



## Portfolio Selection Optimization Problem Under Systemic Risks

Mohammad Ali Dehghan Dehnavi, Mohammad Mahdi Bahrololoum\* , Moslem Peymani Foroushani, Sayed Ali Raeiszadeh

*Department of Finance and Banking, Faculty of Accounting and Management Allameh Tabataba'i University, Tehran, Iran*

Received: 12 April 2021, Revised: 20 May 2021, Accepted: 20 May 2021

© University of Tehran 2020

### Abstract

Portfolio selection is of great importance among financiers, who seek to invest in a financial market by selecting a portfolio to minimize the risk of investment and maximize their profit. Since there is a covariant among portfolios, there are situations in which all portfolios go high or down simultaneously, known as systemic risks. In this study, we proposed three improved meta-heuristic algorithms namely, genetic, dragonfly, and imperialist competitive algorithms to study the portfolio selection problem in the presence of systemic risks. Results reveal that our Imperialist Competitive Algorithm are superior to Genetic algorithm method. After that, we implement our method on the Iran Stock Exchange market and show that considering systemic risks leads to more robust portfolio selection.

### Keywords:

Genetic Algorithm;  
Imperialist Competitive Algorithm;  
Portfolio Selection;  
Systemic Risks

### Introduction

The set of stocks bought by the investor is called portfolio selection. In fact, portfolio refers to the fact that the investor must divide his capital between several different financial assets to reduce his investment risk. But more important than the concept of portfolio selection are the points that a successful investor should be aware of securities. The first and most important point that makes the term securities very important is a concept called risk. Risk can be defined as risk.

Portfolio selection involves the allocation of capital among a large number of securities so that the investor seeks the most profitable return, while carrying the least risk. Investors in the stock market always make their decisions to choose a portfolio for the future, precisely because of the uncertainty of future markets, it is not easy to predict the realized value of each stock.

Therefore, investors usually have no choice but to trust the data gained from the past and through experience. Most of the literature assumes that the stock rate of return is a random variable and the distribution parameters can be estimated from past data. Based on this assumption, a large number of portfolio selection models are based on probability theory.

Markowitz [27] published his pioneering work, which served as the basis for the development of modern portfolio theory over the past few decades. The Markowitz model used variance to describe risk by the degree of bias between the effective rate of return and the expected rate of return. However, the variance calculated by the total deviation from the expected return describes both downside and downside risk. In fact, investors do not like

\* Corresponding author: (M. Bahrololoum)

downside risk, but they actually want to take upward risk. Therefore, the use of variance may also limit the potential benefit. He introduced the first safety work on the sample, which only minimized the risk of falling. Another standard measure of risk across the company is Value at Risk (VaR). For a given time horizon and  $\beta$  level of confidence, VaR is a portfolio loss of portfolio market value over a time horizon that exceeds the probability of  $1 - \beta$ . However, there are many factors including social, economic, political and investor psychology in addition to the potential factor. More and more researchers have found that the theory of fuzzy sets proposed by Zadeh [39]. It is a good tool for controlling ambiguity or ambiguity in stock markets.

Dubois and Pradd [9] introduced the possibility space, which was similar to the model space proposed by Nahmias[31], and first defined a fuzzy variable as mapping a possible space into a real number. Dubois and Prad [9] then did research on the theory of possibility and provided definitions for two basic criteria of a fuzzy variable: possibility and necessity. For a fuzzy event, different decision makers may have different perceptions of the probability of occurrence. If the decision maker is more aggressive, it is more valuable when measuring probability. If Carter is a conservative decision maker, he will have less value when measuring. Since the probability of a fuzzy event occurring is always greater than its necessity, so possibility can be considered a criterion with an aggressive attitude and necessity can be considered as a criterion with a conservative attitude. In addition, Liu and Liu [22;23] defined the dual measurement criterion that can be used to quantify probability under a neutral attitude.

In this study, the problem of portfolio selection under systemic risk is described. Systemic risk in portfolio selection was introduced by Biglova et al. [5]. Systemic risk is the risk that affects not just specific market participants, but an entire financial market or system due to the interlinkages and interdependences of financial institutions throughout the world. To solve a portfolio selection under systemic risk, we, for the first time, proposed meta-heuristic algorithms. Three meta-heuristic algorithms including genetic, dragonfly, and imperialist competitive algorithms to study the portfolio selection problem in the presence of systemic risks. To improve the meta-heuristics, a learning operator and a new mutation operator are added to these meta-heuristics.

This paper is structured as follows. In Section 2, the literature of portfolio selection is reviewed. In Section 3, our proposed portfolio selection problem is explained. In Section 4, meta-heuristic algorithms are developed to solve the problem. In Section 5, results of Iran Stock Exchange market are discussed. Finally, in Section 7, this study is concluded.

## Literature review

In this section literature of portfolio selection is reviewed. There are a number of studies on the use of fuzzy set theory to solve the portfolio selection problem. Xu et al. [38] developed optimistic and pessimistic portfolio selection models in a fuzzy random environment. Kocada and Keskin [16] introduced a new fuzzy portfolio selection model that tailored risk preferences to market trends as well as risk-return risk, allowing decision makers to prioritize among their goals.

Abdelaziz et al. [1] set a multi-objective stochastic plan for portfolio selection in which the decision maker considered conflicting goals such as rate of return, liquidity and risk. They also used techniques to meet ideals and priorities. Objective programming and agreement programming have been used, of course, assuming the parameters are random and based on programming models with possible limitations. The case study has been the selection of a portfolio from the Tunisian stock market. Abdelaziz et al.[2], they proposed a randomized ideal planning approach to create a satisfactory portfolio for the UAE stock market, assuming that the shareholding range was abnormal, along with a number of scenarios. They considered total

returns, current income and risk and compared the results with the traditional Markowitz model, and finally proved the superiority of random ideal programming over other methods in optimizing portfolio selection. Bermuders et al. proposed a new method for portfolio selection in which it extends the genetic algorithm from its traditional realm of optimization to a fuzzy rating strategy for selecting an efficient cardinality portfolio. Has been from the Spanish stock market.

Ghahtarani and Najafi [13] used a strong ideal planning approach to the portfolio selection problem, in which they examined the parameters that had uncertainties with a strong optimization approach, which was analyzed using the analysis method. Data coverage has been used. The model proposed by them is widely used in the real world because the uncertainty and the decision-maker's idea in selecting the portfolio are considered simultaneously.

Recently, many researchers have argued that the selection of a beta portfolio should be considered as an indeterminate parameter. Abdelaziz et al. [3] have created a random variable and presented a multi-objective random portfolio selection model with random beta. To solve it, they have used the ideal planning model, which has already proven the superiority of this method in optimal portfolio selection. Li et al. [20] examined a fuzzy portfolio selection model with a background risk based on the definitions of return and potential risk, which follows the LR type probability distribution for asset returns. So they obtained the ground and compared it to the efficient frontier of a portfolio without background risk, and finally concluded that background risk could better reflect the investment risk of the economic environment, which makes investors a portfolio. Choose more suitable for them.

Najafi et al. [30] developed an efficient innovative approach to dynamic stock portfolio selection by considering transaction costs and uncertainty conditions from a single-period model to a multi-period model. Considering uncertainty conditions and costs has made the problem more realistic and complex by using an innovative (metaheuristic) method to solve it. The results have proven the superiority of this method. Zhao et al. [41] introduced a tool for the convenience of investors that allows them to express their priority in choosing a portfolio at two levels: 1) by comparing the criteria of a similar nature 2) by comparing the two criteria of the top level (objectives) Financial and strategic) who have used the fuzzy method to solve the problem due to conflicting goals with modeling.

Jean and King also proposed a model for selecting a portfolio with value-at-risk constraints in which the asset price process is modeled by non-extensive statistical mechanisms instead of the classical Wiener process. Reduces the risk of investing in high-risk assets and, at the same time, at the same level of confidence, reduces the ratio of capital invested in high-risk assets under the proposed model faster than the model based on the Wiener process. It can be a good reference for investors to decide on portfolio selection. Arshadi et al., to deal with the inherent complexity and uncertainty of project portfolio construction, have developed a robust optimization algorithm to maximize the combined options of project portfolio selection, which is very effective for uncertainty conditions. It should be noted that the results of this study have been very useful as a managerial point of view.

Zhou et al. [42] examined the issue of stock portfolio selection under various protective, neutral, and aggressive approaches in a fuzzy environment, reviewing 10 data from the Chinese stock market using Pareto optimization solutions. Maximizing returns and minimizing risk, and considering the transaction cost and value constraints at risk, have examined the effectiveness of the proposed model. Also, Lansman et al. [17] have introduced a new class of functions for selecting the optimal stock portfolio, in which the class uses important metrics such as mean, variance, Sharp ratio and standard deviation, and more.

Masoudi and Abdul Aziz [24] have provided a comprehensive overview of definite and indefinite multi-objective planning models in stock portfolio selection. Expression of pricing models and risk selection criteria for stocks. Finally, they show how to use these models to

select a portfolio. Portfolio selection has always had two violations: 1) Projects that have already been started have always been ignored. 2) Select parameters have always been accurate. Li et al. [19] have examined a case in which two scenarios are examined. As a result, they realized that the larger the scale of existing projects, the better the company's revenue.

When several goals are considered in the portfolio selection process, the issue becomes somewhat complicated, as Garcia et al. [11] have used genetic algorithms to solve the model. But when prices in a portfolio are interrelated, portfolio selection and bilateral trading become complicated, with Pascal et al. conducting a numerical experiment in 2019 in which they have proven a competitive ratio and optimal conditions.

Evaluating the quality of solutions is very important in selecting a portfolio in the project. This feature examined portfolio options. Including a stability function to show the strength of portfolios against changes in any initial data whose optimality threshold is obtained. The result indicates the ability of the stability function to evaluate the quality of portfolio selection.

Sandra et al. presented a new way to choose a portfolio based on minimizing regret. The finding is described by considering a robust strategy in which the minimum expected profit of the investor is maximized in the worst case, which uses a genetic algorithm to solve this problem. In his book, Markin devotes a chapter to how portfolio selection is managed, which has greatly helped the investor in product decision-making and development. Frej [10] in Germany introduced a new model for selecting portfolios of projects based on cost-benefit ratios with incomplete information provided by them. This ranking of criteria is done on a fixed scale. A case study conducted by a Brazilian power company. Investment priority is important in estimating the risk of portfolio selection issues because it affects investment strategies. Theo and Yang have considered a mini-max criterion that specifically aims to simply limit the standard deviation for each of the existing stocks, the related optimization problem has been formulated as a linear program, so it can be easily implemented in the real world.

## Problem description

In this section, the problem of portfolio selection under systemic risk is described. To formulate this problem, the proposed performance measure by Biglova et al. [5] is used. Selecting a portfolio is generally based on Markowitz model by which we aim to minimize the risk and maximize the expected profit. A general form of portfolio selection problem is as follows.

$$\begin{aligned} & \text{Max}_x \frac{v(x'r - r_f)}{\rho(x'r - r_f)} \\ & \text{s.t.} \\ & \sum_{i=1}^n x_i = 1 \\ & x_i \geq 0, \quad \forall i \end{aligned} \tag{1}$$

where  $v$  is the expected profit indicator,  $\rho$  is the risk parameter and  $r$  is the index of  $n$ -stock market log-return set of vectors, denoted by  $[r_1, r_2, \dots, r_n]'$ . Furthermore,  $r_f$  is the benchmark return that is risk-free.

Sharp proposed reward-to-variability ratio as the objective function  $\left(\frac{E(x'r - r_f)}{\text{var}(x'r - r_f)}\right)$  to be used in this problem. Rachev proposed a value at risk based ratio for performance measuring of a portfolio, which is defined as  $RR(\alpha, \beta) = \frac{ETL_\alpha(r_f - x'r)}{ETL_\beta(x'r - r_f)}$ . R-Ratio is a measure of the risk-return of a portfolio.  $r_f$  is a benchmark return and ETL or conditional value-at-risk (CVaR) is defined in Eq. 2.

$$ETL_{\alpha}(X) = \frac{1}{\alpha} \int_0^{\alpha} VaR_q(X) dq \tag{2}$$

which  $VaR_q(X) = -F_X^{-1}(q) = -inf\{x|P(X \leq x) > q\}$  is the  $VaR$  of random return  $X$  and  $1 - \alpha$  is the confidence level of  $X$  losses. If  $X$  is considered as a continuous parameter, then  $AVaR_{\alpha}(X) = -E(X|X \leq -VaR_{\alpha}(X))$  and  $AVaR_{\alpha}$  is the average loss by  $\alpha$  percent.

However, this two measures do not account systemic risks. Therefore, Biglova et al. [5] proposed another measure based on the Rachev measure that considers systemic risks. This measure is a co-measure named Co-ETL to select a portfolio.

$$CoETL_{\alpha}(x'r) = -E(x'r|r_1 \leq -VaR_{\alpha}(r_1), \dots, r_n \leq -VaR_{\alpha}(r_n)) \tag{3-1}$$

Given that Co-ETL evaluates the losses for all stocks during periods of financial system instability, it can assess the systemic risk of the portfolio. Also, the average portfolio returns can be evaluated using the Co-Reward measure when all stocks rise in price, due to the simultaneous rise in prices in financial markets. They called it Co-Expected Tail Profit.

$$CoETP_{\beta}(x'r) = -E(x'r|r_1 \geq -VaR_{1-\beta}(r_1), \dots, r_n \geq -VaR_{1-\beta}(r_n)) \tag{3-2}$$

Biglova et al. [5] lastly proposed a measure to create a correlation and pay-off between profit and loss. The measure is named Co-Rachev and it's a development of R-Ration.

$$CoRR(x'r; \alpha, \beta) = \frac{CoETP_{\beta}(x'r - r_f)}{CoETL_{\alpha}(x'r - r_f)} \tag{4}$$

In this study, we aim to use the Co-Rachev measure to analyze Iran Stock Exchange market.

### Mathematical model

To develop the mathematical model of the problem, we need some preliminaries. The main constraint of the problem based on Eqs. 3 and 4 is as follows:

$$-VaR_{1-\beta}(r_i) \leq r_i \leq -VaR_{\alpha}(r_i) \tag{5}$$

There are three different states for  $r_i$ , expressed in Eqs. 6 to 8.

$$r_i \leq -VaR_{1-\beta}(r_i) \tag{6}$$

$$-VaR_{1-\beta}(r_i) \leq r_i \leq -VaR_{\alpha}(r_i) \tag{7}$$

$$-VaR_{\alpha}(r_i) \leq r_i \tag{8}$$

Similar to Eqs. 7 to 11 of Rabbani et al.[32], these equations can be linked using thee binary auxiliary variables. The binary auxiliary variables are as follows.

$$y_{1,i} \quad 1, \text{ if } r_i \leq -VaR_{1-\beta}(r_i) ; 0, \text{ otherwise}$$

$$y_{2,i} \quad 1, \text{ if } -VaR_{1-\beta}(r_i) \leq r_i \leq -VaR_{\alpha}(r_i) ; 0, \text{ otherwise}$$

$$y_{3,i} = \begin{cases} 1, & \text{if } -VaR_{\alpha}(r_i) \leq r_i; \\ 0, & \text{otherwise} \end{cases}$$

New constraints are:

$$r_i \leq -VaR_{1-\beta}(r_i) + M(1 - y_{1,i}) \quad (9)$$

$$-VaR_{1-\beta}(r_i) - M(1 - y_{2,i}) \leq r_i \leq -VaR_{\alpha}(r_i) + M(1 - y_{2,i}) \quad (10)$$

$$-VaR_{\alpha}(r_i) \leq r_i + M(1 - y_{3,i}) \quad (11)$$

$$y_{1,i} + y_{2,i} + y_{3,i} = 1 \quad (12)$$

where  $M$  is a very big number.

We also need another binary auxiliary variable to linked all  $n$  conditions.

$$z = \begin{cases} 1, & \text{if all } n \text{ conditions are met;} \\ 0, & \text{otherwise} \end{cases}$$

Therefore, the mathematical model is:

$$\max \frac{v(x'.r.z - r_b)}{\rho(x'.r.z - r_b)} \quad (13)$$

s.t.

$$\sum_{i=1}^n x_i = 1 \quad (14)$$

$$r_i \leq -VaR_{1-\beta}(r_i) + M(1 - y_{1,i}) \quad (15)$$

$$-VaR_{1-\beta}(r_i) - M(1 - y_{2,i}) \leq r_i \quad (16)$$

$$r_i \leq -VaR_{\alpha}(r_i) + M(1 - y_{2,i}) \quad (17)$$

$$-VaR_{\alpha}(r_i) \leq r_i + M(1 - y_{3,i}) \quad (18)$$

$$y_{1,i} + y_{2,i} + y_{3,i} = 1 \quad (19)$$

$$z = \prod_{i=1}^n y_{2,i} \quad (20)$$

$$y_{1,i}, y_{2,i}, y_{3,i} \in \{0, 1\} \quad (21)$$

$$z \in \{0, 1\} \quad (22)$$

$$x_i \geq 0 \quad (23)$$

Eq. 6 expresses the objective function, which maximizes a function of expected rewards over a function of risk. Eq. 7 expresses that summation of  $x_i$ s must be equal to one. Eqs. 8 and 9 check if  $r_i \leq -VaR_{1-\beta}(r_i)$ . Eq. 10 checks if  $-VaR_{1-\beta}(r_i) \leq r_i \leq -VaR_{\alpha}(r_i)$ . Eq. 11 checks if  $-VaR_{\alpha}(r_i) \leq r_i$ . Please be noted that we look for  $-VaR_{1-\beta}(r_i) \leq r_i \leq -VaR_{\alpha}(r_i)$ . Eq. 12 expresses that only one of the three states for  $r_i$  can come true. Eq. 13 expresses that if all  $n$  conditions come true, then  $z$  can be 1. Eqs. 14 to 16 define the variables.

## Solution methods

The presented problem in Section 3 is a non-linear mathematical problem, which is Np-hard. To solve this problem, three meta-heuristic algorithms including genetic algorithm (GA), dragonfly algorithm (DA), and imperialist competitive algorithm (ICA) are developed.

### Imperialist competitive algorithm

The imperialist competitive algorithm (ICA) is inspired not by a natural phenomenon but a social-human one. In particular, ICA views the imperial process as a stage of socio-political evolution of human being, and it develops a mathematical model for this historical phenomenon in order to use it as a powerful tool for optimization purposes. ICA was first introduced by Atashpaz-Gargari and Lucas [4].

This algorithm starts by several ‘countries’ in their initial position. Countries are, in fact, possible answers to the problem [at hand] and are equivalent to chromosomes in genetic algorithms and particles in particle swarm optimization. All countries are divided into two categories: imperialist and colony. Depending on their power, the colonizers absorb these colonies through a specific process, which is described below. The total power of any empire depends on both its constituent parts, i.e., the imperial state (as the central core) and its colonies. This dependence is mathematically modeled by defining imperial power as the sum of the power of the imperial state in addition to a percentage of the average power of its colonies.

Imperial competition begins following the formation of the early empires. Any empire that fails in this competition and cannot increase its power (or at least prevent it from diminishing) will be removed from the scene. Therefore, the survival of an empire is predicated on its power to absorb and dominate the colonies of rival empires. Thus, in the course of imperial competitions, the power of larger empires will gradually expand and that of weaker ones will shrink. To increase their power, empires will have to develop their own colonies. Over time, the colonies will approximate the empires in terms of power and a convergence will take place. The end of colonial rivalry is when there remains a single empire in the world along with colonies that are very close in position [and authority] to the imperial state. In what follows, the different parts of ICA are fully examined.

To start the algorithm, we create a number of countries (equal to the number of initial countries in the algorithm). Thus, a matrix of all countries is randomly created as the initial solution.

$$COUNTRY = \begin{bmatrix} country_1 \\ country_2 \\ country_3 \\ \cdot \\ \cdot \\ \cdot \\ country_{N_{country}} \end{bmatrix} = \begin{bmatrix} KP_1 & KI_1 & KD_1 \\ KP_2 & KI_2 & KD_2 \\ KP_3 & KI_3 & KD_3 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ KP_{N_{country}} & KI_{N_{country}} & KD_{N_{country}} \end{bmatrix} \quad (24)$$

The cost of a country is determined through evaluating the function fitness in the as:

$$cost_i = f(country_i) = f(p_1, p_2, p_3, \dots, p_{N_{var}}) \quad (25)$$

To start the algorithm, we create  $N_{country}$  countries. Next,  $N_{imp}$  best members of this population (countries with the lowest cost function) are selected as imperialists. The remaining  $N_{col}$  countries form the colonies, each belonging to an empire. To divide the initial colonies among the imperial states, we assign each state a number of colonies (this number corresponds to the power of each imperialist). To this end, the cost of all imperialists is calculated and their normalized costs are determined as follows:

$$C_n = \max_i \{c_i\} - c_n \quad (26)$$

where the cost of the  $n$ -th imperialist is the highest cost among its rivals, and  $C_n$  is the normalized cost of this imperialist. An imperialist with a higher cost (i.e., a weaker imperialist) will have a lower normalized cost. Considering the normalized cost, we can obtain the relative normalized power of each imperialist (as given below); accordingly, the colonies are divided among the imperial states.

$$p_n = \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \quad (27)$$

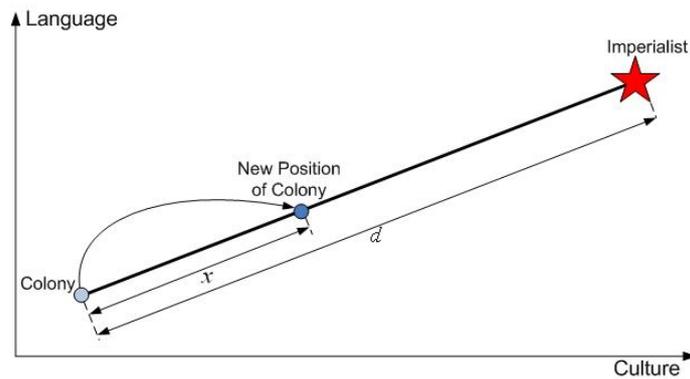
From another point of view, the normalized power of an imperialist is in proportion to the colonies over which it rules. Therefore, the initial number of colonies of an imperialist is equal to:

$$N.C_n = \text{round}\{p_n \cdot (N_{col})\} \quad (28)$$

where  $N.C_n$  is the initial number of colonies of an empire, and  $N_{col}$  expresses the total number of colonies in the population of the initial countries; also, *round* is a function that obtains the integer closest to a decimal number. Considering  $N.C_n$  for each empire, this number of primary colony countries is randomly selected and assigned to the  $n$ th imperialist. Having determined the initial state of all empires, we can start the ICA. The evolutionary process is in a loop that continues until a stop condition is met.

#### *Modeling the absorption policy: the movement of colonies toward the imperial state*

The policy of assimilation (absorption) was adopted with the aim of integrating the culture and social structure of the colonies into the culture of the central state. Given the particular performance of a country in solving an optimization problem, the imperial state adopted an absorption policy to bring the colonized country closer in terms of various socio-political dimensions. This part of the colonial process in the optimization algorithm is modeled as the movement of colonies toward the imperial state. Fig. 1 shows an overview of this movement.

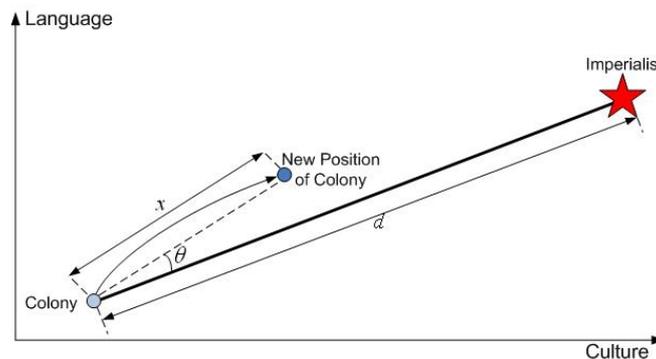


**Fig. 1.** An overview of colonies' movement toward the imperial state

Accordingly, the imperial state absorbs a colony in the direction of the axes of culture and language. As shown, the colony moves  $x$  units in the direction of the line connecting it to the imperialist and is pushed to a new position. In this figure,  $d$  represents the distance between the imperialist and the colony, and  $x$  denotes a random number with a uniform distribution (or any other suitable distribution). In other words, for  $x$  we have:

$$x \sim U(0, \beta \times d) \tag{29}$$

where,  $\beta$  is a number greater than 1 and close to 2; thus,  $\beta = 2$  can be a good choice. The coefficient,  $\beta > 1$  causes the colony to approach the imperial state from different directions.



**Fig. 2.** The real movement of colonies towards the imperialist

Also, in order to increase diversity, instead of simply moving up to  $x$ , we make a  $\theta$  deviation in the path and continue to move toward the imperial state and in the direction of the colony-imperialist vector. We randomly consider  $\theta$  with a uniform distribution (but any other desired, appropriate distribution can also be used). Therefore:

$$\theta \sim U(-\gamma, \gamma) \tag{30}$$

In this relation,  $\gamma$  is an arbitrary parameter whose increase leads to a rise in the search space around the imperialist, and its decrease causes the colonies to move as close as possible to the colony-imperialist vector. Considering the radian for  $\theta$ , a number close to  $\pi/4$  has proved a good choice in most implementations.

### *Transfer of information between colonies*

In order to transfer information between colonies, we used the crossover operator in the genetic algorithm. The so-called tournament selection method was employed to choose the colonies.

### *Revolution*

This process is similar to the mutation method in the genetic algorithm and is performed to escape local searches.

### *Updated colonies*

In each period, the initial population of the colonies, the simulated population, the population resulting from the transfer of information between the colonies, and the population obtained from the revolution are merged for each empire; then, the best colonies, which are equal to the population of the considered colonies, are selected for each imperialist.

### *Displacement of colonial and imperialist position*

As the colonies move toward the imperialist, some of them may reach a better position than the imperialist. In this case, the colony and the imperialist swap their positions, and the algorithm continues with the imperial state in a new position; this time it is the new imperialist that begins to implement the policy of assimilation to its colonies.

### *The total power of an empire*

The power of an empire is equal to the power of the imperial state in addition to a percentage of the total power of its colonies. Thus, to calculate the total cost of an empire, we use the following relation:

$$T.C._n = Cost(imperialist_n) + \xi \text{ mean}\{Cost(colonies\ of\ empire_n)\} \quad (31)$$

where  $T.C._n$  is the total cost of the  $n$ th empire and  $\xi$  is a positive number, usually between zero and one and close to zero. Considering a small  $\xi$  causes the total cost of an empire to be approximately equal to the cost of its central state (the imperial country), and increasing  $\xi$  amplifies the effect of the cost of an empire's colonies in determining its total cost. Typically,  $\xi = 0.02$  has led to favorable answers in most cases.

### *Colonial competition*

As mentioned earlier, any empire that fails to expand its power and loses its competitive power will be removed from imperial rivalries. This elimination occurs gradually, meaning that over time, weak empires lose their colonies and stronger empires take over these colonies and increase their power.

To model this fact, we assume the empire that is in the process of being eliminated is the weakest empire available. In this way, by repeating the algorithm, we select one or more of the weakest colonies of the weakest empire and create a rivalry between all the empires to seize these colonies. These colonies will not necessarily be conquered by the strongest empire, yet stronger empires are more likely to seize them. To this end, we first calculate the normalized total cost of the empire from its total cost.

$$N.T.C._n = \max_i \{T.C._i\} - T.C._n \quad (32)$$

where  $T.C._n$  is the total cost of the  $n$ th empire, and  $N.T.C._n$  is the normalized total cost of that empire. Any empire with a lower  $T.C._n$  will have a higher  $N.T.C._n$ . Indeed,  $T.C._n$  corresponds to the total cost of an empire and  $N.T.C._n$  represents its total power. The empire with the lowest cost will possess the highest power. Using the normalized total cost, we may calculate the probability (power) of an empire to seize the colony of another empire as follows:

$$p_{p_n} = \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \quad (33)$$

After obtaining this probability for each empire, we need a mechanism, such as the Roulette Wheel, in order to allocate a colony over which there is a competition to an empire that possesses an appropriate probability.

#### *Fall of weak empires*

As stated, during imperial rivalries weak empires—perforce—gradually collapse and their colonies fall into the hands of stronger empires. Different conditions might be considered for the dissolution of an empire. In the proposed algorithm, an empire is eliminated when it has lost its colonies.

#### *Convergence*

The algorithm continues until a convergence condition is met or the total number of iterations terminates. After a while, all the empires will collapse and we will have only one empire (with the rest of the countries being under the authority of this single empire) and the algorithm will end.

### **Genetic algorithm**

The onset innovation of genetic algorithm (GA) goes back to 1970, where Holland [15], by inspiring the nature and imitation of the natural selection process in breeding organisms, introduced this method. Later, the GA has been developed by Goldberg [14] to solve various combinatorial problems.

#### *Initial Population*

For utilization of GA method, at the first step it should be generated an initial population of solutions (called chromosome). Since the quantity of this method is affected by the population size, it should be taken into account as one of the crucial parameters in the applications of this method. If this population is too small, this method unable to generate qualified solutions, and if this population has a large size, the GA method takes a long time, and consequently, this method would be time-consuming. As a result, to determine the effective population size, two main parameters, including the crossover rate and mutation rate, should be set appropriately. For this purpose, the Taguchi method is employed in this study in Section 5 to tune genetic

algorithm parameters. The more suitable initial population led to the use of the GA method more efficiently.

### Selection

To select parents in the GA method, two different approaches are proposed. A percent of parents is selected through the tournament approach, and the rest of them are chosen from the best feasible solutions of the current generations. The latter approach guarantees the properties of a qualified solutions that transfer to the next generation. Also, this approach provides more accuracy and prevents the GA method converged quickly.

### Genetic Operators

The child production process is carried out in the genetic algorithm using the crossover and mutation operators on the selected parents. These operators are illustrated in the following.

*Crossover operator:* the crossover operators combine two parents' properties. Several types of these operators were presented for different problems (e.g., see Gen and Cheng [12]). In this study, uniform crossover operator is utilized with the identical probability in the proposed GA method.

*Mutation operator:* in the GA, the mutation operator prevents the GA solution becomes converged quickly. To implement mutation in the proposed GA, the two cells are selected randomly, then, their values are swapped with each other.

## Numerical results

In this section, numerical experiments are conducted to compare the performance of the proposed algorithms. First, we analyzed 15 stock indexes from Iran Stock Exchange market. Table 1 summaries the statistical information of these 15 stock indexes. Full detail of these stock indexes are available from the author upon request.

**Table 1.** Statistical summary of the 15 stock indexes implemented in this study

No.	Index name	Mean (%)	Standard deviation (%)	Skewness	Kurtosis	Alpha*	Beta*	Sigma (%)*	Delta (%)*
1	petroleum products	2.12	4.1	0.81	5.62	1.18	0.31	6.38	-0.95
2	basic metals	-1.48	2.81	0.01	10.22	1.05	-0.23	4.26	0.32
3	metal ores extraction	-2.08	3.07	-0.27	10.84	1.24	-0.3	5.64	-0.06
4	automobile	-0.44	60.9	37.22	1769.37	0.85	-0.1	4.6	0.36
5	industrial multidisciplinary	-1.54	6.01	-0.33	4.18	1.96	-0.89	13.68	-0.87
6	computer	-1.95	1.71	-0.81	7.64	0.96	-0.32	2.53	0.08
7	pharmacological	-1.51	2.99	-1.63	24.62	0.9	-0.17	3.02	0.39
8	chemical	-1.79	2.88	-1.07	13.37	1.08	-0.21	4.36	-0.19
9	transportation and warehousing	-1.76	2.24	-0.51	5.38	1.62	-0.39	6.92	-0.43
10	food	-1.53	2	-0.68	5.02	1.2	-0.36	4.63	0.78
11	investment	-2.01	2.44	-0.33	7.08	1.13	-0.28	4.38	-0.23
12	radio	-1.79	2.79	-1.37	19.61	1.35	-0.26	6.15	-0.3
13	cement	-1.81	8.98	17.3	627.48	1.27	-0.23	7.43	-0.36
14	bank	-1.9	3.17	-0.28	5.3	1.39	-0.34	6.93	0.05
15	engineering	-1.35	7.46	-0.08	9.09	0.61	0	4.06	0.07

\* these are the maximum likelihood estimates of the parameters of the stable Paretian

As we mentioned before, there was no study to solve a portfolio selection problem under systemic risks. Thus, the performance of the proposed algorithms is compared to each other. To evaluate proposed method (Co-Rachev ration) under systemic risk, a great number of observations should be in hand, where all assets are jointly in tail. The procedure proposed by Biglova et al. [5] is implemented to generate 30000 return scenario for each of the 10 data sets in accordance with data of 15 Iran Stock Exchange market indexes. Table 2 summarized the data sets. To generate data sets, we considered different combinations of these indexes.

**Table 2.** Indexes considered to each generated data set

Data set	No. Indexes	Included indexes
1	5	petroleum products, basic metals, metal ores extraction, automobile, industrial multidisciplinary
2	5	computer, pharmacological, chemical, transportation and warehousing, food
3	5	investment, radio, cement, bank, engineering
4	8	basic metals, metal ores extraction, automobile, transportation and warehousing, food, cement, bank, engineering
5	8	investment, radio, cement, computer, pharmacological, petroleum products, basic metals, metal ores extraction
6	8	automobile, industrial multidisciplinary, chemical, transportation and warehousing, radio, cement, bank, engineering
7	12	computer, pharmacological, chemical, transportation and warehousing, radio, cement, bank, engineering, basic metals, metal ores extraction, automobile, industrial multidisciplinary
8	12	petroleum products, automobile, industrial multidisciplinary, investment, radio, cement, bank, engineering, pharmacological, chemical, transportation and warehousing, food
9	12	investment, radio, cement, bank, engineering, petroleum products, basic metals, metal ores extraction, automobile, industrial multidisciplinary, pharmacological, chemical
10	15	petroleum products, basic metals, metal ores extraction, automobile, industrial multidisciplinary, computer, pharmacological, chemical, transportation and warehousing, food, investment, radio, cement, bank, engineering

In the following, first, the parameters of proposed metaheuristics are tuned. Then, their performances are compared to elicit the best one.

### Parameter Tuning

The efficiency of meta-heuristic algorithms is directly related to the adjustment of their parameters, so the incorrect selection of a meta-heuristic algorithm parameters causes its inefficiency. These parameters should be tuned through experimental tests. A variety of statistical methods have been proposed for designing experiments. A naive way is a full-factor experiment which is not always effective because increasing the number of factors studied makes calculations complex and extremely time-consuming. Taguchi introduced a series of fractional factor experiments that significantly reduce the number of experiments required while maintaining the information required for display (Taguchi, [37]). Besides a full-factor experiment, a better way to adjust the parameters of a meta-heuristic algorithm is to use Taguchi method, which is implemented here. This method is widely used in the literature of meta-heuristics (Rabbani et al., [32]; Mokhtarzadeh et al., [26]).

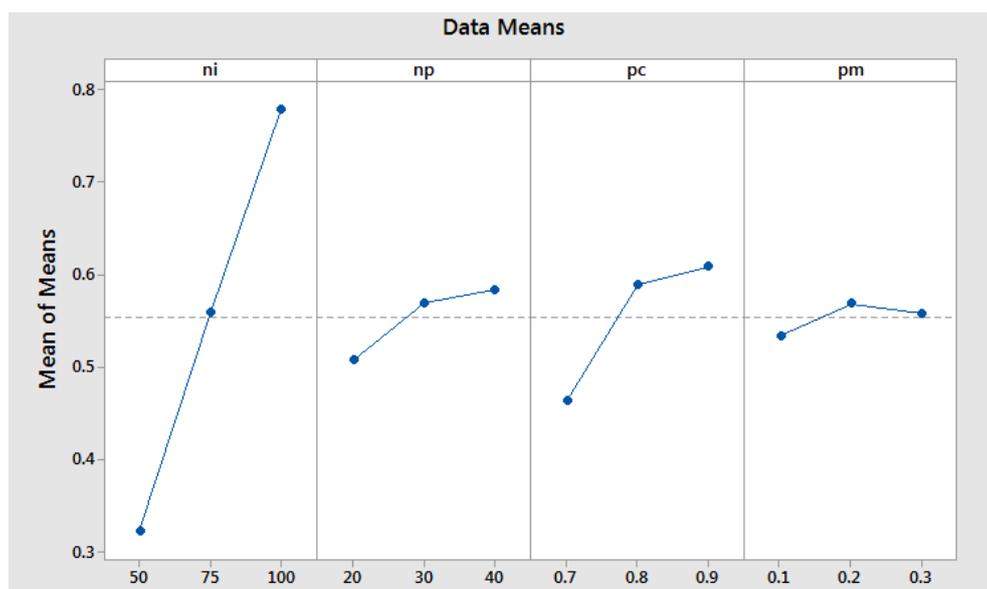
Taguchi stated that factors (agents) are divided into two categories: controllable factors and uncontrollable ones. The purpose of the method is to find the optimal levels of controllable important factors and minimize the effect of uncontrollable factors. In this method, the qualitative characteristics of the values measured from the experiments are converted from signal to noise ratio ( $S / N$ ). This rate indicates the amount of deviations displayed in the response variable. Here, the objective value is considered as the response variable.

GA has 4 parameters, maximum number of iteration ( $ni$ ), the number of population ( $np$ ), percent of crossover children ( $pc$ ), and percent of mutation children ( $mc$ ), that should be tuned. DA has 2 parameters,  $ni$  and  $np$ . ICA has 6 parameters,  $ni$ ,  $np$ , the number of countries ( $nc$ ), the assimilation coefficient ( $ac$ ), revolution probability ( $rp$ ), and colonies mean cost coefficient ( $cmcc$ ). A three level Taguchi design is considered. Therefore 3 different values for each parameter in considered based on the literature and our expertise, which are shown in Table 3.

**Table 3.** Parameters levels for Taguchi design

Algorithm	Parameters levels							
	$ni$	$np$	$nc$	$pc$	$mc$	$ac$	$rp$	$cmcc$
GA	50, 75, 100	20, 30, 40	-	0.7, 0.8, 0.9	0.1, 0.2, 0.3	-	-	-
ICA	50, 75, 100	20, 30, 40	40, 60, 80	-	-	1, 2, 3	0.1, 0.2, 0.3	0.1, 0.2, 0.3

The Taguchi tests for each algorithm is determined using MiniTab software and the experiments are conducted. Each experiment is run 5 times to remove the effect of randomness. Therefore, the average of objective value of the 5 run of each experiment is considered as its response level value. Then, MiniTab is used to analyzed the test. The results of analyzing are in Figs. 3 and 4. The Best value for each parameter of each algorithm is in Table 4. Also, it is worth mentioning that the adaptive form of these algorithm has no new parameter; therefore, the obtained best value for parameters of each algorithm is also used for its adaptive form.



**Fig. 3.** Analysis diagrams of GA parameters tuning based on Taguchi method

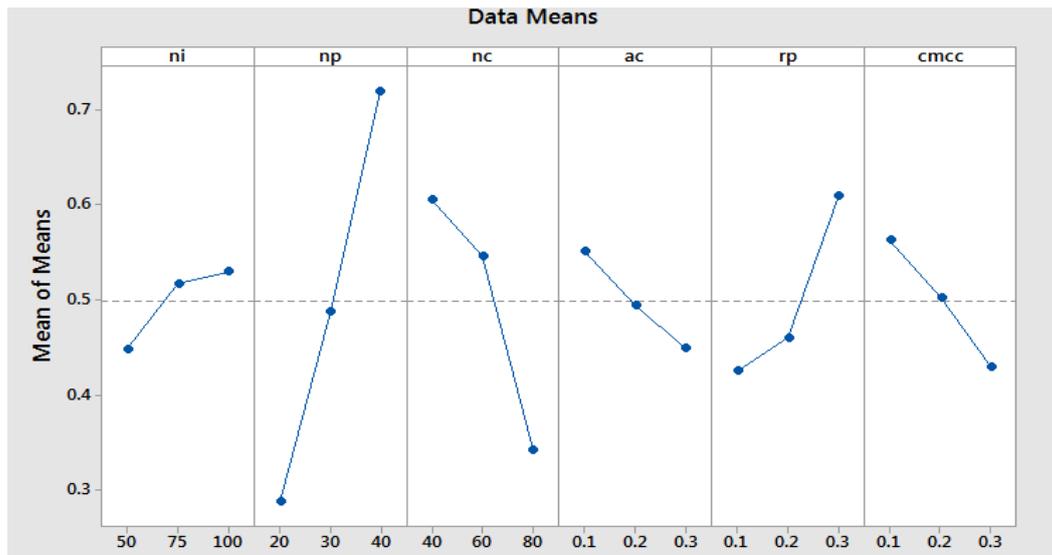


Fig. 4. Analysis diagrams of ICA parameters tuning based on Taguchi method

Table 4. Tuned value of each parameter of meta-heuristics

Algorithm	Tuned value							
	ni	np	nc	pc	mc	ac	rp	cmcc
GA	100	40	-	0.9	0.2	-	-	-
ICA	100	40	40	-	-	1	0.3	0.1

### Performance Evaluation

In this section, we compare the performance of three proposed algorithms, GA, and ICA. For this purpose, the relative increase percentage (RPI) measures the performance.

$$RPI_s = \frac{f_s - f_b}{f_b} * 100, \quad \forall s \in \{GA, DA, ICA, AGA, ADA, AICA\} \tag{34}$$

where  $f_s$  is the objective function value obtained by meta-heuristics  $s$ .  $s$  is either GA, DA, ICA, or their adaptive forms (AGA, ADA, AICA).  $f_b$  is the best objective value obtained from all algorithms.

To make our result more confident, each test instance is solved 10 times using each meta-heuristics. The RPI from the best results among 10 runs of each algorithm is reported in Table 5. Also, the RPI from the average results of 10 runs for each algorithm is in Table 6.

Table 5. RPI from the best results among 10 runs

Data set	GA	ICA
1	0.77	8.88
2	8.28	3.18
3	4.94	6.49
4	0	5.75
5	6.68	9.39
6	7.91	0.48
7	5.01	9
8	5.06	0.85
9	3.34	4.72
10	7.41	8.85
Average	4.94	5.759

Table 6. RPI from the average results among 10 runs

Data set	GA	ICA
----------	----	-----

1	2.17	1.97
2	0.95	6.28
3	1.2	3.15
4	0	8.05
5	0	4.31
6	5.61	0.65
7	4.03	8.09
8	3.17	0
9	5.11	9.38
10	4.95	6.09
Average	2.719	4.797

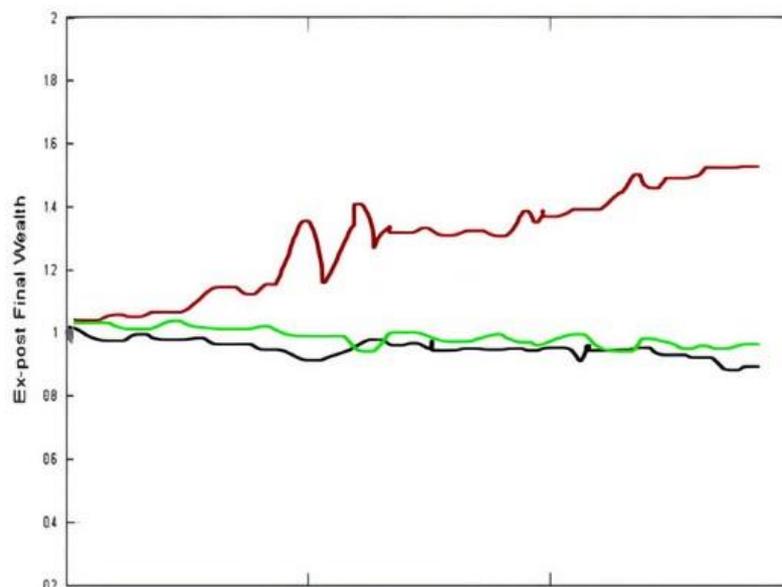
Table 6 summarizes average results of test instances. Similar to the previous explanation, it can be seen that adaptive form of proposed algorithms are superior to the simple form of the algorithms. Therefore, it can be concluded that it is better to use proposed adaptive form of these algorithms instead of their simple form to obtain high-quality solutions. Also, among adaptive form of these algorithms, it can be seen that the three algorithms work equally. Therefore, there is no differences between average results of them.

### Sensitivity Analysis

In this section, the 15 market indexes are executed using ICA and the results are discussed. In this part, an experimental analysis is provided to put the products of maximizing the diverse portfolio indices, the ex-post portfolio wealth, into an analogy. Specifically, for all trading days within the period of 2010/03/27 and 2020/06/10, a moving window of 5062 daily historical return is used for assessing the parameters of the model and, in the afterwards, creating 30000 different return scenarios for any of the portfolio elements.

The compendium of ex-post experimental comparison is explained in this section. In all of the 1250 trading days the future scenarios are generated from the vector of returns. The efficacy of substitutive performance indicators put into a comparison.

Fig. 5 depict the results of this analysis. The ex-post ultimate wealth and total return acquired by maximizing the Sharpe, Rachev, and Co-Rachev ratios. As it can be observed, only the Co-Rachev ratio can apparently be accountable for the system risk and by selecting a strategy on the ground of this measure, the greatest ultimate wealth and total return can be acquired.



**Fig. 5.** Ex-post comparison of the final wealth processes (green: Rachev, black: Sharp, Red: Co-Rachev)

For assessing the variegation and the turnover of diverse policies, both the average of the optimal portfolio weights sized within the back testing period and the ex-post wealth are reported in Table 7. It can be fairly concluded from Table 7 that the policy of using the Co-Rachev performance index has more diversification than both Rachev and Sharpe ratios. As a result, it can be concluded that the Co-Rachev racial generates a greater deal of portfolio variegation among less correlated returns.

**Table 7.** Comparison of Sharpe, Rachev and Co-Rachev average optimal weights

Index	Sharpe	Rachev	Co-Rachev
1	1.98	5.13	7.37
2	9.46	12.64	8.76
3	0.95	0.95	11.52
4	4.65	0.01	2.76
5	12.55	15.32	4.15
6	5.33	5.68	10.6
7	8.19	9.62	3.69
8	11.55	14.95	3.23
9	4.26	6.52	7.83
10	9.58	12.64	5.99
11	1.64	0.37	4.15
12	3.52	1.12	3.23
13	5.2	6.61	7.37
14	4.03	0.35	11.52
15	6.06	8.09	7.83

## Conclusion

In this paper, the importance of systemic risks is discussed and a reward-risk performance measure is borrowed from Biglova et al. [5], in which the co-movement of returns for financial indexes is considered to represent the systemic risks in the portfolio. Then three self-adaptive meta-heuristics namely GA and ICA are proposed to optimize the portfolio selection problem in order to maximize benefits and minimize risks.

Ten simulated sets of data using the method developed by Biglova et al. [5] and in accordance with the Iran Stock Exchange last 10 years' data are generated to access the performance of the proposed algorithms. To assess the performance of the algorithms, the average of the objective function (the reward-risk performance measure) values of a one-year day-by-day moving time window are considered.

Results indicated that the ICA algorithm is superior to the GA method. Thus, this algorithm is proposed to solve real-world portfolio selection problems. After determining the outstanding algorithm, the algorithm is applied to the data of 15 indexes of the Iran Stock Exchange to determine the daily portfolio for a 2-year time window and the performance of the proposed performance measure is evaluated against the Sharp ratio and the Rachev ratio. As with Biglova et al. [5], the results showed that the proposed measure can lead to a better decision for daily portfolio selection in the presence of systemic risks. Therefore, this measure is proposed to select portfolios during periods of financial instability.

Future directions for this study can be as follows. For some portfolio selection problems, there are constraints such as budget and return constraints, cardinality constraints, floor and

ceiling constraints (Di Tollo and Roli, [8]) that can be integrated with the proposed model. Also, other meta-heuristics can be tested to find out if any other meta-heuristic algorithm can outperform our ICA.

## References

- [1] Abdelaziz, F. B., Aouni, B., & El Fayedh, R. (2007). Multi-objective stochastic programming for portfolio selection. *European Journal of Operational Research*, 177(3), 1811-1823.
- [2] Abdelaziz, F. B., El Fayedh, R., & Rao, A. (2009). A discrete stochastic goal program for portfolio selection: The case of United Arab Emirates equity market. *INFOR: Information Systems and Operational Research*, 47(1), 5-13.
- [3] Abdelaziz, F. B., & Masmoudi, M. (2014). A multiple objective stochastic portfolio selection problem with random Beta. *International Transactions in Operational Research*, 21(6), 919-933.
- [4] Atashpaz-Gargari, E. and Lucas, C. *Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition*. in 2007 IEEE congress on evolutionary computation. 2007. Ieee.
- [5] Biglova, A., Ortobelli, S., & Fabozzi, F. J. (2014). Portfolio selection in the presence of systemic risk. *Journal of Indicator Management*, 15(5), 285-299.
- [6] Caçador, S., Dias, J. M., & Godinho, P. (2021). Portfolio selection under uncertainty: a new methodology for computing relative-robust solutions. *International Transactions in Operational Research*, 28(3), 1296-1329.
- [7] Chen, S., & Ge, L. (2021). A learning-based strategy for portfolio selection. *International Review of Economics & Finance*, 71, 936-942.
- [8] Di Tollo, G., & Roli, A. (2008). Metaheuristics for the portfolio selection problem. *International Journal of Operations Research*, 5(1), 13-35.
- [9] Dubois D., H. Prade, (1988). *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York.
- [10] Frej, E. A., Ekel, P., & de Almeida, A. T. (2021). A benefit-to-cost ratio based approach for portfolio selection under multiple criteria with incomplete preference information. *Information Sciences*, 545, 487-498.
- [11] Garcia, F., González-Bueno, J., Oliver, J., & Tamošiūnienė, R. (2019). A credibilistic mean-semivariance-PER portfolio selection model for Latin America. *Journal of Business Economics and Management*, 20(2), 225-243.
- [12] Gen, M. and R. Cheng, *Genetic algorithms and engineering optimization*. Vol. 7. 1999: John Wiley & Sons.
- [13] Ghahtarani, A., & Najafi, A. A. (2013). Robust goal programming for multi-objective portfolio selection problem. *Economic Modelling*, 33, 588-592.
- [14] Goldberg, D.E., *Genetic algorithms in search, optimization, and machine learning*. Addison. Reading, 1989.
- [15] Holland, J.H., Genetic algorithms. *Scientific American*, 1992. 267(1): p. 66-73.
- [16] Kocadağlı, R. Keskin, (2015). A novel portfolio selection model based on fuzzy goal programming with different importance and priorities, *Expert Syst. Appl.* 42 (20), 6,898-6,912.
- [17] Landsman, Z., Makov, U., & Shushi, T. (2018). A generalized measure for the optimal portfolio selection problem and its explicit solution. *Risks*, 6(1), 19.
- [18] Li, X., Huang, Y. H., Fang, S. C., & Zhang, Y. (2020). An alternative efficient representation for the project portfolio selection problem. *European Journal of Operational Research*, 281(1), 100-113.
- [19] Li, X., Wang, Y., Yan, Q., & Zhao, X. (2019). Uncertain mean-variance model for dynamic project portfolio selection problem with divisibility. *Fuzzy Optimization and Decision Making*, 18(1), 37-56.

- 
- [20] Li, T., Zhang, W., & Xu, W. (2015). A fuzzy portfolio selection model with background risk. *Applied Mathematics and Computation*, 256, 505-513.
- [21] Li, B., Zhu, Y., Sun, Y., Aw, G., & Teo, K. L. (2018). Multi-period portfolio selection problem under uncertain environment with bankruptcy constraint. *Applied Mathematical Modelling*, 56, 539-550.
- [22] Liu B., Y. Liu, (2002). Expected value of fuzzy variable and fuzzy expected value models, *IEEE Trans. Fuzzy Syst.* 10, 445–450.
- [23] Liu Y., B. Liu, (2003). A class of fuzzy random optimization: expected value models, *Inf. Sci.* 155 (1), 89–102.
- [24] Masoudi, M., & Abdelaziz, F. B. (2018). Portfolio selection problem: a review of deterministic and stochastic multiple objective programming models. *Annals of Operations Research*, 267(1), 335-352.
- [25] Mirjalili, S. (2016). Dragonfly algorithm: a new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems. *Neural Computing and Applications*, 27(4), 1053-1073.
- [26] Mokhtarzadeh, M., Tavakkoli-Moghaddam, R., Triki, C., & Rahimi, Y. (2021). A hybrid of clustering and meta-heuristic algorithms to solve a p-mobile hub location–allocation problem with the depreciation cost of hub facilities. *Engineering Applications of Artificial Intelligence*, 98, 104121.
- [27] Markowitz H., (1952) Portfolio selection, *J. Finance* 7 (1) 77–91.
- [28] Markowitz H. (1959). *Portfolio Selection: Efficient Diversification of Investments*, John Wiley & Sons.
- [29] Montajabiha, M., Khamseh, A. A., & Afshar-Nadjafi, B. (2017). A robust algorithm for project portfolio selection problem using real options valuation. *International Journal of Managing Projects in Business*.
- [30] Najafi, A. A., & Pourahmadi, Z. (2016). An efficient heuristic method for dynamic portfolio selection problem under transaction costs and uncertain conditions. *Physica A: Statistical Mechanics and its Applications*, 448, 154-162.
- [31] Nahmias S., (1978). Fuzzy variables, *Fuzzy Sets Syst.* 1 (2) (1978) 97–110.
- [32] Rabbani, M., Heidari, R., & Farrokhi-Asl, H. (2018a). A bi-objective mixed-model assembly line sequencing problem considering customer satisfaction and customer buying behaviour. *Engineering Optimization*, 50(12), 2123-2142.
- [33] Rabbani, M., Mokhtarzadeh, M., & Farrokhi-Asl, H. (2018b). A new mathematical model for designing a municipal solid waste system considering environmentally issues. *International Journal of Supply and Operations Management*, 5(3), 234-255.
- [34] Relich, M. (2021). *Model for Formulating Decision Problems Within Portfolio Management. Decision Support for Product Development* (pp. 27-50). Springer, Cham.
- [35] Roy A.D. (1952) Safety-first and holding of assets, *Economics* 20, 431–449.
- [36] Schroeder, P., Kacem, I., & Schmidt, G. (2019). Optimal online algorithms for the portfolio selection problem, bi-directional trading and-search with interrelated prices. *RAIRO-Operations Research*, 53(2), 559-576.
- [37] Taguchi, G. (1986). *Introduction to quality engineering: designing quality into products and processes* (No. 658.562 T3).
- [38] Xu J., X. Zhou, S. Li. (2011). A class of chance constrained multi-objective portfolio selection model under fuzzy random environment, *J. Optim. Theory Appl.* 150 (3), 530–552.
- [39] Zadeh L.A. (1965). Fuzzy sets, *Inf. Control* 8, 338–353.
- [40] Zhang, Y., Gong, D. W., Sun, X. Y., & Guo, Y. N. (2017). A PSO-based multi-objective multi-label feature selection method in classification. *Scientific reports*, 7(1), 1-12.
- [41] Zhao, P., & Xiao, Q. (2016). Portfolio selection problem with Value-at-Risk constraints under non-extensive statistical mechanics. *Journal of computational and applied mathematics*, 298, 64-71.
- [42] Zhou, X., Wang, J., Yang, X., Lev, B., Tu, Y., & Wang, S. (2018). Portfolio selection under different attitudes in fuzzy environment. *Information Sciences*, 462, 278-289.



This article is an open-access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license.