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RESEARCH PAPER

Optimizing a Hub Arc Location Problem with Set-up Cost and Isolated Hub Nodes

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Abstract

Efficient flow management is critical in transportation and logistics, with hub networks playing a key role in optimizing these processes. The hub arc location problem has recently emerged as a new framework that emphasizes hub arcs while allowing for isolated hubs. This paper extends the hub arc location problem by incorporating set-up costs into the optimization model. A heuristic algorithm is developed to enhance hub network design, considering both the flow of goods and the associated hub set-up costs. Additionally, a detailed sensitivity analysis is conducted to assess the impact of strategic adjustments on optimization outcomes. By reducing the discount factor for inter-hub flows and increasing the number of exogenous hub arcs, significant improvements in route optimization and cost reduction are achieved. This research challenges traditional approaches to hub network design and opens the door for further exploration of the dynamics within hub networks. A deeper understanding of these networks can lead to more efficient and resource-optimized transportation systems, potentially transforming flow management into a more cost-effective and sustainable process.

Keywords:
Hub Arc Location; Hub Network; Greedy Heuristic Algorithm; Isolated Hub

Introduction

Hub networks play an important role in communications between origins and destinations through the hub facilities (such as transportation terminals, sorting centers, etc.) by collecting (i.e. sending the flow from the origin to its allocated hub node), transmitting (i.e. passing flow between hubs) and distributing flows (i.e. sending the flow from the hub node to the destination node) across the network (Rekabi, Sazvar et al. 2024). Each of these transmissions would impose a cost on the distribution system, depending on the distance, the volume of the flow, and the type of transportation vehicle (Farahani, Hekmatfar et al. 2013). Each hub location problem has three main elements: non-hub nodes (which are the origin or destination of the flow), hub nodes, and flows. Each non-hub node is connected to at least one hub and there is at least one route between every two hub nodes (Sener and Feyzioglu 2023). Considering setup costs in hub network problems is crucial for optimizing the overall cost of establishing and

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maintaining the network (Blanco, Fernández et al. 2023). By factoring in setup costs, decision-makers can make informed choices that balance the initial investment required to set up hubs with ongoing operational costs, leading to cost-effective network configurations (Hu, Liu et al. 2024). Ignoring setup costs may result in suboptimal solutions that do not accurately reflect the real-world expense of network establishment. By including setup costs, a more efficient and realistic model can be developed, fostering better resource allocation and network design decisions (Chanta, Sangsawang et al. 2024).

In most classical location problems, a discount factor is considered for all hub arcs to create economies of scale benefits for flows on hub arcs (Sener and Feyzioglu 2023). This assumption may lead to unrealistic results, as optimal solutions in hub location problems may result in flows on hub arcs that are significantly smaller compared to arcs between a hub and a non-hub, while the discount factor is only considered between hub points (Ghaffarinasab and Kara 2022). On the other hand, the assumption of complete connectivity of hub points in most classical hub location problems also imposes additional constraints on such models, and while simplifying network design and flow routing, it imposes a specific cost structure and topology that may not be desirable in real conditions (Nasiri, Khaleghi et al. 2023). Unlike air transportation networks where direct flights exist among all hubs to prevent passenger time wastage, large transportation networks and long-haul communication networks are exceptions to this rule. Therefore, in practice, most hub networks do not have complete connections between hub nodes (Atay, Eroglu et al. 2023).

Studying the hub arc location problem with set-up cost and isolated hub nodes is essential in the field of operations research and logistics because it addresses critical challenges in optimizing transportation networks. The incorporation of set-up costs adds a realistic dimension to the problem, reflecting the expenses incurred in establishing transportation hubs. Moreover, the presence of isolated hub nodes introduces complexities that require innovative solutions to ensure efficient connectivity within the network. By studying this problem, we can develop strategies to minimize costs, enhance network resilience, and improve overall system performance. Therefore, this paper involves the intricacies of this problem for advancing logistics and supply chain management practices.

The continuation of this article is structured as follows. In the second section, a review of the literature on the topic has been presented, highlighting gaps and innovations. In section 3, a mathematical model has been formulated and presented. Section 4 elaborates on the solution method and the greedy algorithm. Sections 5 and 6, respectively, encompass computational results and sensitivity analysis of the model. Section 7 mentions the results and the future research direction.

Literature Review

Hub networks have the advantage of transmitting flows with economies of scale through consolidation of flows and this leads to service level improvement. This concept is indicated by an inter-hub discount factor, which ranges from 0 to 1. The discount is applied to all inter-hub connections, regardless of the amount of transferring flow. This inter-hub discount factor reduces the cost of conveying flows across the hub arcs and creates a motivation for bundling the flows (O'Kelly and Bryan 1998).

Generally, studies on the classic hub location problems are mainly categorized as follows (Alumur, Campbell et al. 2021).

- *Hub-covering problems*: In this class of problems, the demand will be covered if it is within a specific distance from its servers. This kind of problem is studied in the form of two main problems, as hub covering and maximum hub covering problem.
- *P-hub center problems*: In this category of problems, non-hub nodes are assigned to the P

number of hubs in such a way that the maximum time or distance between each pair of origin and destination in the network is minimized.

- *P-hub median problems*: In this type of problems, *P* (the number of hubs) is exogenous and non-hub nodes are allocated to the hubs in such a way that the total cost of transportation in the network is minimized. Transportation cost from origin *i* to destination *j*, when passing through the hub nodes *k* and *l*, is $C_{ijkl} = C_{ik} + \alpha C_{kl} + C_{lj}$, and α ($0 \leq \alpha < 1$) is the discount factor between the hubs.

All classic hub location problems include two simplifying assumptions: (1) a fixed discount factor is used on all inter-hub connections to reduce the cost of transferring flow on them. (2) All hub nodes are fully interconnected. (Campbell, Ernst et al. 2005) introduced the Hub Arc Location Problems which relax these unrealistic. The first one is the use of a fixed discount factor (independent of the flow) on all connections between each pair of hub nodes. This assumption may lead to unrealistic results, because the optimal solutions to the hub location problem may yield much smaller flows on hub arcs than flows between a hub and a non-hub node, while the discount factor is considered only on the flows between the hub nodes (Campbell and O'Kelly 2012). The second restriction is assuming a fully connected graph of hub nodes that would simplify the network design and the routing of flows while imposing an extra cost, which is not desirable in real conditions (Campbell, Stiehr et al. 2003). Unlike some of the air transportation networks that contain direct flights between all hubs in order to avoid passengers' time-wasting, major transportation and telecommunication networks do not contain full connectivity between the hub nodes in practice.

Releasing the hub network's full connectivity assumption was the first challenge that was discussed in the literature. (Chou 1990) proposed a hierarchical-hub model for airline networks by using a spanning-tree network to overcome this restriction. (Jaillet, Song et al. 1996) considered an integer linear programming and a heuristic approach to model and solve flow-based models of capacitated airline networks. Another model was proposed by (Nickel, Schöbel et al. 2001), who studied urban traffic networks and applied shortest path algorithms to solve it. Also, the advantages of considering isolated hubs were examined by (Gelareh and Nickel 2011), (Korani and Eydi 2021) and (Atay, Eroglu et al. 2023). More recently, Wu, Qureshi et al. (2024) investigated a hub routing problem and developed a branch and cut algorithm for solving the model. They also applied the model n an Australian post dataset to prove their research application and superiority. Arbabi, Nasiri et al. (2021) presented a hub and spoke architecture including a distribution center and different cross-docks. They also defined numerous heuristic algorithms to solved the proposed model.

Taking into account different forms of incomplete graphs of hub networks was another idea for relaxing the aforesaid assumption. In this regard, a single allocation of incomplete hub networks was applied in 2009 by (Alumur, Kara et al. 2009). In addition, (Karimi and Setak 2014) offered a model for multiple allocations of incomplete hub location problems considering routing costs and developed its lower bounds using the Lagrangian relaxation approach and valid inequalities. (Contreras, Tanash et al. 2017) suggested a cycle hub location problem and provided an exact and heuristic approaches for this type of hub graphs in another paper. Also, a tight bound for a path-based formulation was considered for the tree of hub location problem (Contreras, Fernández et al. 2010), (Oliveira, de Sá et al. 2022), (Fernández and Sgalambro 2020), (Bütün, Petrovic et al. 2021), and its valid inequity was proposed in another paper (Contreras, Fernández et al. 2010). Similar models using a tree of hub location was considered and solved by the minimum spanning (Mohajeri and Taghipourian 2011) and improved Benders decomposition algorithm (de Sá, de Camargo et al. 2013), (Ramamoorthy, Vidyarthi et al. 2024) and (Muffak and Arslan 2023), respectively. Also, a new multi-objective model was presented to design a multi-modal tree hub location network under uncertainty and it was solved by a multi-objective imperialist competitive algorithm (Sedehzadeh, Tavakkoli-Moghaddam et al.

2016). The hub line location problem was another form of the incomplete hub network which was developed by Martins et al. and was solved by a Benders decomposition algorithm (Martins de Sá, Contreras et al. 2015).

The second challenge of the literature was relaxing the discount factor restriction. In order to overcome this shortcoming, concave costs and flow-dependent costs (O'Kelly and Bryan 1998) were proposed and were solved by a piecewise linear function. Bryan and O'Kelly considered a minimum threshold on a capacitated network, and (Klincewicz 1998) used the same model with exogenous hub number on an uncapacitated network. (Horner and O'Kelly 2001) introduced a nonlinear cost function, and intermodal freight hubs with a concave cost function were investigated by (Racunica and Wynter 2005). In other papers, A threshold-based discounting was conducted (Podnar, Skorin-Kapov et al. 2002), and a piecewise cost function was applied to the hub network formulation to help using discount factor for all routes (Kimms 2006). Finally, a tighter uncapacitated formulation using Benders decomposition was illustrated by considering released discount factor assumption (De Camargo, de Miranda Jr et al. 2009).

In an important paper, (Campbell, Ernst et al. 2005) introduced four types of Hub Arc Location Problems and released both of the aforesaid restricting assumptions simultaneously. They studied the definition and concept of these models and compared the results with classic hub median problems. In another paper, authors provided hub arc location integer programming formulations and benefited from an enumeration based algorithm to solve them (Campbell, Ernst et al. 2005). Authors then applied the first type of this classification of hub location problems on a cluster of workstations (Campbell, Stiehr et al. 2003) and used a parallel implementation of enumeration algorithm to study the United States air transportation problem and postal operations in Australia. Then (Campbell 2009) used the first type of the hub arc location problem to formulate a time-definite transportation network while considering service level constraints. The author compared the result with optimal solutions of multiple allocation hub median problems and indicated that by increasing the service level, hub arcs become shorter in length and the location of hub nodes become more centralized. Then, (Sasaki, Campbell et al. 2009), Taherkhani, Alumur et al. (2021), Ghaffarinasab (2022) and Sener and Feyzioglu (2023) considered the hub arc location under the condition of competition. (Roozkhosh and Motahari Farimani 2023) designed a hub location-allocation problem. They tried to add tardiness and earliness while investigating uncertainty in this research.

In this paper, we take advantage of Campbell's hub arc location problem and develop and optimize it considering isolated hub nodes and set-up cost to approach real-world conditions. Due to the fact that there is just one small-scale enumeration-based solution method (Campbell, Ernst et al. 2005) in the hub arc location literature, we solve this type of hub location problem in large-scale by a greedy heuristic approach for the first time in the literature.

This study aims to address the limitations present in classical hub location problems by exploring a hub-location problem that relaxes restrictive assumptions, leading to enhanced results that better align with real-world scenarios. By deviating from traditional methodologies, we cover the gap between theoretical models and practical applications, offering a more comprehensive and adaptable solution. The primary innovation of our research lies in the inclusion of isolated hubs alongside connected hub facilities within the hub location problem framework. Isolated hubs, despite not being directly connected to other hubs, play a pivotal role in optimizing routing efficiency and facilitating seamless flow transfer, ultimately improving service levels by establishing more efficient origin-destination routes.

Furthermore, our study breaks new ground by successfully solving a deterministic model on a large scale, a feat not previously explored in the literature. To tackle this challenge, we developed and implemented a heuristic Greedy algorithm coupled with local search techniques, as well as a Genetic algorithm. The innovative use of these methods allowed us to achieve efficient solutions while balancing solution time and accuracy. To validate the efficacy of our proposed solution approaches, we conducted extensive comparative analyses. This involved benchmarking the results obtained against exact solutions on a smaller scale and comparing them with lower bound solutions on a larger scale. Through rigorous experimentation and benchmarking, we were able to demonstrate the effectiveness and practicality of our novel approach in tackling complex hub location problems.

Overall, by combining theoretical advancements with practical applicability, our study introduces a new perspective to hub location optimization, delivering more robust and adaptable solutions in transportation and logistics management.

Problem Definition

As mentioned in the previous section, hub arc location models are discussed to resolve the shortcomings of the classic hub median location problems. Generally, in hub location problems, all origin-destination routes contain at least one hub node. This assumption states that (with the triangle inequality theorem) there is no direct link between every two non-hub origins and destinations (Campbell, Ernst et al. 2005).

Moreover, in the classic hub location problems, there is an assumption stating that the unit flow cost between all pairs of hubs uses a discount factor, α ($0 \leq \alpha < 1$). However, in the hub arc location models, the latter assumption is released. Thus, this would not necessarily lead to a fully connected graph of hub nodes and would utilize the hub arcs wherever is appropriate. According to the above description, an alternative assumption in the hub arc location problems suggests that the unit flow cost on the hub arcs uses a discount factor, α ($0 \leq \alpha < 1$). So given that q is the number of hub arcs, for every $q > 0$, the possible number of hubs is achieved by, $p^2 - p >= 2q$ and $p <= 2q$, (As a hub arc can be generated by connecting two hubs. So, the number of hubs in the maximal form is less than $2q$. In addition, sometimes these hubs may be shared, meaning hub number 1 connects two other hubs, reducing the count from $2q$ to less than p) i.e.

$$\left\lceil \frac{1 + \sqrt{1 + 8q}}{2} \right\rceil \leq p \leq 2q \tag{1}$$

The assumptions of the proposed multiple allocation hub arc location model in this study are as follows:

- The network includes N nodes and there is a flow between each origin-destination by a many-to-many relationship. The distances between nodes, that follow triangle inequality theorem, are defined as costs.
- The number of hub arcs and a maximum number of hub nodes on the network are exogenous.
- Set-up costs for all hub facilities are considered and added to the total cost.
- The hub node capacity is assumed unlimited.
- The origin-destination route contains at least one and at most two hub facilities.
- The flow cost on the hub arcs is discounted by an α factor ($0 \leq \alpha < 1$).
- The isolated hubs, that are not adjacent to any hub arc, are allowed in the network.
- The objective is locating q hub arcs and a maximum number of p hub nodes so that the total cost of transportation and establishment of the hub facilities is minimized.

Mathematical Formulation

The hub arc location problem is designed to double the service levels in the p-hub median problem by limiting each origin-destination route to a maximum of one hub arc. So, each origin-destination route is limited to a maximum of three arcs, that the middle one if exists, is a hub arc. In airlines' network design, this issue ensures that passengers do not change more than two flights. Therefore, in this paper, a mixed-integer linear formulation for hub arc location problems is developed by adding set-up cost and considering isolated hub nodes.

Consider the graph, $G=(V, E)$ which, $V=1,\dots, N$ is the vertex set and E is the set of possible arcs between each node. The demand or flow between node o and node d is W_{od}, and the cost of travel from node i to node j is dis_{ij}. The hub arcs, that connect two hub nodes k and l, have αdis_{kl} discounted costs per flow unit.

Indices

o: set of origin nodes
d : set of destination nodes
i : set of the first hubs
j : set of the second hubs (if needed)

Parameters

W_{od} : flow between origin and destination
dis_{ij} : the distance between node i and j
f_i : set-up cost for hub i
α : discount factor between two hubs
q : number of hub arcs
p : the maximum number of hubs

Decision variables

X_{ijod}: If the flow from origin o to the destination d passes through hub i and hub j, 1; otherwise, 0.
Y_{ij}: If there is a hub arc between hub i and hub j, 1; otherwise, 0.
Z_i: If the node i, is a hub node, 1, otherwise, 0.

$$Min \sum_{\substack{o,d,i,j \\ o<d}} (W_{od}+W_{do})\left(dis_{oi}+\alpha dis_{ij}+dis_{jd}\right)X_{ijod}+\sum_i f_i Z_i \quad (2)$$

St.

$$\sum_i \sum_j X_{ijod}=1, \quad \forall o,d,(\,o<d) \quad (3)$$

$$\sum_i \sum_{\substack{j \\ j<i}} Y_{ij}=q \quad (4)$$

$$\sum_i Z_i \le p \quad (5)$$

$$X_{iiod}+\sum_{\substack{j \\ j\neq i}} (X_{ijod}+X_{jiod}) \le Z_i, \quad \forall i,o,d,(\,o<d) \quad (6)$$

$$X_{ijod} \le Y_{ij}, \quad \forall i,j,o,d,(\,o<d),(\,i\neq j) \quad (7)$$

$$Y_{ij} \le Z_i, \quad \forall i,j,(\,i\neq j) \quad (8)$$

$$Y_{ij} \le Z_j, \quad \forall i,j,(\,i\neq j) \quad (9)$$

$$X_{ijod} \ge 0, \qquad \forall i,j,o,d \quad (10)$$

$$Z_i, Y_{ij} \in \{0,1\}, \qquad \forall i,j \quad (11)$$

The objective function is to minimize the total costs of transportation and set-up costs. Constraints 3 ensure that flow between every pair of origin i and destination d passes through a pair of hubs (hub i and hub j) or possibly a single hub X_{iiod}, and direct paths between origins and destinations are not allowed. Constraints 4 and 5 are related to the exogenous number of hub arcs and the maximum number of hub nodes, respectively. Constraints 6 make sure that hub i is open for each path between pair of origin i and destination d that uses this hub. Constraints 7 mean that for each flow transmission between pair of origin i and destination d through a hub arc i-j, that hub arc is open. Constraints 8 and 9 ensure that the two ends of a hub arc should be hub nodes. Finally, Constraints 10 and 11 describe the characteristics of decision variables.

It should be noted that the routing decision variable X_{ijod} is formulated as a continuous variable, but it is binary in nature. Because when a set of open hub nodes and hub arcs became known, the optimal path between each pair of origin-destination is the route with the least transportation cost. Since the hub facilities do not have a capacity limitation, only one path for current routing will be selected. Therefore, despite the continuousness of the decision variable, X_{ijod}, the formulation of the problem forces it to take a zero or one value.

Unlike the original model provided by (Campbell, Ernst et al. 2005), the proposed formulation in this paper for hub arc location problem includes set-up cost for hub facilities and opens isolated (or individual) hubs in the network. Isolated hubs are not connected to any hub arcs and they play a very important role in transferring flows. They provide sorting, switching and connecting functions to relate origins and destinations with fewer vehicles and routes than it would be needed with a direct linkage between every origin-destination pair. Moreover, utilizing an isolated hub facility may lead to a lower cost than establishing a hub arc to link some origins and destinations with less flow in the network. In other words, we would achieve better service levels and more reasonable network structure through shorter origin-destination routes by using isolated hubs (Yang, Liu et al. 2011). Obviously, if $q \leq p/2$, there will be at least one isolated hub facilities in the network. The effect of using isolated hub nodes will be studied in the sensitivity analysis section by comparing the optimum solution of a hub network with and without isolated hubs.

Solution Approach

Hub location problems are classified as NP-Hard categories and even if the location of hub nodes are known, still the allocation parts remain NP-Hard (Rabbani, Zameni et al. 2013). This complexity arises due to simultaneously considering the facility location problem and the quadratic assignment problem. Due to the fact that these two issues are very difficult individually, combining them makes the hub location problems more complicated. Therefore, a heuristic method is developed to solve large-scale problems.

The general approach of this greedy heuristic method is based on a proposed distance/flow scale and then finding the least cost network structure according to allocating non-hubs to the hubs through shortest distance logic (see **Fig** 1). According to the proposed scale, the nodes are sorted in descending order. The minimum number of required nodes for forming q hub arcs (see equation 1) is selected from the sorted sequence as a set of hub nodes. Then q arcs with the larger flows are selected from possible arcs. These arcs are formed by dual combinations of selected hub nodes. Now the total cost is calculated according to the logic of "the shortest distance" by assigning non-hub nodes to the hubs so that the optimal path between each origin-destination is the path of the least distance.

The next step is shifting members of the selected hub arcs with unselected possible arcs. The set with the greatest improvement in the objective function is selected. To reduce the number of calculations for $q>3$, the two-stage displacement rule is proposed as below:

Rule #1: Only the members of the new set of hub arcs that the flows through them are less than A are replaced by the previous set of hub arcs that are obtained from Rule #2.

A = The total flow through the first member of the previous set of hub arcs × (α)

Rule #2: Only the members of the previous set of hub arcs that the flows through them are more than B, are replaced by the new set of hub arcs that are obtained from Rule #1.

B = The total flow through the last member of the previous set of hub arcs × (α)

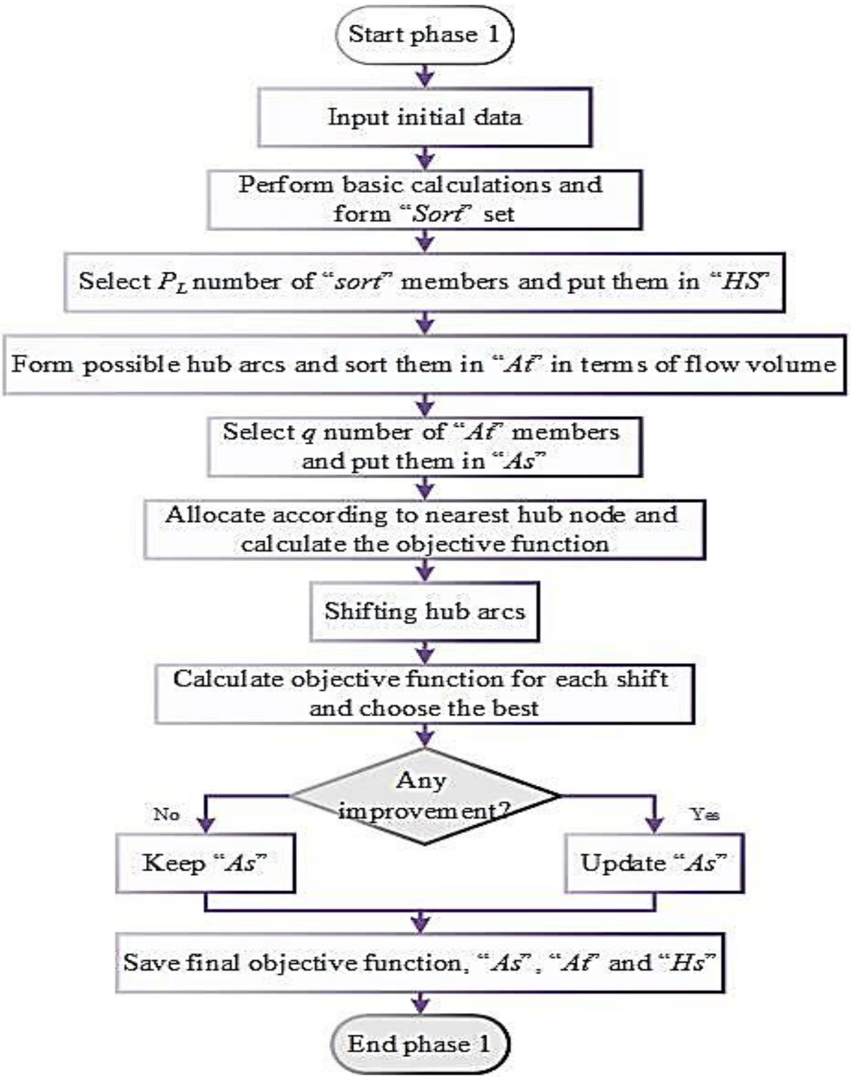


Fig 1. Phase 1 of the greedy heuristic method

Then, as long as the number of hub nodes has not reached the maximum of *p*, the next node is added to the set of selected hubs. Initially, the new hub node is entered into the model individually (as an isolated hub). If it makes an improvement, possible arcs between the new hub and the previously selected hubs are formed. After calculating the current flow on the new arcs and shifting them respectively with previous hub arcs, the best answer is selected and both sets of hub node and hub arc will be updated. If there is no improvement after the shifting phase, the previous answer is hold and the new node is added to the set of the isolated hubs (hubs that are not connected to any hub arc).

If the maximum number of hub nodes, *p*, is reached, a new hub node is shifted with existing isolated hubs (if any). If it makes an improvement, the set of isolated hubs will be updated. The stopping criterion is adding all the existing nodes to the algorithm.

In the following, symbols and parameters that are used in this heuristic algorithm are explained and the steps are described in the flowchart.

- The scale for sorting nodes: Base on this scale, all of the nodes are arranged in descending order.

$$\text{Sorting Ratio} = \frac{\sum_{j=1}^{n} (W \times dis)_{ij}}{F_i} \tag{12}$$

- Set “*Hs*”: selected hub nodes
- Set “*As*”: previously selected hub arcs
- Set “*At*”: a set of possible new hub arcs
- Set “*isolated*”: a set of isolated hubs

Proposed greedy heuristic algorithm’s steps

Proposed greedy heuristic algorithm’s steps can be summarized as follows (see **Fig** 2):

Step 1: Form the sequence “*Sort*” and put the minimum number of hub nodes (P_L) in the set “*Hs*”.

Step 2: Form binary combinations from the members of “*Hs*”, calculate the flow passing through them and sort them in descending order in the set “*At*”.

Step 3: Insert q number of “*At*” members to the set “*As*” and calculate the objective function with “*As*” hub arcs and the hub nodes on the ends of them.

Step 4: Replace the remaining unelected arcs of the “*At*” with the selected arcs of "*As*”, one by one, by using displacement law.

Step 5: Calculate the objective function for each replacement, choose the best set of hub arcs and hub nodes and update the sets if there is an improvement.

Step 6: Select a new node if there is an unexplored node in the sequence of "*Sort*”, otherwise, stop.

Step 7: If the number of “*Hs*” members does not reach to the maximum number of p, add the selected nodes from sixth step as an isolated hub to the sets “*Hs*” and “*Isolated*”, and calculate the objective function with previous set of hub arcs and new isolated hubs, otherwise go to the step twelfth.

Step 8: If there is no improvement, remove the last added node to the set “*isolated*” and “*Hs*” and return to Step Six. Otherwise, go to the next step.

Step 9: Form the possible hub arcs with the newly added hub node, calculate passing flow through them and sort them in descending order in the set “*At*”.

Step 10: Replace the arcs in “*At*”, with the selected arcs in “*As*”, one by one, using the aforesaid “displacement law”.

Step 11: Calculate the objective function for each replacement, choose the best set of hub arcs and hub nodes, update the sets if there is an improvement, and return to Step Six.

Step 12: If the set “*isolated*” is not empty, replace the newly added member with every member of “*isolated*” set one by one, calculate the objective function and choose the best set of hubs and hub arcs. Otherwise, go to step fourteenth.

Step 13: If there is an improvement, update sets of “*Hs*” and “*isolated*” and go to the ninth step. Otherwise, go to step eight.

Step 14: Form binary combinations with the newly added member and the members of “*Hs*”, calculate the passing flow through them and sort them in descending order in “*At*”.

Step 15: Replace the arcs in “*At*” with the previously selected arcs in “*As*”, one by one, using the “displacement law”. In this step, it should be noted that in every movement, by eliminating any arc of “*As*”, the related hub nodes should be removed from “*Hs*” to keep the number of “*Hs*” members equal to p. If the number of “*Hs*” members is less than p, between two deleted hub nodes, add the node with higher priority in the “*sort*” sequence as an isolated hub in the set "*isolated*”.

Step 16: Calculate the objective function for each replacement, choose the best set of hub arcs and hub node and update the sets if there is an improvement. Back to Step Six.

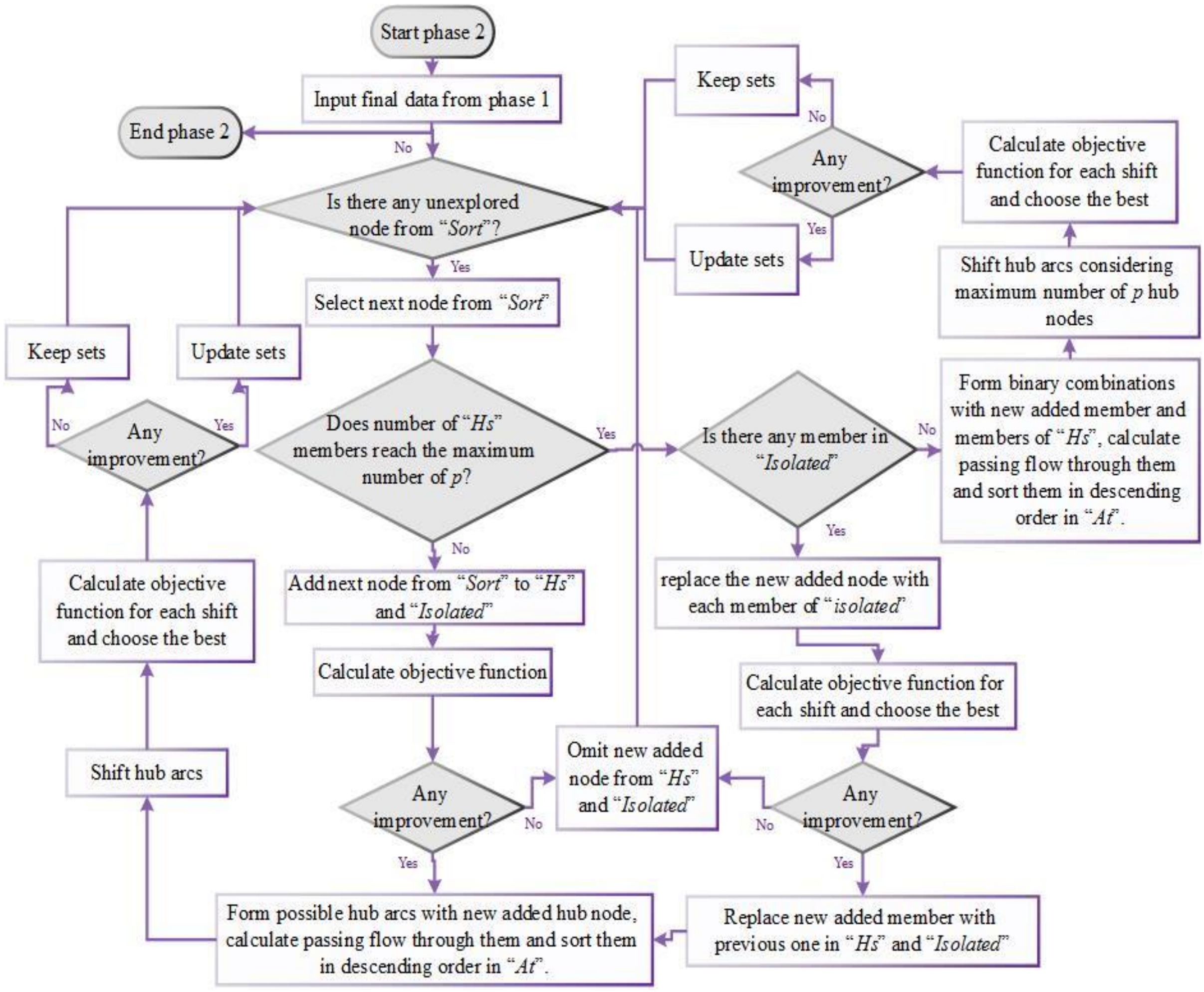


Fig 2. Phase 2 of the greedy heuristic method

Computational results

The proposed hub arc location problem is solved by the aforesaid greedy heuristic algorithm in this section. So, the performance of the proposed heuristic algorithm will be evaluated in terms of the obtained solution's quality and run time. In order to validate the results, the obtained outcomes are compared to the exact optimal solution from *GAMS* optimization software in terms of average error and runtime. Finally, in order to analyze the sensitivity of the proposed formulation, a hub arc location with and without isolated hub nodes are compared and the sensitivity of the obtained results are also analyzed with regards to the changes in discount factor α and the number of hub arcs.

To ensure a robust evaluation of the proposed solution method, we conducted testing across a diverse set of sample problems. Specifically, we utilized four sample problems sourced from the widely recognized AP data sets commonly employed in hub location research studies. In gathering the data, we accessed the AP data sets containing flow matrices that are not symmetric (i.e., Wij ≠ Wji) and where the flow from each node to itself may not necessarily be zero, as previously noted by (Abyazi-Sani and Ghanbari 2016). These distinctive characteristics of the AP data set provided a suitable foundation for testing the performance of our solution method across various scenarios. The data analysis involved rigorous examination of the sample problems, incorporating the unique features of the AP data sets to assess the efficacy of our proposed method. By applying our solution method to these selected problems, we were able to evaluate its performance, efficiency, and applicability in real-world scenarios. Furthermore, the management of the data involved ensuring consistency, accuracy, and integrity throughout the analysis process to maintain the credibility and reliability of our findings.

The proposed heuristic algorithm is implemented in *Matlab* software R2014b and exact solutions are obtained from *GAMS* software 24.1.2 *Cplex* solver. All of the calculations related to the sample problems have been done on a computer with specs *Intel Core i7-2670 QM 2.2 GHz* Memory *8 GB*.

The performance of the proposed heuristic algorithm for a discount factor of 0.4, 0.6 and 0.8, on a small-scale and large-scale problem, is presented in **Table 1** and **Table 2**, respectively. It should be mentioned that the time limit of *GAMS* software for the exact solution is set to 55000 seconds, so the best possible solution of *GAMS* is reflected in the table for the size *N*=55, with a relative gap of 0.04%.

Table 1. Relative deviation and run times of small-scale problems

		GAMS Solution	Greedy Heuristic Solution	
α=0.4		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=5	*q*=1, *p*=2	0.04	0.00	**0.14**
N=10	*q*=3, *p*=4	0.25	0.00	**0.24**
N=15	*q*=4, *p*=6	1.34	0.24	**0.92**
N=20	*q*=5, *p*=8	8.59	1.70	**3.78**
N=25	*q*=6, *p*=10	27.53	1.90	**11.51**
Average		**7.55**	**0.76**	**3.31**
		GAMS Solution	**Greedy Heuristic Solution**	
α=0.6		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=5	*q*=1, *p*=2	0.04	0.00	**0.15**
N=10	*q*=3, *p*=4	0.27	0.00	**0.25**
N=15	*q*=4, *p*=6	1.34	0.00	**0.94**
N=20	*q*=5, *p*=8	8.42	0.81	**3.75**
N=25	*q*=6, *p*=10	48.16	0.44	**9.78**
Average		**11.64**	**0.25**	**2.97**
		GAMS Solution	**Greedy Heuristic Solution**	
α=0.8		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=5	*q*=1, *p*=2	0.04	0.00	**0.15**
N=10	*q*=3, *p*=4	0.19	0.00	**0.24**
N=15	*q*=4, *p*=6	2.23	0.98	**0.86**
N=20	*q*=5, *p*=8	7.52	0.72	**3.68**
N=25	*q*=6, *p*=10	31.31	0.75	**10.40**
Average		**8.25**	**0.49**	**3.06**
Total Average		**9.15**	**0.5**	**3.11**

Table 2. Relative deviation and run times of large-scale pronlems

		Gams Solution	**Greedy Heuristic Solution**	
α = 0.4		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=30	*q=7, p=12*	247.30	1.26	**21.93**
N=35	*q*=8, *p*=14	586.28	0.24	**55.16**
N=40	*q*=9, *p*=16	640.27	1.82	**113.44**
N=45	*q*=10, *p*=18	1566.63	1.51	**180.79**
N=50	*q*=11, *p*=20	55000.00	0.96	**349.41**
Average		**11608.10**	**1.15**	**144.14**
		Gams Solution	**Greedy Heuristic Solution**	
α = 0.6		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=30	*q=7, p=12*	270.98	1.04	**23.19**
N=35	*q*=8, *p*=14	361.19	0.36	**60.75**
N=40	*q*=9, *p*=16	503.95	1.72	**101.37**
N=45	*q*=10, *p*=18	1382.77	1.02	**183.36**

		Gams Solution		Greedy Heuristic Solution
N=50	*q*=11, *p*=20	55000.00	0.84	**335.38**
Average		**11503.77**	**0.99**	**140.81**
α **= 0.8**		**Average CPU Time (s)**	**% Gap Average**	**Average CPU Time (s)**
N=30	*q=7, p=12*	211.23	1.16	**24.98**
N=35	*q*=8, *p*=14	560.36	0.94	**57.61**
N=40	*q*=9, *p*=16	682.42	1.06	**103.59**
N=45	*q*=10, *p*=18	1179.94	0.53	**183.08**
N=50	*q*=11, *p*=20	55000.00	1.25	**344.05**
Average		**11526.79**	**0.98**	**142.66**
Total Average		**11546.22**	**1.04**	**142.53**

According to the computational results, by increasing the size of the problem, the optimal solution's runtime increases exponentially that confirms that this problem is NP-Hard. The average error of the heuristic algorithm on the small-scale problem is equal to 0.5% on the average runtime of 3.11 seconds, while the exact solution from GAMS software is achieved on average run-time of 9.15 seconds. Also, the performance of the proposed heuristic algorithm on the large-scale problems is acceptable due to the average deviation of 1.04% on the average run-time of 142.53 seconds, while it takes an average of 11546 seconds for GAMS software to result in the exact solution. The achieved results indicate the ability of the heuristic algorithm and confirm that it is able to eventuate in satisfactory answers in appropriate runtimes. **Fig 3** compares the trends of the exact and heuristic solution and indicates that the heuristic algorithm's runtime is preferable.

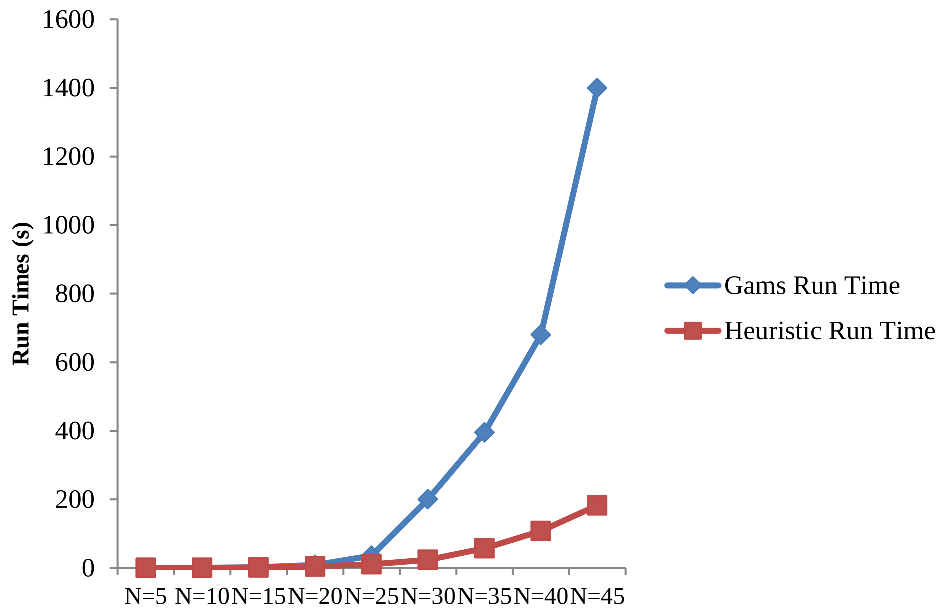


Fig 3. Run time comparison between using GAMS and Hruristic approach

Sensitivity analyses

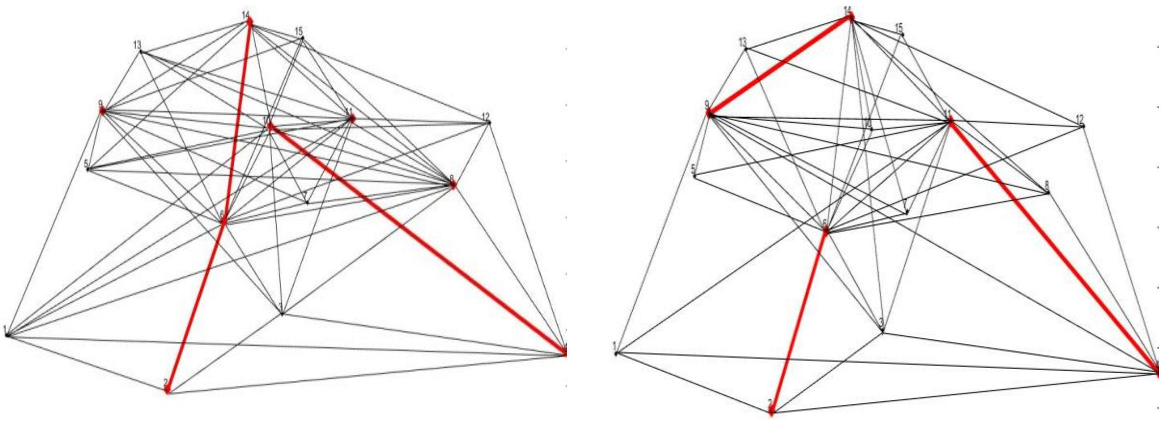
In order to illustrate the effect of isolated hubs in the hub network, an example of 15-nodes network using the *AP* dataset is presented and solved to optimality with and without isolated hub nodes. To prevent the network from opening isolated hubs, the following restriction is added to the mentioned formulation to model a hub arc location problem without isolated hub nodes. The Constraints (13) ensure that every hub node is connected to at least a hub arc and there are no isolated hubs in the network.

$$Z_i \leq \sum_{i<j} Y_{ij} + \sum_{j<i} Yji, \quad \forall i \tag{13}$$

Fig 4 represents the hub arc location optimal solution without and with isolated hub nodes, respectively, with *N*=15, *α*=0.5, *q*=3, and *p*=8. In the visual representation, hub arcs are denoted by a striking red color, while hub nodes are highlighted in a discernible manner. Analysis of the optimal solution reveals significant insights into the effectiveness of incorporating isolated hub nodes within the network configuration.

Comparing the two scenarios, the optimal solution featuring isolated hub nodes showcases a notable enhancement, with a 3.6% improvement in total cost over the solution that excludes isolated hub nodes. In the configuration devoid of isolated hubs, 6 hub nodes out of the total capacity of 8 hubs are utilized to form 3 unconnected hub arcs, reflecting a different approach to network optimization.

Conversely, in the optimal solution that integrates isolated hub nodes, all 8 hub nodes are strategically employed, with 3 of them functioning as isolated hubs. This configuration alteration presents advantages in terms of route efficiency, leading to shorter origin-destination routes throughout the network. The decision to leverage isolated hubs not only optimizes cost-effectiveness but also enhances the overall performance and connectivity within the network, highlighting the strategic significance of considering diverse hub configurations to achieve improved operational outcomes.



(a) Hub network with isolated hub nodes **(b) Hub network without isolated hub nodes**

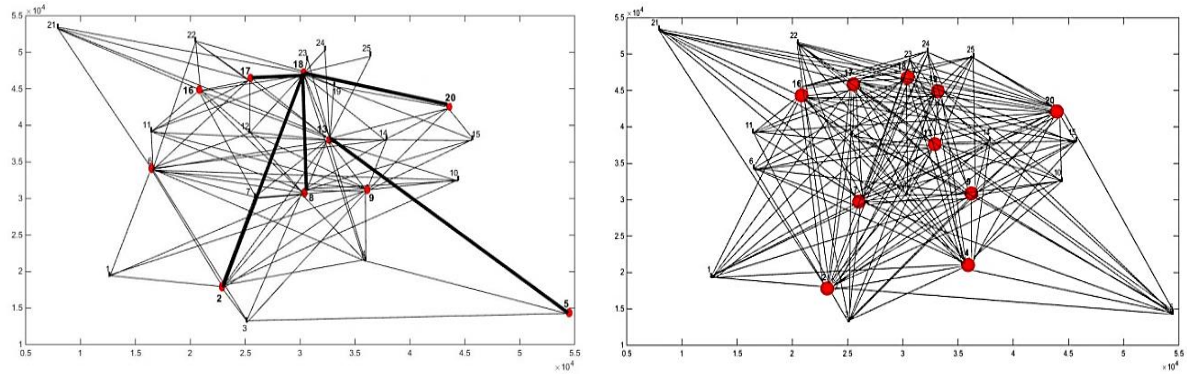
Fig 4. Comparison on hub network with isolated hub nodes

To investigate the impact of varying the parameter α on the network's cost and structure, we conducted a thorough analysis using a problem size consisting of 25 nodes from the well-known AP standard data set. The findings from our investigation, detailed in **Table 3**, shed light on the relationship between the discount factor and the overall cost of the network. As α, the discount factor, increases, a corresponding rise in the total cost of the network is observed in a predictable manner. This escalation in costs is accompanied by a notable shift in the utilization of hub arcs within the network structure, as illustrated in **Fig 5**. With higher values of α, there is a decreased inclination towards utilizing hub arcs, leading to significant alterations in the hub network's configuration.

Moreover, the adjustment of the discount factor impacts the routing of node pairs through hub arcs. A decrease in the discounted cost between hubs (signifying a smaller α value) results in a higher number of node pairs being routed through these hub arcs. Consequently, this leads to the establishment of less costly hub networks that may offer greater feasibility in real-world scenarios. This observation emphasizes the dynamic nature of hub network design optimization and underscores the interplay between cost considerations, routing efficiency, and network structure in decision-making processes.

Table 3. Sensitivity analysis on ***α*** variation

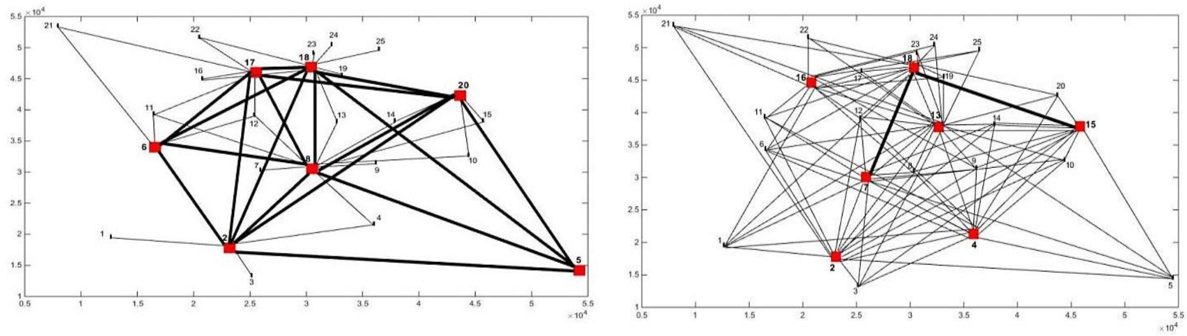
N **= 25, *q*** **= 5, *P*** **= 10**	**Cost**	**Hubs Nodes**	**Hub Arcs**
$\alpha = 0.2$	42575653	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	**2-18, 5-13, 8-18, 17-18, 18-20**
$\alpha = 0.4$	48825139	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	**2-18, 5-13, 8-18, 17-18, 18-20**
$\alpha = 0.6$	53866990	2, 5, 6, 8, 9, 16, 17, 18, 19, 20	**2-18, 5-9, 8-18, 17-18, 18-20**
$\alpha = 0.8$	57516566	2, 4, 7, 10, 13, 16, 17, 18, 19, 20	**2-7, 4-13, 7-18, 10-19, 19-20**
$\alpha = 1.0$	59759492	2, 4, 7, 9, 13, 16, 17, 18, 19, 20	**2-9, 17-19, 18-19, 18-20, 19-20**



(a) *N*=25, *q*=5, *p*=10, *α*=0.2 **(b) *N*=25, *q*=5, *p*=10, *α*=1**

Fig 5. Analyzing the effect of discount factor on hub structure

Furthermore, our study included a comprehensive sensitivity analysis to examine the impact of variations in the number of hub arcs on the network structure and associated costs, as detailed in **Table 4**. Decreasing the number of hub arcs resulted in noticeable increases in overall costs, necessitating a restructuring of the hub network. This restructuring led to a notable expansion in the number of origin-destination routes within the network, consequently augmenting its complexity, as illustrated in



(a) *N*=25, *q*=18, *p*=7, *α*=0.2 **(b) *N*=25, *q*=2, *p*=7, *α*=0.2**

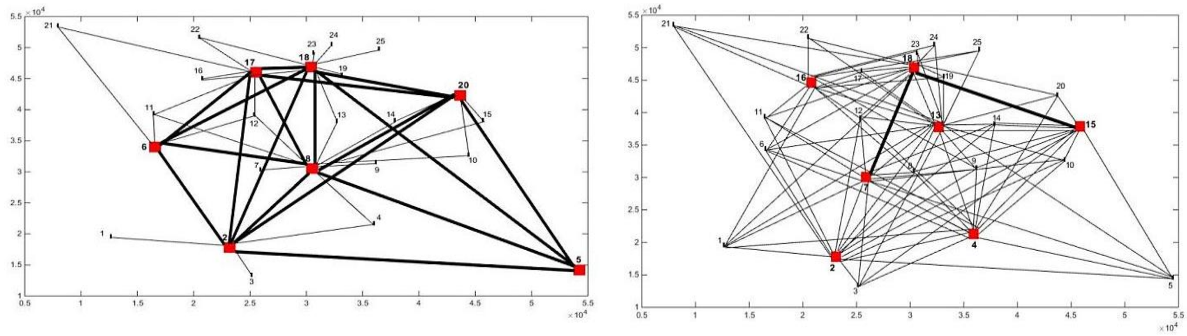
Fig 6.

Notably, as the number of hub arcs was decreased, the network exhibited a higher level of intricacy with an increased number of origin and destination routes. This increased complexity underscores the importance of careful consideration when making changes to the hub network configuration. As highlighted previously, when the number of hub arcs is reduced to a level below twice the maximum number of hub nodes, the potential for utilizing isolated hubs within the network becomes more pronounced. This nuanced observation underscores the strategic significance of balancing hub network design considerations to optimize operational efficiency and cost-effectiveness.

Table 4. Sensitivity analysis on the number of hub arcs

***N*=25, *P* =**	**Cost**	**Hubs Nodes**	**Hub Arcs**

7, α = 0.2			
q = 21	36531630	2, 5, 6, 8, 17, 18, 20	**2-5, 2-6, 2-8, 2-17, 2-18, 2-20, 5-6, 5-8, 5-17, 5-18, 5-20, 6-8, 6-17, 6-18, 6-20, 8-17, 8-18, 8-20, 17-18, 17-20,18-20**
q = 18	36817532	2, 5, 6, 8, 17, 18, 20	**2-5, 2-6, 2-8, 2-17, 2-18, 2-20, 5-8, 5-18, 5-20, 6-8, 6-17, 6-18, 8-17, 8-18, 8-20, 17-18, 17-20,18-20**
q = 15	37543890	2, 5, 6, 8, 17, 18, 20	**2-6, 2-8, 2-17, 2-18, 5-8, 5-18, 6-8, 6-17, 6-18, 8-17, 8-18, 8-20, 17-18, 17-20,18-20**
q = 12	38558780	2, 5, 6, 8, 17, 18, 20	**2-6, 2-8, 2-17, 2-18, 5-8, 5-18, 6-18, 8-17, 8-18, 8-20, 17-18, 18-20**
q = 9	40126085	2, 5, 6, 8, 17, 18, 20	**2-8, 2-18, 5-8, 5-18, 6-18, 8-18, 8-20, 17-18, 18-20**
q = 6	42527529	2, 5, 8, 13, 16, 18, 20	**2-8, 2-18, 5-13, 8-18, 16-18, 18-20**
q = 4	45631823	2, 5, 8, 13, 16, 18, 20	**2-18, 5-13, 8-18, 16-18, 18-20**
q = 2	50782979	2, 4, 7, 13, 15, 16, 18	**7-18, 15-18**
q = 1	54666827	2, 7, 9, 16, 18, 19, 20	**7-18**
q = 0	61106737	2, 7, 9, 16, 18, 19, 20	**-**



(a) ***N*=25, *q*=18, *p*=7, *α*=0.2** **(b)** ***N*=25, *q*=2, *p*=7, *α*=0.2**

Fig 6. Analyzing the effect of number of hub arcs on hub structure

Embracing a more realistic representation of hub network scenarios by allowing for isolated hubs and applying discounts exclusively to hub arcs can help in making decisions closer to real-world conditions. Managers could consider such models for better insights and strategies as follows:

- **Addressing Research Gaps for Optimal Solutions:**
  Recognizing the limitations of classical hub location models and developing innovative solutions like hub covering location-allocation problems can lead to optimized hub and node allocations. Managers should be open to exploring new problem-solving approaches to enhance efficiency and reduce costs.
- **Computational Efficiency and Quality Solutions:**
  Implementing heuristic algorithms for large-scale optimization challenges can offer high-quality solutions within reasonable time frames. Managers can leverage such computational tools to streamline decision-making processes and achieve efficient outcomes.
- **Impact of Isolated Hub Nodes:**
  Conducting sensitivity analyses on factors like discount factor and the number of exogenous hub arcs can provide valuable insights into system cost reductions. Managers should assess and manipulate these parameters to optimize origin-destination routes and access arcs for cost savings.
- **Validation and Comparison for Decision Confidence:**
  Validating proposed methods through comparisons with exact solutions and introducing alternative problem sizes for validation can enhance decision confidence. Managers should ensure the reliability of proposed algorithms by benchmarking against established optimization methods.

Overall, embracing realistic modeling, innovative solutions, computational efficiency, sensitivity analyses, and validation processes can empower managers to make informed decisions in hub network optimization, leading to cost reductions and enhanced operational efficiency.

Conclusion

In this study, a novel formulation of the hub arc location problem incorporating the presence of isolated hubs and set-up costs has been introduced. Unlike traditional hub location models, the hub network in this formulation allows for isolated hubs and applies the discount factor exclusively to hub arcs, resulting in a more realistic representation of real-world scenarios.

In this paper, the location-allocation problems of q-hub covering are proposed as a solution to overcome the deficiencies of classical hub location problems, which are not optimum under real-world conditions. Limiting assumptions, such as applying a discount factor to all hub-to-hub arcs and assuming a complete graph for the hub network, impose additional costs and specific topologies on the hub network, distancing the problem from real-world conditions. Additionally, many issues in the literature of classical hub research neglect the inaccuracies of real-world conditions and the uncertainty of problem parameters, often presenting their models under deterministic conditions to simplify the modeling process.

Therefore, aimed to address this research gap and developed a hub covering location-allocation problem considering isolated hubs. The objective was to optimize the allocation of hubs and hub nodes, as well as the assignment of non-hub points to minimize the total costs of establishing hub facilities and transporting flows. In this model, the hub network is not necessarily a complete graph, allowing for isolated or single hubs, and the discount factor is only applied to hub arcs.

In this regard, for the first time, we presented two innovative and metaheuristic algorithms to solve the deterministic model on small, medium, and large scales. A heuristic approach has been devised to address this complex optimization challenge at scale, with computational efficiency being a key focus. Through rigorous experimentation comparing the performance of the heuristic against exact optimization solutions, the effectiveness of the proposed method has been demonstrated, showcasing the ability to generate high-quality solutions within reasonable time frames. Furthermore, an investigation into the impact of isolated hub nodes has unveiled valuable insights through sensitivity analysis. By manipulating parameters such as the discount factor and the number of exogenous hub arcs, the study reveals a direct correlation to the reduction of origin-destination routes and access arcs, ultimately leading to decreased system costs. To validate the proposed methods, we compared the solution times and errors of the solutions with the exact solutions obtained from optimization software. On a large scale, where the optimization software was unable to solve the problem or even formulate it, we introduced an equivalent mid-size hub location problem as a lower bound and compared the results of the proposed algorithms with it. The results indicated the satisfactory quality of the solutions within a reasonable time for both proposed algorithms in solving the deterministic problem for each scenario. Sensitivity analyses demonstrated that by reducing the discount factor and increasing the number of hub arcs, origin-destination routes and access arcs decreased, leading to cost reductions in the system.

Building upon these findings, avenues for future research have been delineated to expand the understanding and applicability of the hub arc location problem. Suggestions include involving to develop alternative models under conditions of uncertainty, incorporating additional cost factors like arc establishment costs, and integrating constraints such as limited capacity for hub nodes and arcs. Moreover, a call is made to account for real-world complexities by considering factors such as congestion, environmental impacts, risk probability, multivehicle operations, and service level considerations to advance the development of more robust and realistic models in hub network optimization.

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