criteria decision-making model for determination of constructive elements of weapons. *Facta Universitatis Series Mechanical Engineering*, 18(3), 399.
- [34] Torkayesh, A. E., Pamucar, D., Ecer, F., & Chatterjee, P. (2021). An integrated BWM-LBWA-CoCoSo framework for evaluation of healthcare sectors in Eastern Europe. *Socio-Economic Planning Sciences*, 78(101052), 101052.
- [35] Jokić, Ž., Božanić, D., Pamučar, D. (2021). Selection of fire position of mortar units using lbwa and fuzzy mabac model. *Operational Research in Engineering Sciences: Theory and Applications*, 4(1).
- [36] Pamucar, D., & Faruk Görçün, Ö. (2022). Evaluation of the European container ports using a new hybrid fuzzy LBWA-CoCoSo'B techniques. *Expert Systems with Applications*, 203(117463), 117463.
- [37] Ögel, İ. Y., Ecer, F., & Özgöz, A. A. (2023). Identifying the leading retailer-based food waste causes in different perishable fast-moving consumer goods' categories: application of the F-LBWA methodology. *Environmental Science and Pollution Research International*, 30(12), 32656–32672.
- [38] Božanić, D., Pamucar, D., Badi, I., & Tešić, D. (2023). A decision support tool for oil spill response strategy selection: application of LBWA and Z MABAC methods. *Opsearch. Quarterly Journal of the Operational Research Society of India*, 60(1), 24–58.
- [39] Ogundoyin, S. O., & Kamil, I. A. (2023). An integrated Fuzzy-BWM, Fuzzy-LBWA and V-Fuzzy-CoCoSo-LD model for gateway selection in fog-bolstered Internet of Things. *Applied Soft Computing*, 143(110393), 110393.
- [40] Özekenci, E. K. (2024). Personnel selection based on the LBWA, TOPSIS and GRA methods: A case study on foreign trade company. *Fiscaoeconomia*, 8(2), 646–665.
- [41] Pamučar, D., Vasin, L., & Lukovac, V. (2014). Selection of railway level crossings for investing in security equipment using hybrid dematel-maric model: *Application of a new method of multi-criteria decision-making*.
- [42] Boral, S., Howard, I., Chaturvedi, S. K., McKee, K., & Naikan, V. N. A. (2020). A novel hybrid multi-criteria group decision making approach for failure mode and effect analysis: An essential requirement for

- sustainable manufacturing. *Sustainable Production and Consumption*, 21, 14–32.
- [43] Gul, M., & Ak, M. F. (2020). Assessment of occupational risks from human health and environmental perspectives: a new integrated approach and its application using fuzzy BWM and fuzzy MAIRCA. *Stochastic Environmental Research and Risk Assessment: Research Journal*, 34(8), 1231–1262.
- [44] Hadian, S., Shahiri Tabarestani, E., & Pham, Q. B. (2022). Multi attributive ideal-real comparative analysis (MAIRCA) method for evaluating flood susceptibility in a temperate Mediterranean climate. *Journal Des Sciences Hydrologiques [Hydrological Sciences Journal]*, 67(3), 401–418.
- [45] Gul, M., Mete, S., Serin, F., & Celik, E. (2021). Fine–Kinney-based occupational risk assessment using fuzzy best and worst method (F-BWM) and fuzzy MAIRCA. In *Fine–Kinney-Based Fuzzy Multi-Criteria Occupational Risk Assessment* (pp. 13–30). Springer International Publishing.
- [46] Ecer, F. (2022). An extended MAIRCA method using intuitionistic fuzzy sets for coronavirus vaccine selection in the age of COVID-19. *Neural Computing & Applications*, 34(7), 5603–5623.
- [47] Nguyen, H.-Q., Nguyen, V.-T., Phan, D.-P., Tran, Q.-H., & Vu, N.-P. (2022). Multi-criteria decision making in the PMEDM process by using MARCOS, TOPSIS, and MAIRCA methods. *Applied Sciences (Basel, Switzerland)*, 12(8), 3720.
- [48] Eti, S., & Baş, H. (2023). Determining the social exclusion levels of international students from different regions using the MAIRCA and TOPSIS method. *Globalisation Societies and Education*, 1–20.
- [49] Tepe, S., Eti, S., Moda, H. M., & Kahraman, Z. (2024). Evaluation of psychosocial risk factors encountered for energy sector employees by MAIRCA method. In *Contributions to Management Science* (pp. 65–83). Springer Nature Switzerland.
- [50] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, 8(3), 199–249.
- [51] Zimmermann, H.-J. (1986). Fuzzy set theory and mathematical programming. In *Fuzzy Sets Theory and Applications* (pp. 99–114). Springer Netherlands.
- [52] Grattan-Guinness, I. (1976). Fuzzy membership mapped onto intervals and many-valued quantities. *MLQ. Mathematical Logic Quarterly*, 22(1), 149–160.
- [53] N. Karnik, N., & M. Mendel, J. (2001). Operations on type-2 fuzzy sets. *Fuzzy Sets and Systems. An International Journal in Information Science and Engineering*, 122(2), 327–348.
- [54] Ashtiani, B., Haghghirad, F., Makui, A., & Montazer, G. A. (2009). Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. *Applied Soft Computing*, 9(2), 457–461.
- [55] Vahdani, B., Hadipour, H., Sadaghiani, J. S., & Amiri, M. (2010). Extension of VIKOR method based on interval-valued fuzzy sets. *The International Journal of Advanced Manufacturing Technology*, 47(9–12), 1231–1239.
- [56] Gorzalczy, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems. An International Journal in Information Science and Engineering*, 21(1), 1–17.
- [57] Yao, J.-S., & Lin, F.-T. (2002). Constructing a fuzzy flow-shop sequencing model based on statistical data. *International Journal of Approximate Reasoning: Official Publication of the North American Fuzzy Information Processing Society*, 29(3), 215–234.
- [58] Chen, S.-M. (2002). Fuzzy system reliability analysis based on vague set theory. *1997 IEEE International Conference on Systems, Man, and Cybernetics. Computational Cybernetics and Simulation*.
- [59] Hong, D. H., & Lee, S. (2002). Some algebraic properties and a distance measure for interval-valued fuzzy numbers. *Information Sciences*, 148(1–4), 1–10.
- [60] Chen, S.-J., & Chen, S.-M. (2008). Fuzzy risk analysis based on measures of similarity between interval-valued fuzzy numbers. *Computers & Mathematics with Applications (Oxford, England: 1987)*, 55(8), 1670–1685.
- [61] Ashtiani, B., Haghghirad, F., Makui, A., & ali Montazer, G. (2009). Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. *Applied Soft Computing*, 9(2), 457–461.
- [62] Dorfeshan, Y., Mousavi, S. M., Zavadskas, E. K., & Antucheviciene, J. (2021). A new enhanced ARAS method for critical path selection of engineering projects with interval type-2 fuzzy sets. *International Journal of Information Technology & Decision Making*, 20(01), 37–65.
- [63] Dorfeshan, Y., Jolai, F., & Mousavi, S. M. (2023). A multi-criteria decision-making model for analyzing a project-driven supply chain under interval type-2 fuzzy sets. *Applied Soft Computing*, 148, 110902.
- [64] Aramesh, S., Mousavi, S. M., & Mohagheghi, V. (2021). A new comprehensive project scheduling, monitoring, and management framework based on the critical chain under interval type-2 fuzzy uncertainty. *Iranian Journal of Fuzzy Systems*, 18(1), 151–170.



This article is an open-access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license.