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RESEARCH PAPER

The Impact of a Quantity Flexibility Contract on Disruption Management in a Dual-Sourcing Supply Chain

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Abstract

This paper focuses on addressing resilience in a two-tier supply chain under supply disruption and demand uncertainty by integrating dual-sourcing and flexibility strategies. The supply chain comprises two suppliers and one manufacturer. In the event of a disruption with the primary supplier, a reliable backup source is chosen to supply some of the orders to the manufacturer at a higher price and lower quality than the primary source. This study aims to answer the following questions by developing a Stackelberg game model between the backup supplier and the manufacturer: How should the order quantity from each supplier be determined, and how can the backup supplier set the selling price of the component under supply disruption? This study also looks into coordinating the backup supplier and the manufacturer using a Quantity Flexibility Contract (QFC) to create flexibility in addition to redundancy for the manufacturer to manage supply disruption and demand uncertainty. Analytical results show that the component's selling price by the backup supplier increases with the disruption probability in the decentralized system but remains independent of the disruption probability under QFC. Numerical calculations demonstrate that when the disruption probability is not very high, accepting the contract prevents the backup supplier's exploitation of supply disruption and improves profits. Although the feasible range of flexibility rate for agreeing on QFC gets tighter with the increased disruption probability, the penalty price in QFC will decrease, indicating that the contract offers the manufacturer more resilience in the case of a more likely supply disruption.

Keywords:
Supply Disruption,
Dual-Sourcing,
Supply Chain Coordination,
Quantity Flexibility Contract,
Resilience.

Introduction

The concept of disruption in the supply chain context has been a topic of interest for researchers and experts across various industries for more than three decades. The rise of globalized supply chains and innovative industry approaches has significantly influenced the evolution of disruption management strategies. The importance of implementing proactive and reactive strategies for disruption management has become increasingly evident in this context. A disruption can interfere with the normal flow of materials and goods in the supply chain, leading to irregularities that expose supply chain members to various risks depending on the disruption's extent, intensity, and duration (Joshi and Luong, 2023).

Numerous incidents over different periods have caused widespread disruptions and long-

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term effects on various supply chains' performance. The COVID-19 pandemic was one of the recent events that significantly impacted various businesses and industries, including automotive, electronics, and pharmaceuticals. The severity of this event prompted most countries worldwide to implement strict measures to prevent the spread and expansion of the virus. In some cases, disease control measures, such as border restrictions, brought about fundamental changes in businesses, the closure of some and the emergence of new ones (Moosavi et al., 2022; Joshi and Luong, 2023; Kamalahmadi et al., 2021). Under these circumstances, supply and procurement operations in supply chains were disrupted, and labor shortages due to quarantines were sometimes experienced. Large companies with a primary global sourcing strategy, like Apple, could not receive essential parts from their suppliers due to government bans and restrictions in their countries of origin (Ivanov and Das, 2020). This supply disruption caused the shutdown or disorder of production and distribution operations in specific supply chains. In such situations, understanding the concept and characteristics of supply chain resilience becomes increasingly important.

As a response to disruptions and the need for efficiency in dealing with them, resilience has been widely accepted as a complementary approach to risk management and vulnerability reduction in various fields (Kamalahmadi and Parast, 2016; Adobor and McMullen, 2018). Resilience in supply chain management has been discussed with various definitions and interpretations. According to the literature on supply chain management, the level of resilience of a supply chain depends on essential characteristics such as redundancy, flexibility, visibility, speed, and adaptability (Namdar et al., 2017). Achieving each of these characteristics requires adopting proactive or reactive approaches in supply chains to develop appropriate solutions before disruptions occur and to recover the diminished performance of the supply chain after disruptions (Chowdhury and Quaddus, 2017).

The occurrence of disruptions in the upstream supply chain is highly critical (Kamalahmadi and Parast, 2016). Global supply chain approaches have caused cross-border disruptions in the upstream supply chain, impacting downstream company performance (Li et al., 2021). While companies often focus on disruptions in the downstream chain members, disruptions in the upstream members have made supply chain resilience crucial (Pereira et al., 2020). Implementing a multi-sourcing strategy in supply chains has become an essential responsive measure for resilience (Namdar et al., 2017), helping to reduce vulnerability to supply disruptions from suppliers (Chowdhury and Quaddus, 2017). Given the nature of disruptions, manufacturers cannot afford to wait and see what will happen to purchase the required components. Utilizing alternative suppliers with capacity reservations or using multiple suppliers simultaneously are resilient supply strategies (Fattahi, 2021).

In addition to the multi-sourcing strategy, negotiating flexible contracts with suppliers can also help minimize the impact of disruptions. Different coordination contracts, such as buyback and flexible order contracts, offer flexibility for supply chains and can be used to manage disruptions and uncertainties (Fattahi, 2021). Several industries, for example, automotive (Yuan et al., 2021) and electronic (Knoblich et al., 2015), benefit from flexible contracts, such as QFC. QFC could share the downstream risk with the upstream entity in the supply chain (Yuan et al., 2021). Combining multi-sourcing with coordination contracts can enhance supply chain flexibility and resilience. This study focuses on the dual-sourcing strategy and creating coordination to mitigate disruption-related consequences. The study considers a supply chain with one manufacturer and two suppliers, where the primary supplier may face disruption. In addition to the unreliable primary source (e.g., a global source), the manufacturer also considers a reliable backup supplier (e.g., a domestic source). The parts are purchased from the leading supplier at a lower price and are of higher quality than the backup supplier. In addition to the dual-sourcing strategy, a QFC is formulated between the manufacturer and the backup supplier. This contract allows the manufacturer to adjust the order quantity from the backup supplier within a specific range under certain conditions, considering supply disruption and demand uncertainty. The study investigates the optimal ordering policy for the manufacturer and pricing for the backup supplier. The results indicate that the flexible contract can create coordination between the manufacturer and the backup supplier for low and medium disruption probabilities, benefiting both parties. Although the selling price of the part by the backup supplier is affected by disruption probabilities in decentralized decision-making without a flexible contract, QFC eliminates this dependency.

The article is structured as follows: The second section will examine the research background in two areas: resilient multi-sourcing strategies for managing supply disruptions and coordination under disruption. The third section will define the problem set, clarify the assumptions, and then describe the models in three scenarios: centralized, decentralized, and coordinated. The fourth section will present the computational results and sensitivity analysis. Finally, the fifth section will include the conclusion and suggestions for further development of the problem discussed in this paper.

Literature Review

The defined problem in this study is related to two research fields, which are separately investigated in this section. Supply chain coordination under disruption has an extensive background in supply chain management. The resilience multi-sourcing strategy under disruption is not actually a new research stream, but the concept of "resilience" has brought new insights into the field of disruption management. All articles reviewed in both subsections investigated the problem of optimal ordering for a dual-sourcing supply chain under disruption.

Resilience Multi-Sourcing Strategy under Disruption

Using a multi-sourcing approach as a resilient strategy for managing disruptions has gained attention in literature and practice. This strategy is observed through two approaches in the supply chain. In the proactive approach, the buyer orders from multiple sources simultaneously to prepare for potential disruptions. In the reactive approach, backup sources are used to fulfill orders not received due to disruptions in other sources. Yin and Wang (2017) examined both approaches and found that simultaneous ordering from two sources is the best solution for a high probability of disruption occurrence. Chakraborty et al. (2019) studied conditions under which both suppliers may be disrupted. They found that reserving capacity from a backup supplier benefits the retailer even if the probability of disruption occurrence is low. The problem of using a backup supplier under supply disruption was investigated by Pan et al. (2022) in an assembly system. They considered a partial disruption for one of the components and investigated optimal orders from suppliers when the assembler orders from the backup supplier after disruption realization.

Some studies investigated another resilience strategy in addition to the dual-sourcing. For example, Kamalahmadi et al. (2021) examined the impact of two approaches—redundancy and flexibility—on the supply chain's resilience against supply and environmental disruptions. They found that the backup supplier strategy is more effective in improving cost-related resilience metrics and service levels in the supply chain than the flexible supplier strategy. Additionally, a combined approach with a degree of capacity flexibility to the backup supplier was found to be more effective. Lou et al. (2024) developed two resilience strategies for managing supply disruptions in a two-level supply chain. The first strategy focuses on the manufacturer's investment in a sustainable supply chain from a single (unreliable) supplier, while the second strategy involves selecting an alternative supplier for procurement. The analytical results of this paper show that if supply sustainability is a priority for the manufacturer, choosing both strategies in combination will have a more effective impact on maximizing average sales.

However, if profit is a priority for the manufacturer and the investment cost is either less or more than a certain threshold, the dual-sourcing strategy will bring higher profits. Our study would be placed in this stream because of the development of a flexible contract to add flexibility for the manufacturer to manage supply disruption in a dual-sourcing supply chain.

Several studies have investigated horizontal competition between manufacturers or supply chains when they use a dual-sourcing strategy. Li et al. (2021a) explored the dual-sourcing strategy within a supply chain involving two suppliers and two manufacturers, with one being unreliable. The study assumed that one of the manufacturers only orders from a reliable supplier. Meanwhile, the study examined a dual-sourcing strategy for the second manufacturer's ordering, considering both reactive and proactive forms. The results revealed that choosing an appropriate sourcing strategy for the second supplier depends on the disruption probability, the order ratio from the unreliable supplier, and the market size. Li et al. (2022) considered two competing supply chains, each including a manufacturer and a tier 1 supplier. They considered a pair of tier 2 suppliers, which may be prone to disruption with unequal probabilities. The authors investigated the impact of suppliers' degree of reliability on the optimal pricing for manufacturers. They found that highly reliable or unreliable suppliers lead to a low wholesale price for the manufacturer.

Supply Chain Coordination under Disruption

Hou et al. (2010) and Hu et al. (2013) analyzed a supply chain comprising a single buyer, one unreliable source, and one reliable source. In their research, Hou et al. (2010) formulate a buyback contract between the manufacturer and the reliable supplier. Similarly, Hu et al. (2013) employed a combination of two risk-sharing and buyback contracts to establish coordination between the buyer and the reliable supplier. Their studies focused on determining optimal contract requirements such that the optimal policies of supply chain members in decentralized decision-making are consistent with centralized conditions.

Zhang et al. (2015) investigated the coordination between a manufacturer and a retailer under demand and production cost disruptions by adjusting three parameters through a contract. The findings revealed that, in most cases, adjusting the parameters of the same contract related to the pre-disruption period enables coordination of the chain even after a disruption occurs. Giri and Sarker (2016) examined the coordination of a two-level supply chain under production disruption, considering price and service level competition among retailers. Their study demonstrated that a wholesale price discount scheme can effectively coordinate the chain. Heydari et al. (2020) investigated a QFC to address demand uncertainty in a two-tier supply chain comprising a manufacturer and a retailer, wherein the retailer can adjust its initial order. They also explored conditions under which the manufacturer could subcontract a portion of its production. The study revealed that outsourcing a part of the production would improve each member's profits. Tang et al. (2018) developed an enhanced revenue-sharing contract to coordinate a two-channel supply chain under cost and demand disruptions. Their findings suggested that chain members' profits under disruption conditions using the enhanced revenue-sharing contract are higher than those before the disruption when coordinated. Using QFC and capacity reservation contracts, Li et al. (2021b) established coordination in a supply chain comprising one manufacturer and one retailer under demand and product price uncertainty. The results indicated that the retailer prefers QFC to coordinate the chain.

In a study by Giri et al. (2021), coordination was examined in situations involving demand and supply uncertainty, with both suppliers being unreliable. The authors introduced a new revenue-sharing contract to establish coordination between the manufacturer and suppliers. In a study by Mohammadzadeh and Zegordi (2016), coordination under supply disruption and demand uncertainty in a three-source supply chain with two reliable and one unreliable source was examined using a cooperation-based approach. The results indicated that adopting a cooperation-based approach among supply chain members and profit sharing through Nash bargaining achieved coordination and eliminated the dependence of the reliable suppliers' selling price on the disruption probability. Joshi and Luong (2023) considered a dual-sourcing supply chain where the primary supplier may face disruption. In this scenario, the backup supplier is a reliable supplier with whom the retailer enters into a two-stage capacity reservation contract. A recent paper by Zhou et al. (2023) focused on a two-level supply chain comprising one manufacturer and one retailer, with the manufacturer potentially facing supply disruption. Therefore, the retailer also has the option to purchase the product from another manufacturer, albeit at a higher price. The main challenge in this paper was designing the contract between the retailer and the unreliable manufacturer. The authors examined two contracts: wholesale price and fraction-committed procurement. The results suggested that the fraction-committed procurement contract is favorable for the main manufacturer and the entire chain, while the retailer prefers the wholesale contract.

Garai and Paul (2023) developed a coordination model for a supply chain consisting of a retailer and two suppliers under supply disruption from the primary supplier. They investigated a risk-sharing contract and a buy-back contract with a side payment. Heydari et al. (2024) used a call option contract to coordinate a supply chain with a retailer and two suppliers where the primary supplier is prone to disruption. They investigated the contract under two conditions and concluded that as the backup supplier has a higher authority to decide on the prices of the contract, the retailer could benefit from the lower option price set by the backup supplier. Chen and Liu (2024) investigated a price-responsive and information-sharing approach for proactive disruption management in a dual-sourcing supply chain. The findings indicated that the manufacturer's responsive pricing encourages the selection of the dual-sourcing strategy. The authors also highlighted that the manufacturer's willingness to share demand information with the supplier depends on the type of pricing (responsive or committed) and the supplier's production cost.

According to the literature, a few studies have considered both flexibility and redundancy concepts to cope with supply disruption. This paper differs from the reviewed studies regarding the type of contract designed to mitigate supply disruption and demand uncertainty while providing flexibility to the manufacturer in addition to redundancy. In this paper, a QFC is designed to create coordination between the manufacturer and the backup supplier, which has not been addressed in previous articles. Moreover, this study considers the quality cost for the components supplied by the backup supplier, which is not included in other studies. Furthermore, the parameters of this contract have been defined in such a way that distinguishes it from other existing QFCs in the literature and brings new insights into this field of study.

Problem Definition and Formulation

This article discusses a supply chain with one manufacturer and two suppliers. The manufacturer procures one of the components from the primary supplier for the final product. This supplier provides information about the likelihood of a supply disruption occurrence, meaning there's a probability that the ordered parts may not be received due to a specific disruption like sanctions. The manufacturer has a long-term relationship with the primary supplier, and previous contracts determine purchasing conditions. Although the manufacturer prefers the primary supplier due to favorable pricing and quality, the risk of production halts due to potential supply disruptions compels the manufacturer to seek a reliable alternative supplier. This strategy, known as the dual-sourcing strategy, involves using two supply sources simultaneously. The backup supplier produces the same component at a higher price and lower quality compared to the primary supplier. This results in additional costs for the manufacturer, as the total production cost of the component from the backup supplier, including quality costs, exceeds the purchase price from the primary supplier ($c_b + c_q > w_m$).

The study focuses on determining the best ordering policy for the manufacturer and the pricing of a backup supplier when faced with supply disruption and demand uncertainty. The purchase price of the component from the primary supplier is fixed, and only the manufacturer and the backup supplier are involved. In addition to examining a resilient dual-sourcing strategy, this study introduces a QFC to coordinate the manufacturer and the backup supplier. Under this contract, the manufacturer orders components from the alternative source before the sales season begins. Once the sales season commences, the manufacturer can return a portion of the orders to the backup supplier or place a new order. Depending on the return or new order, a value is deducted from or added to the initial selling price of the component by the backup supplier. If the manufacturer returns part of the initial order to the backup supplier, the backup supplier agrees to deduct a value from the initial selling price for the returned components and refund it to the manufacturer. Conversely, if the manufacturer adds to the initial order, the backup supplier commits to supplying the requested components by adding a specified value to the initial selling price. It's important to note that the manufacturer can only adjust the initial order within a specified range.

In the following section, we will describe the centralized, decentralized, and coordinated (QFC) models after explaining the assumptions and introducing the symbols.

Assumption and Notations

The problem is defined based on the following assumptions:

- The model is single-period and single-product.
- Demand for the manufacturer is uncertain and is considered probabilistic with a continuous uniform distribution in the range $[0, D_{max}]$.
- All decisions are made before the start of the selling season and before the manufacturer is informed of the actual demand and the occurrence or non-occurrence of the supply disruption.
- The manufacturer orders one of the required components for the final product from the suppliers, and the consumption rate of this component in the final product is one unit.
- The quality of the components produced by the backup (reliable) supplier is lower than that of the primary (unreliable) supplier. Therefore, the manufacturer incurs the cost of this lower quality.
- The selling price of the final product and the purchase price of the component from the unreliable supplier are predetermined for the manufacturer due to previous contracts. Therefore, the first supplier does not play a role in the game.
- The capacity of the suppliers is considered unlimited.
- The manufacturer's problem is formulated as the single-period newsvendor problem. The objective function of the manufacturer and the backup supplier is to maximize the expected average profit.

The parameters and variables used are as follows:

Table 1: Notations

Notation	Parameter
p	Selling price of the final product per unit
c_p	Production cost of the final product per unit
c_b	Component's production cost of the backup supplier per unit
c_l	Lost sales cost per unit
h	Holding cost per unit
c_q	Quality cost per unit supplied by the backup supplier
α	Supply disruption probability
w_m	Primary supplier's selling price per unit
v	Salvage value per unit

Notation	Decision variables
Q_m	Order quantity from the primary supplier
Q_b	Order quantity from the backup supplier
w_b	Backup supplier's wholesale price per unit
d	Penalty price under contract
β	Flexibility rate of the ordering from the backup supplier under contract

Centralized Decision-Making

Centralized decision-making is examined as a basis for achieving coordination between the manufacturer and the backup supplier using a QFC. In the centralized decision-making system, all chain decisions are made by a central decision-maker based on information from the entire chain in such a way that the total profit of the chain is maximized. As stated in the problem assumptions, demand for the manufacturer is considered uncertain. The random variable *x* with density function *f(x)* and cumulative function *F(x)* represents the manufacturer's demand, which follows a continuous uniform distribution in the range of [0, D_{max}]. Uniform distribution is a common distribution function for market demand and is proper for estimating the demand uncertainty in a conservative form (Heydari et al., 2020). To formulate the disruption from the primary supplier, a Bernoulli random variable *Y* is defined. The value of *Y* will be zero if a disruption occurs with probability α, and one if a disruption does not occur with probability *1−α*. Once the disruption occurs, the manufacturer will not receive any ordered component from the primary supplier (i.e., complete disruption). In this study, considering that the primary supplier is actually outside the decision-making system, integration is only considered between the manufacturer and the backup supplier. Based on the defined parameters and variables, the expected profit of the integrated chain is presented in Eq. (1).

$$\max \pi_{SC}(Q_m, Q_b) = \left(\sum_{Y=0}^{1} [(1-Y)\alpha + (1-\alpha)Y]\right) \left(\int_0^{YQ_m+Q_b} \left(\left(p - c_p\right)x - h(YQ_m + Q_b - x)\right) f(x)dx + \int_{YQ_m+Q_b}^{D_{max}} \left(p - c_p\right)(YQ_m + Q_b) - c_l\left(x - (YQ_m + Q_b)\right)f(x)dx - \left(C_b + C_q\right)Q_b - w_m YQ_m \right) \tag{1}$$

The first integral in Eq. (1) calculates the chain profit under two conditions: whether a disruption occurs. The first integral calculates the chain profit when the demand is at most equal to the sum of the manufacturer's orders. Any excess components beyond the demand are subject to holding costs in this situation. The second integral calculates the chain profit when the demand exceeds the sum of the orders. In this case, the manufacturer will incur lost sales costs. The third term inside the parentheses represents the cost of purchasing and the quality of the components bought from the backup supplier, while the fourth term inside the parentheses represents the cost of purchasing components from the primary supplier. Proposition 1 demonstrates that the profit function in the centralized case is a concave function w.r.t the quantities ordered from each supplier.

Proposition 1: The expected profit function of the supply chain in the centralized decision-making model is concave with respect to the order quantities, and the optimal order quantities are:

$$Q_m^0 = \frac{D_{max}\left(c_b + c_q - w_m\right)}{\alpha\left(p + h + c_l - c_p\right)} \tag{2}$$

$$Q_b^0 = \frac{D_{max}\left((1-\alpha)w_m - c_b - c_q + \alpha\left(c_l - c_p + p\right)\right)}{\alpha\left(p + h + c_l - c_p\right)} \tag{3}$$

All proofs are provided in the Appendix.

Optimal order quantities from the two suppliers show that an increase in the disruption probability will decrease the order quantity from the primary supplier and increase the order quantity from the backup supplier. Corollary 1 discusses the rate of increase and decrease in the order quantities, respectively.

Corollary 1: The increase in the order quantity from the backup supplier and the decrease in the order quantity from the primary supplier occur at the same rate ($\frac{D_{max}(c_b+c_q-w_m)}{2\alpha^2(p+h+c_l-c_p)}$) as the disruption probability increases.

Corollary 1 indicates that in a centralized decision-making system, the decrease in the order quantity from the primary supplier is matched by an increase in the order quantity from the backup supplier when there is an increase in the disruption probability. This means that as the disruption probability rises, the backup supplier can compensate for any shortages in orders from the primary supplier. However, in the decentralized model, it will be demonstrated that this result is not the case.

The centralized model serves as a benchmark for achieving coordination through QFC. The following section will describe optimal decisions when the manufacturer and the backup supplier are involved in a decentralized setting.

Decentralized Decision-Making

In this section, the decisions made by the manufacturer and the backup supplier are analyzed using the Stackelberg equilibrium in a decentralized decision-making system. A Stackelberg equilibrium game is developed between the backup supplier and the manufacturer, where the backup supplier acts as the leader and the manufacturer as the follower. The profit functions of the backup supplier and the expected profit of the manufacturer are represented by Eqs. (4-5), respectively. The backup supplier first determines its optimal selling price by anticipating the manufacturer's optimal strategy. Then, the manufacturer determines the optimal order quantities from two suppliers based on the selling price of the backup supplier. Using a backward approach to determine the optimal strategies of the Stackelberg game, Proposition 2 indicates optimal order quantities for the manufacturer.

Proposition 2: The expected profit function of the manufacturer in the decentralized model is concave with respect to the order quantities, and the optimal order quantities are:

$$\max \pi_{BS}^{d}(w_b) = (w_b - c_b)Q_b \tag{4}$$

$$\begin{aligned}\max \pi_{M}^{d}(Q_m, Q_b) = &\left(\sum_{Y=0}^{1}[(1-Y)\alpha + (1-\alpha)Y]\right)\Bigg(\int_{0}^{YQ_m+Q_b}\Big((p-c_p)x \\ &- h(YQ_m + Q_b - x)\Big)f(x)dx \\ &+ \int_{YQ_m+Q_b}^{D_{max}}(p-c_p)(YQ_m+Q_b) - c_l\big(x-(YQ_m+Q_b)\big)f(x)dx - (w_b+c_q)Q_b \\ &- w_mYQ_m\Bigg)\end{aligned} \tag{5}$$

$$Q_m^* = \frac{D_{max}(w_b + c_q - w_m)}{\alpha(p + h + c_l - c_p)} \tag{6}$$

$$Q_b^* = \frac{D_{max}\big((1-\alpha)w_m - w_b - c_q + \alpha(c_l - c_p + p)\big)}{\alpha(p + h + c_l - c_p)} \tag{7}$$

Eqs. (6-7) represents the optimal order quantities of the manufacturer from the suppliers in the decentralized system. To determine the optimal selling price of the component by the backup supplier, it is first necessary to examine the concavity of the profit function of the backup supplier by substituting the optimal order quantity into it. The following proposition shows that under the Stackelberg equilibrium, the profit function of the backup supplier is a concave function w.r.t the selling price variable. The optimal price can be calculated by solving the equation obtained from the first-order derivative of the function concerning the price variable.

Proposition 3: In decentralized decision-making with Stackelberg equilibrium, the profit function of the backup supplier is a concave function w.r.t the selling price of the component, and the optimal price is indicated by Eq. (8):

$$w_b^* = \frac{1}{2}\left(\alpha\left(p + c_l - c_p\right) + (1-\alpha)w_m + c_b - c_q\right) \tag{8}$$

Eq. (8) reveals the optimal selling price of the component the backup supplier offers in a decentralized decision-making system with Stackelberg equilibrium. This equation shows that the backup supplier's optimal selling price of the component depends on the disruption probability. An increase in the disruption probability results in a higher selling price by the backup supplier. The probability of supply disruption for the manufacturer from the primary supplier allows the backup supplier to raise its price and gain a higher profit.

Corollary 2: In a decentralized decision-making system, the manufacturer's order quantity from the alternative supplier is half that of the centralized model. The rate at which the manufacturer's order quantity from the backup supplier increases ($\frac{D_{max}(c_b+c_q-w_m)}{4\alpha^2(p+h+c_l-c_p)}$) is also half that of the centralized model and half of the rate at which the orders from the primary supplier decrease.

By substituting Eq. (8) into Eq. (7), it is found that decentralization leads to a lower order quantity from the backup supplier compared to the centralized model. In this scenario, the manufacturer will increase its order from the backup supplier with a higher disruption probability but at a slower rate than in the centralized system.

Coordination between the Manufacturer and The Backup Supplier Using a Quantity Flexibility Contract

Contracts like buyback and quantity flexibility are mainly used to handle supply chain demand uncertainty and incentivize the buyer to increase order quantities. This study introduces a QFC between the manufacturer and the backup supplier. It will be shown that QFC, combined with a multi-sourcing strategy, can improve the supply chain's resilience in managing supply disruptions.

In the decentralized model, the manufacturer decides on the optimal orders after receiving the component price from the backup supplier. These decisions are made before the selling season without sufficient information about the exact demand or potential disruptions in the primary source. Under QFC, the manufacturer initially orders Q_b from the alternative supplier before the selling season begins. After the selling season starts, the manufacturer can adjust the initial order by up to βQ_b based on the information received about the demand and any disruptions in the primary source. This means that the manufacturer's contract with the backup supplier allows for flexibility in the order quantity within the range of $(1-\beta)Q_b$ to $(1+\beta)Q_b$. If the manufacturer needs to return components, the backup supplier pays the manufacturer $w_b - d$ for the returned components, and then salvages them at a value of v. If additional components are needed, the manufacturer pays the backup supplier $w_b + d$ for the additional ordered components. Therefore, QFC in this paper is influenced by two important variables: first, the flexibility rate "β" which determines the allowable range of order change, and second, the penalty price "d," which indicates the amount of decrease or increase in the initial selling price of the components w.r.t a reduction or increase in the order quantity.

$$X \sim [0, D_{max}] \rightarrow \begin{cases} 0 \leq x < YQ_m + (1-\beta)Q_b \\ YQ_m + (1-\beta)Q_b \leq x < YQ_m + Q_b \\ YQ_m + Q_b \leq x < YQ_m + (1+\beta)Q_b \\ YQ_m + (1+\beta)Q_b \leq x < \infty \end{cases} \tag{9}$$

$$\begin{aligned} \max \pi_M^{co}(Q_m, Q_b, w_b, d, \beta) = (&\sum_{Y=0}^{1} [(1-Y)\alpha + (1-\alpha)Y]) \\ &* \left(\int_0^{YQ_m+(1-\beta)Q_b} \Big((P - C_p)x + (w_b - d)\beta Q_b - h(YQ_m + (1-\beta)Q_b - x) \Big) f(x)dx \right. \\ &+ \int_{YQ_m+(1-\beta)Q_b}^{YQ_m+Q_b} (P - C_p)x + (w_b - d)(YQ_m + Q_b - x)f(x)dx \\ &+ \int_{YQ_m+Q_b}^{YQ_m+(1+\beta)Q_b} (P - C_p)x - (w_b + d)(x - YQ_m - Q_b)f(x)dx \\ &+ \int_{YQ_m+(1+\beta)Q_b}^{\infty} (P - C_p)(YQ_m + (1+\beta)Q_b) - C_l(x - YQ_m - (1+\beta)Q_b) \\ &\left. - (w_b + d)\beta Q_b f(x)dx \right) - (w_b + C_q)Q_b - YQ_m w_m \end{aligned} \tag{10}$$

$$\begin{aligned} \max \pi_{BS}&(Q_m, Q_b, w_b, d, \beta) \\ &= (\sum_{Y=0}^{1} [(1-Y)\alpha + (1-\alpha)Y]) \\ &* \left(\int_0^{YQ_m+(1-\beta)Q_b} \Big((w_b - C_b)Q_b + \big(v - (w_b - d)\big)\beta Q_b \Big) f(x)dx \right. \\ &+ \int_{YQ_m+(1-\beta)Q_b}^{YQ_m+Q_b} ((w_b - C_b)Q_b + \big(v - (w_b - d)\big)(YQ_m + Q_b - x))f(x)dx \\ &+ \int_{YQ_m+Q_b}^{YQ_m+(1+\beta)Q_b} ((w_b - C_b)Q_b + (w_b + d - C_b)(x - (YQ_m + Q_b))f(x)dx \\ &\left. + \int_{YQ_m+(1+\beta)Q_b}^{\infty} (w_b - C_b)Q_b + (w_b + d - C_b)\beta . Q_b f(x)dx \right) \end{aligned} \tag{11}$$

Considering the order flexibility rate, the demand is placed within one of the specified ranges in Eq. (9). The profit functions of the manufacturer and the backup supplier are expressed in Eqs. (10-11) under QFC. Proposition 3 specifies the conditions for the concavity of the manufacturer's profit function under QFC.

Proposition 4: The manufacturer's profit function under QFC will be concave w.r.t the order quantities, provided that the following condition is satisfied:

$$\frac{\big[\big(\beta + \alpha(1-2\beta)\big(p + q - c_p\big) - h(\beta - \alpha) - w_m(1-\alpha)\cdot 2\beta\big]}{\big[\alpha\big(p + c_1 - c_p\big)(1-2\beta) + \alpha h + 2\beta\big(c_{b+}c_q - (1-\alpha)w_m\big)\big]} > 0 \tag{12}$$

To find the optimal order quantities under QFC, a system of Eqs must be solved. (13-14) after establishing Eq. (12). It's crucial to consider the concavity condition of the manufacturer's objective function when determining the range of the flexibility rate β.

$$\begin{aligned} \frac{\partial \pi_M^{co}}{\partial Q_m} = &\frac{(1-\alpha)\big[\big(P + c_l - c_p\big)(D_{max} - (1+\beta)Q_b - Q_m) - h(Q_{m+}(1-\beta)Q_b) + 2\beta Q_b w_b - D_{max} w_m}{D_{max}} \\ &= 0 \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\partial \pi_M^{co}}{\partial Q_b} = \frac{1}{D_{max}} &\big[\big(P + c_l - c_p\big)(D_{max} + (1-\beta)Q_b - (1-\alpha)Q_m)(1+\beta) - (1 \\ &- \beta)h((1-\beta)Q_b + (1-\alpha)Q_m) + \beta((2+\beta)Q_b + (1-\alpha)Q_m)w_b + b\cdot\beta^2 Q_b \\ &- D_{max}\Big((1+\beta)w_b + C_q\Big)\Big] = 0 \end{aligned} \tag{14}$$

To establish coordination, the quantities ordered in QFC must align with the optimal response of the centralized problem. The optimal selling price of the component by the backup supplier can be determined by setting Q_m equal to Eq. (2) and Q_b equal to Eq. (3), and the price variation under QFC is found in Eqs. (15-16). This means that QFC achieves coordination between the manufacturer and the backup supplier when the optimal selling price of the component by the backup supplier matches Eq. (15) and the penalty price matches Eq. (16).

$$w_b^{c_o} = \frac{1}{2}\left(p + h + c_l - c_p\right) \tag{15}$$

$$d^{c_o} = \frac{(p+h+c_l-c_p)\left[2\beta^2(c_b+c_q-w_m)+\alpha\left(h(1+\beta)+2\beta^2 w_m+2c_b-(1-\beta+2\beta^2)(p+q-c_p)\right)\right]}{\alpha(1-2\beta)(p+c_l-c_p)+2\beta(c_b+c_q)+\alpha\cdot h-2\beta(l-\alpha)w_m} \tag{16}$$

Corollary 3: The optimal selling price of the component by the backup supplier under QFC is independent of the supply disruption probability.

When comparing Eq. (15) with Eq. (8), it becomes apparent that in a coordination contract with quantity flexibility, the selling price of the component from the backup supplier is not influenced by the disruption probability. However, in a decentralized decision-making system, an increase in the disruption probability leads to a higher selling price of the component from the backup supplier. This illustrates that QFC can mitigate one of the negative impacts of supply disruption for the manufacturer, more precisely, the increased selling price from the reliable supplier.

Establishing coordination has other conditions as well. One of the essential conditions for achieving coordination among members of a supply chain is its profitability compared to the decentralized decision-making system; this means that how much each member earns under the coordination contract must be equal to the profit they would obtain in the decentralized system. Additionally, when considering QFC, it is crucial to ensure that the value "d" is non-negative and less than the selling price of the component. The flexibility rate "β" should also fall within the range of zero to one. All these conditions, along with the condition for the concavity of the manufacturer's objective function stated in Proposition 4, will be considered to determine the acceptable range of the flexibility rate. Therefore, we have:

$$0 \le \beta \le 1 \tag{17}$$
$$0 \le d^{c_o} \le w_b^{c_o} \tag{18}$$
$$\left|H_{2*2}\left(\pi_M^{c_o}\right)\right| > 0 \tag{19}$$
$$\pi_M^{co} \ge \pi_M^{d} \tag{20}$$
$$\pi_{BS}^{co} \ge \pi_{BS}^{d} \tag{21}$$

Conditions (17) to (21) and Eqs. (15-16) will define the acceptable range for the flexibility rate. These conditions ensure that QFC facilitates coordination between the manufacturer and the backup supplier. They will be considered when solving the models numerically.

Corollary 4: The disruption probability affects the penalty price in QFC.

$$\frac{\partial d^{c_o}}{\partial \alpha} = -\frac{\left(p + c_l - h - c_p - 2c_b\right)\left(p + c_l + h - c_p\right)\left(c_b + c_q - w_m\right)}{\left(\alpha\left(p + c_l + h - c_p\right) + 2\beta\left(c_b + c_q - w_m - \alpha\left(p + c_l - c_p - w_m\right)\right)\right)^2} \tag{22}$$

Eq. (22) describes the derivative of variable "d" w.r.t the disruption probability. This expression can be either positive or negative. If the holding cost of the component is less than $\left(p + c_l - c_p - 2c_b\right)$, an increase in the probability of disruption will result in a decrease in the penalty price "d." In this scenario, a higher probability of disruption means that QFC will better help manage disruptions by reducing penalty prices.

If the holding cost is higher than $\left(p + c_l - c_p - 2c_b\right)$, an increase in the disruption probability will increase the penalty price. In fact, in situations where the holding cost is very high, the increase in penalty price may prevent the manufacturer from over-ordering and increasing the financial burden on the supply chain simultaneously with the rise in the probability of disruption. However, it may be rare for the holding cost to exceed $(p + c_l - c_p - 2c_b)$.

In the following section, we will thoroughly discuss the properties of the optimal solutions through numerical experiments.

Numerical Experiments and Sensitivity Analysis

In this section, we will use a numerical example to analyze the characteristics of the optimal solutions and how they are affected by important parameters. Based on the conditions (17-21) in the study, which is needed to derive optimal solutions to the problem under QFC, as well as specific $(c_b + c_q > w_m)$ and general (e.g., $p > c_p > v$) relations among parameters, a numerical example was designed to verify the characteristics of the closed-form solutions. The values of these parameters can be found in Table (2). The optimal solutions of the decentralized and coordinated models, using the values from the numerical example, are presented in Tables (3) and (4) for various disruption probabilities.

The results indicate that increased disruption probability will lead to a higher order quantity from the backup supplier. However, the order rate from the backup supplier in the coordinated model is significantly higher than in the decentralized model. Fig. (1) demonstrates that an increase in the disruption probability will reduce the slope of the increase in the order rate from the backup supplier in both models.

Table 2: Parameters value in the numerical experiment

P	C_p	C_b	C_l	h	C_q	w_m	v	D_{max}
140	40	25	35	42	15	30	5	1000

Table 3: Optimal solutions of the decentralized model

α	Q_m^*	Q_b^*	Q^*	Q_b/Q	w_b^*	π_{BS}^*	π_M^*	π_{SC}^*
0.1	579.096	14.12429	593.2203	0.02381	25.25	3.531073	10531.43	10534.96
0.2	437.8531	155.3672	593.2203	0.261905	30.5	854.5198	7842.514	8697.034
0.3	390.7721	202.4482	593.2203	0.34127	35.75	2176.318	5389.007	7565.325
0.4	367.2316	225.9887	593.2203	0.380952	41	3615.819	2994.35	6610.169
0.5	353.1073	240.113	593.2203	0.404762	46.25	5102.401	623.2345	5725.636
0.6	343.6911	249.5292	593.2203	0.420635	51.5	6612.524	-1736.11	4876.412
0.7	336.9653	256.255	593.2203	0.431973	56.75	8136.098	-4088.73	4047.367
0.8	331.9209	261.2994	593.2203	0.440476	62	9668.079	-6437.15	3230.932
0.9	327.9975	265.2228	593.2203	0.44709	67.25	11205.67	-8782.76	2422.905

Table 4: Optimal solutions of the coordination model

α	Q_m^{Co}	Q_b^{Co}	Q_b^{Co}/Q	w_b^{Co}	$\beta^{Co}(LB)^1$	$\beta^{Co}(MP)^2$	$\beta^{Co}(UB)^3$	$d^{Co}(MP)$	$\pi_{BS}^{Co}(MP)$	$\pi_M^{Co}(MP)$
0.1	564.97	28.25	0.048	46.5	0.247	0.375	0.528	31.703	816.716	10543.511
0.2	282.49	310.73	0.524	46.5	0.299	0.65	1	33.006	9921.672	11388.164
0.3	188.32	404.90	0.682	46.5	0.332	0.617	0.902	18.858	9787.338	12999.895
0.4	141.24	451.98	0.762	46.5	0.360	0.553	0.746	10.760	9281.712	13274.641
0.5	112.99	480.22	0.809	46.5	0.386	0.521	0.655	5.901	8977.058	13447.439
0.6	94.16	499.06	0.841	46.5	0.414	0.5	0.588	2.662	8778.244	13527.383
0.7	80.71	512.51	0.864	46.5	0.457	0.488	0.518	0.348	8630.125	13629.526
0.8	70.62	522.60	0.881	46.5	No feasible range	No feasible range	No feasible range	No feasible range	No feasible range	No feasible range
0.9	62.77	530.45	0.894	46.5	No feasible range	No feasible range	No feasible range	No feasible range	No feasible range	No feasible range

[1] *Lower Bound of β^{Co} feasible range*
[2] *Midpoint of β^{Co} feasible range*
[3] *Upper Bound of β^{Co} feasible range*

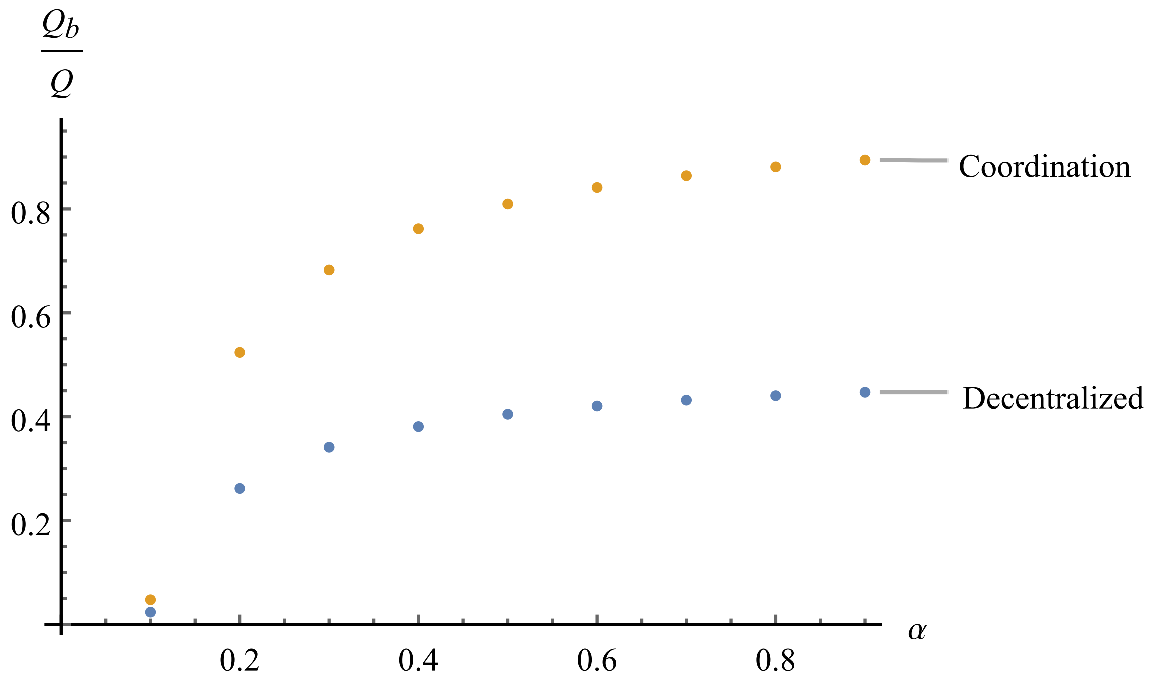


Figure 1: The impact of disruption probability on the order quantity rate from the backup supplier

The impact of using a QFC to coordinate the supply chain on the profits of the manufacturer and the backup supplier is illustrated in Figures 2 and 3. In the decentralized model, an increase in the disruption probability decreases the manufacturer's profit. If the disruption probability exceeds 0.5, the manufacturer will experience losses. However, introducing QFC ensures that an increase in the disruption probability not only does not reduce the manufacturer's profit but also leads to a slight increase in the manufacturer's profit. This increase is due to a significant rise in the order rate from the backup supplier as the disruption probability increases. It's important to note that the higher the disruption probability, the more significant the positive impact of QFC would be on improving the manufacturer's profit. The variations in the profit of the backup supplier show a different pattern. In the decentralized model, an increase in the disruption probability will increase the backup supplier's profit. However, with the establishment of QFC between the manufacturer and the backup supplier, the profit of the backup supplier initially increases with an increase in the disruption probability. Nevertheless, with a further increase in the disruption probability, the profit of the backup supplier decreases.

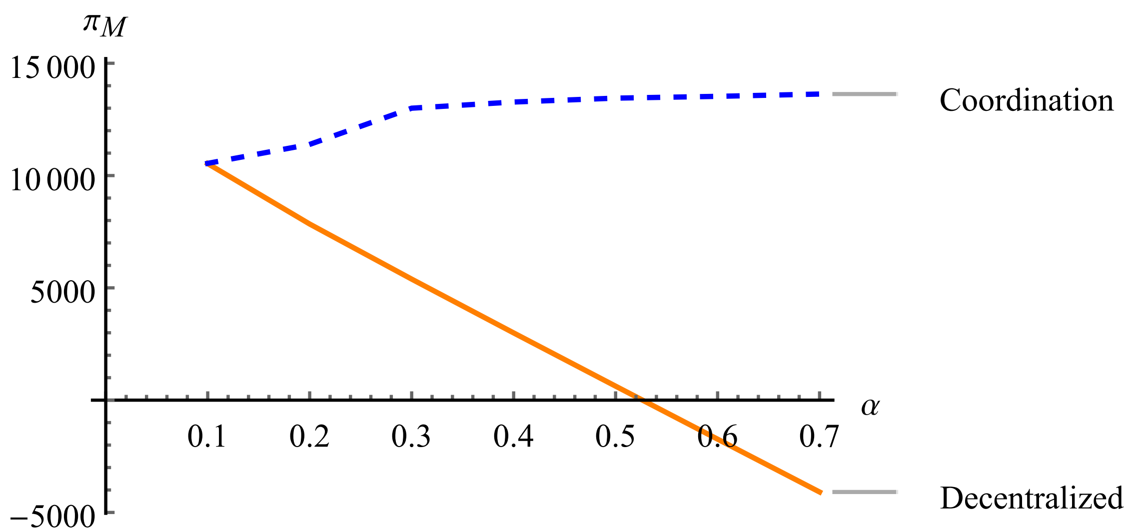


Figure 2: The impact of disruption probability on the manufacturer's profit

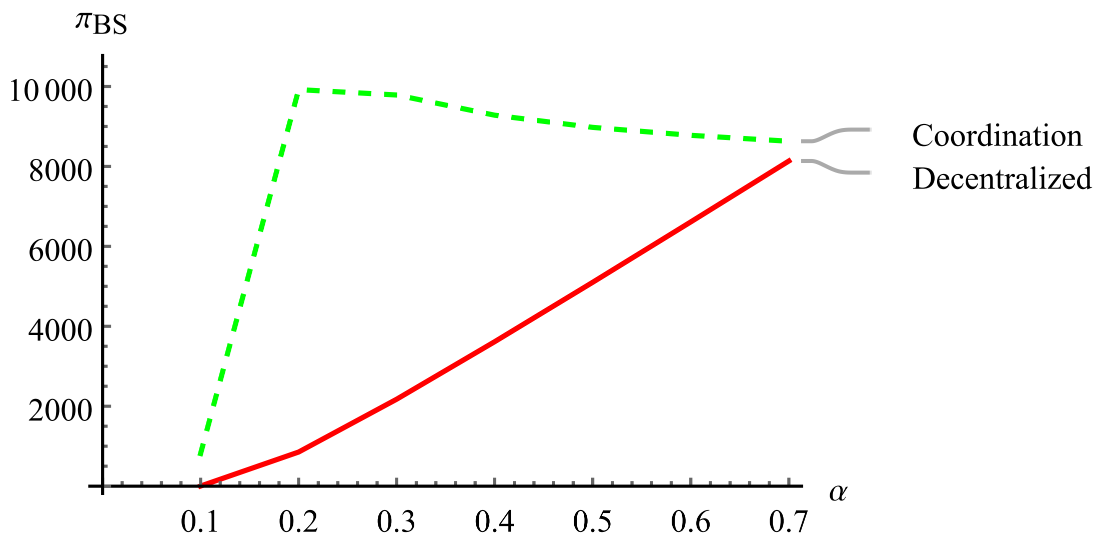


Figure 3: The impact of disruption probability on the backup supplier's profit

Implementing QFC prevents the backup supplier from exploiting supply disruptions. It's important to note that the flexibility rate is considered the midpoint of its acceptable range when calculating the profits of the manufacturer and the backup supplier (Table 4) under QFC. Opting for lower values for the flexibility rate may increase the profit of the alternative supplier. To further explore the impact of the flexibility rate on the earnings of both parties in the supply chain under the contract, Fig. (4) has been included. This figure illustrates the profits of the manufacturer and the backup supplier under various disruption probabilities within the acceptable ranges of the flexibility rate for entering into the coordination contract.

Fig. (4) highlights two critical points. Firstly, as the disruption probability exceeds 0.2, there is a reduction in the acceptable range for establishing the contract. Secondly, increasing the flexibility rate has a different effect on the profits of the manufacturer and backup supplier. An increase in the flexibility rate within its acceptable range will result in higher profit for the manufacturer across all possible disruption probabilities. However, the impact on the profit of the backup supplier varies. If the disruption probability is very low, increasing the flexibility rate will lead to a slight profit increase for the backup supplier. In this scenario, opting for the endpoint of the range for the flexibility rate will maximize profit for both members of the supply chain. However, as the likelihood of disruption increases up to 0.6, the profit function behavior of the backup supplier will initially increase and then decrease with the rise in flexibility rate. The higher the disruption probability, the shorter the increasing segment of the backup supplier's profit curve and the longer the decreasing segment. When the disruption probability is very high, an increase in the flexibility rate will decrease the alternative supplier's profit. For disruption probabilities between 0.2 and 0.6, if the manufacturer's bargaining power is greater, the agreement on the flexibility rate will shift towards the decreasing segment of the backup supplier's profit curve. If the backup supplier can maintain its bargaining power in contracting with the manufacturer, it can steer the agreement on the flexibility rate towards the increasing segment of its profit curve. In brief, for disruption probabilities between 0.2 and 0.6, the agreement on the flexibility rate will depend on the relative bargaining power of the manufacturer and the backup supplier.

Another important factor that influences the acceptable range for setting up QFC is the holding cost. Fig. (5) demonstrates the impact of holding costs on the acceptable range for establishing a contract (assuming a fixed disruption probability). An increase in the holding cost widens the acceptable range for the contract between the manufacturer and the backup supplier. Thus, it can be inferred that there is a wider range for negotiating the flexibility rate and establishing the contract in situations where the holding cost is high.

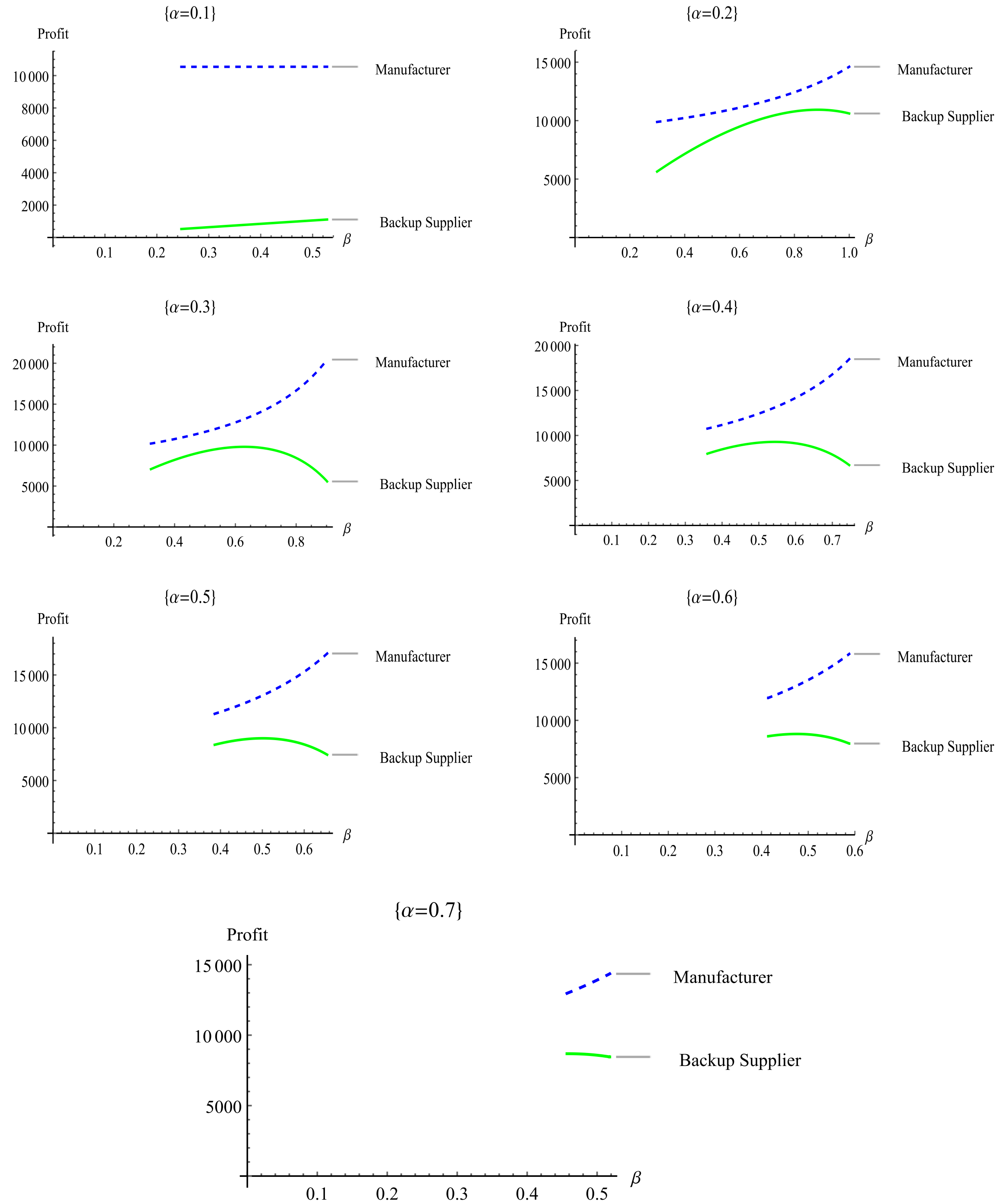


Figure 4: The impact flexibility rate "β" on the optimal profits under various disruption probabilities

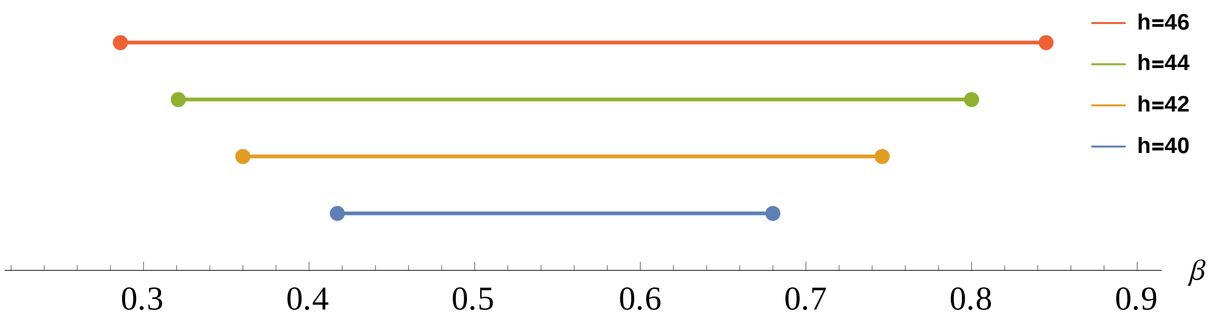


Figure 5: The impact of the holding cost on the feasible flexibility range

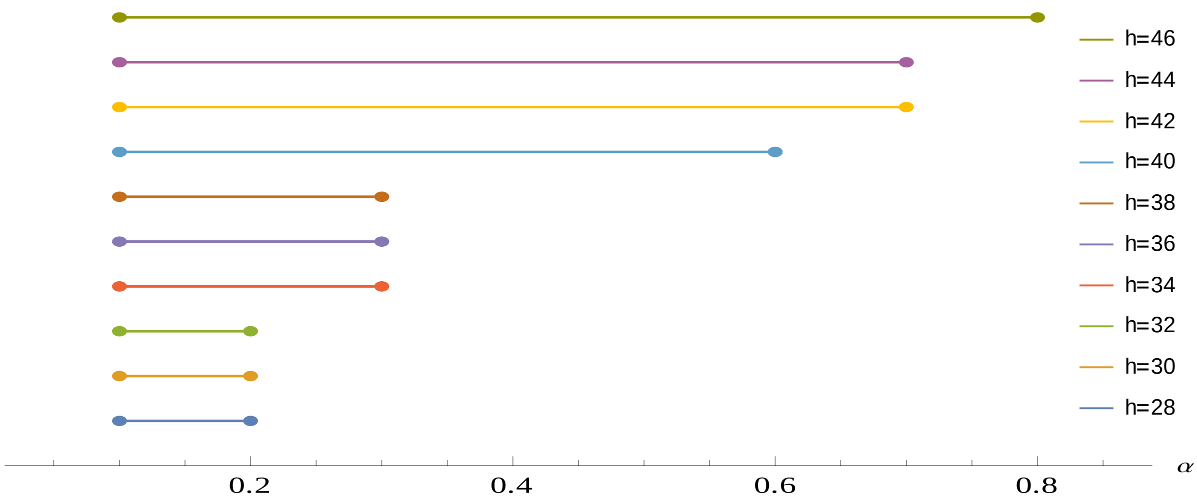


Figure 6: The impact of the holding cost on the range of disruption probability for feasible contracting

It would also be interesting to analyze this effect across different disruption probabilities. Fig. (6) shows acceptable ranges for contract establishment based on the disruption probability and variations in holding costs. This figure illustrates, for any given holding cost, the range of disruption probabilities within which a contract can be established between the manufacturer and the backup supplier; essentially, it indicates the range of disruption probabilities for which there is an acceptable range for the flexibility rate and, as a result, for establishing the contract. This graph confirms that higher holding costs lead to the feasibility of establishing a QFC over a broader range of disruption probabilities. In conclusion, establishing a QFC is more significant when the manufacturer faces high holding costs, both in terms of the acceptable range for negotiating the flexibility rate and managing supply disruptions.

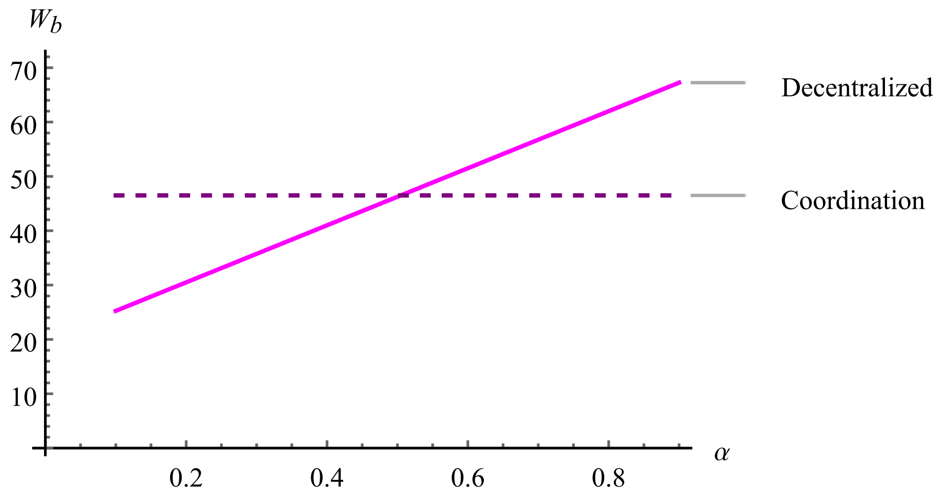
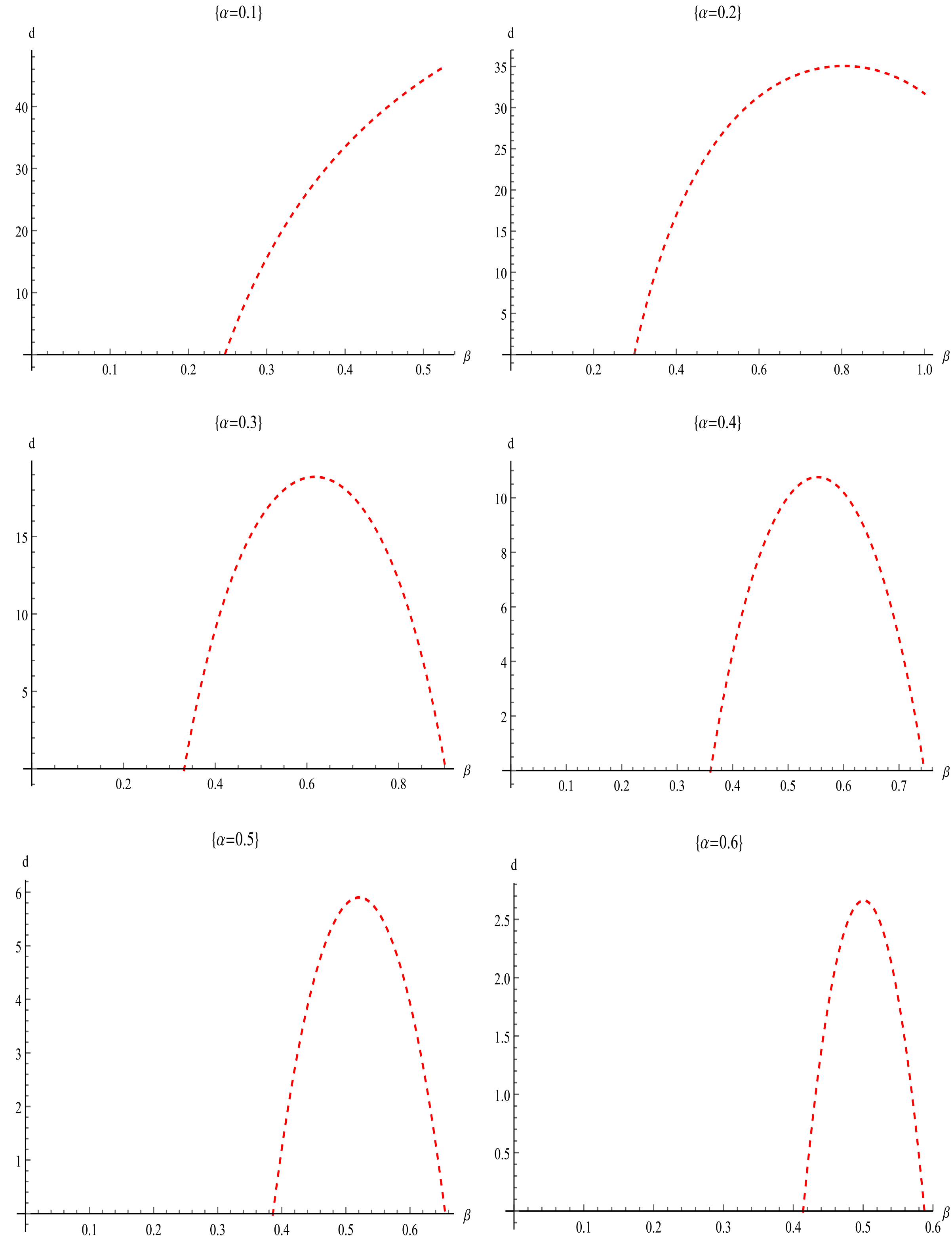


Figure 7: The impact of disruption probability on the optimal component's selling price from the backup supplier

QFC discussed in this paper consists of several important components, including the flexibility rate, the selling price of the component to the manufacturer by the backup supplier, and the penalty price in response to changes in the manufacturer's order quantity. Fig. (7) illustrates the impact of disruption probability on the selling price of the component, both in coordinated and decentralized decision-making situations. In a decentralized scenario, where the manufacturer and backup supplier make decisions independently, an increase in the disruption probability will result in a higher selling price of the component. The occurrence of a supply disruption provides an opportunity for the backup supplier to maximize the component's selling price. The increasing slope of the graph is calculated as $\frac{p+c_l-c_p-w_m}{2}$. Factors such as the selling price of the product, lost sales costs, production costs, and the component's selling price from the primary supplier influence this slope. When the backup supplier and the manufacturer coordinate through a QFC, the optimal component's selling price by the backup supplier becomes independent of the disruption probability. This implies that entering into a QFC prevents the backup supplier from taking advantage of supply disruptions to increase the component price.



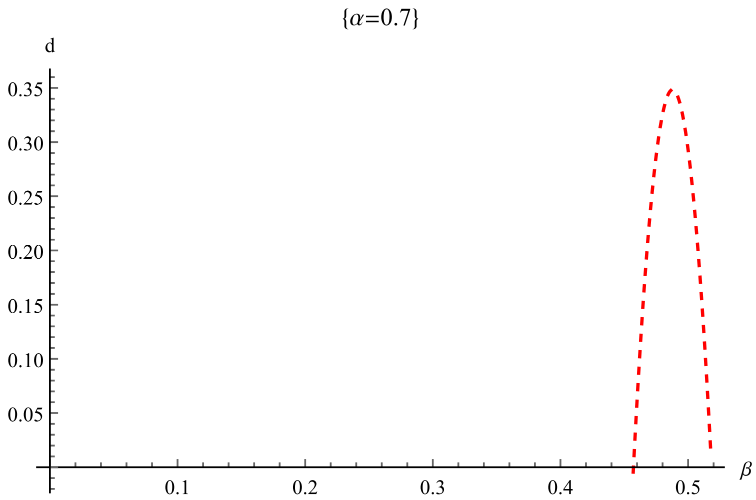


Figure 8: The impact of flexibility rate on the penalty price under various disruption probabilities

Fluctuation in the selling price of the component, denoted as *"d,"* is an influential component of QFC. Analyzing how "*d"* will change with the flexibility rate yields interesting findings. In Fig. (8), you can see how the flexibility rate affects the penalty price for different disruption probabilities. For disruption probabilities other than 0.1, *d* follows a concave function w.r.t. to the flexibility rate, reaching its maximum value at a specific level of flexibility. For disruption probabilities ranging from 0.3 to 0.7, the maximum *d* occurs at the midpoint of the acceptable range for the flexibility rate. Another notable point in Fig. (8) is the decrease in *d* with the increased disruption probability. This observation demonstrates how establishing a coordination contract can help manage disruptions. When the manufacturer and the backup supplier are coordinated through a QFC, reducing the penalty price with increased disruption probability provides stronger resilience for the manufacturer. In essence, the more likely a disruption becomes, the more QFC assists the manufacturer in mitigating the adverse effects of supply disruptions.

Conclusion

The article addressed the issue of improving resilience in a two-tier supply chain under supply disruption and demand uncertainty by using two strategies: dual-sourcing and flexibility. The study considered a two-tier supply chain with two suppliers and one manufacturer, where the primary source could be disrupted. A backup source was available, but it could deliver the same part to the manufacturer at a higher price and lower quality than the primary source. The study proactively examined the dual-sourcing strategy, which involved ordering from both sources simultaneously. Additionally, a QFC was introduced between the backup supplier and the manufacturer to enhance the manufacturer's flexibility in managing disruptions. The study determined the order quantity from the sources and the component's selling price by the backup supplier. The model was solved analytically and numerically in three scenarios: centralized, decentralized with a Stackelberg equilibrium, and coordinated using QFC.

The results of the solutions indicate that if the disruption probability is not very high, there is always a feasible range for contracting between the manufacturer and the backup supplier that improves the situation for both parties compared to the decentralized system. In these circumstances, the higher the disruption probability, the greater its effect on the manufacturer's performance improvement. Analytical results show that in a decentralized system, the higher the disruption probability, the higher the price at which the backup supplier sells the component to the manufacturer. However, accepting QFC eliminates the dependence of the backup supplier's selling price on the disruption probability. Examining the effect of holding costs on optimal solutions reveals that the higher the holding costs, the wider the feasible range for accepting the contract. Sensitivity analysis results indicate that the increase in disruption probability leads to a decrease in the penalty price of the backup supplier's selling price for increasing or decreasing the order quantity by the manufacturer under the contract. It means that under more severe probabilities of disruption occurrence, this contract saves the manufacturer from more severe losses.

This study had limitations that, when examined, could lead to new conclusions. First, the primary supplier was not included as a player in the study. If the primary supplier is added to the model, it will affect the decision-making of other players. The second was determining optimal solutions for a demand function with a specific distribution, but if extracted in general form, it could yield more comprehensive results. Additionally, investigating the capacity constraint for suppliers will change the results since it limits the order quantity from each supplier. Finally, since a monopoly market was considered for the final product in this study, investigating the problem in a duopoly or a competitive market would demonstrate the impact of competition between manufacturers on the design of QFC with the backup supplier. Addressing these limitations in future research could provide more comprehensive and generalizable findings.

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Appendix

Proof of Proposition 1: To prove the concavity of the centralized chain's profit function, the determinants of the minors of this function's Hessian matrix are calculated and presented in Eqs. (A-2) and (A-1). The optimal order quantities are obtained by solving the system of first-order derivative equations of the expected profit function of the centralized chain, shown in Eq. (A-3) and Eq. (A-4).

$$|H_{2*2}(\pi_{SC})| = \frac{\alpha(1-\alpha)\left(p+h+c_l-c_p\right)^2}{{D_{max}}^2} > 0, \qquad \forall\alpha \neq 0,1 \tag{A-1}$$

$$|H_{1*1}(\pi_{SC})| = \frac{-(1-\alpha)\left(p+c_l-c_p\right)}{D_{max}} < 0, \forall\alpha \neq 1 \tag{A-2}$$

$$\frac{\partial\, \pi_{SC}(Q_m, Q_b)}{\partial Q_m} = \frac{(1-\alpha)\left[(D_{max}-Q_b-Q_m)\left(p+c_l-c_p\right)-h(Q_m+Q_b)-w_m D_{max}\right]}{D_{max}} = 0 \tag{A-3}$$

$$\frac{\partial \pi_{sc}(Q_m, Q_b)}{\partial Q_b} = \frac{D_{max}\left(p+c_l-c_b-c_p-c_q\right)-\left(p+h+c_l-c_p\right)(Q_b+(1-\alpha)Q_m)}{D_{max}} = 0 \tag{A-4}$$

Proof of Proposition 2: The same as Proposition 1.

Proof of Proposition 3: The second derivative of the backup supplier's profit function, after substituting the optimal order quantity from Eq. (7), is given in Eq. (A-5). Solving Eq. (A.6) leads to the optimal selling price of the component supplied by the backup supplier.

$$\frac{\partial^2 \pi_{BS}(w_b)}{\partial {w_b}^2} = \frac{-2D_{max}}{\alpha(p+h+c_l-c_p)} < 0, \forall\alpha \neq 0 \tag{A-5}$$

$$\frac{\partial \pi_{BS}(w_b)}{\partial w_b} = Q_b^* + \frac{\partial Q_b^*}{\partial w_b}(w_b-c_b) = \frac{D_{max}\left((1-\alpha)w_m-w_b-c_q+\alpha(c_l-c_p+p\right)}{\alpha(p+h+c_l-c_p)} - \frac{D_{max}}{\alpha(p+h+c_l-c_p)}(w_b-c_b) = \frac{D_{max}\left((1-\alpha)w_m-2w_b+c_b-c_q+\alpha(c_l-c_p+p\right)}{\alpha(p+h+c_l-c_p)} \tag{A-6}$$

Proof of Proposition 4:

$$\left|H_{1*1}\left(\pi_M^{c_0}\right)\right| = -\frac{(1-\alpha)\left[\left(P + c_l - c_p + h\right)\right]}{D_{max}} < 0, \forall\alpha \neq 1 \quad \text{(A-7)}$$

$$\left|H_{2*2}\left(\pi_M^{c_0}\right)\right| = \frac{\alpha(1-\alpha)\left(p + h + c_l - c_p\right)^2\left[\left(\beta + \alpha(1-2\beta)\left(p + q - c_p\right) - h(\beta - \alpha) - w_m(1-\alpha) \cdot 2\beta\right]}{D_{max}^2\left[\alpha\left(p + c_1 - c_p\right)(1-2\beta) + \alpha h + 2\beta\left(c_{b+}c_q - (1-\alpha)w_m\right)\right]} \quad \text{(A-8)}$$

To prove the concavity of the profit function of the manufacturer under the coordination contract, the determinants of the minors of this function's Hessian matrix are calculated and presented in Eqs. (A-7) and (A-8). The first determinant of the Hessian matrix is strictly negative for every $\alpha \neq 1$. The positivity of the second determinant depends on the value of the parameters. According to Eq. (A-8), if $\frac{\left[\left(\beta+\alpha(1-2\beta)\left(p+q-c_p\right)-h(\beta-\alpha)-w_m(1-\alpha)\cdot 2\beta\right]}{\left[\alpha\left(p+c_1-c_p\right)(1-2\beta)+\alpha h+2\beta\left(c_{b+}c_q-(1-\alpha)w_m\right)\right]}$ is positive, the determinant is strictly positive for every $\alpha \neq 0,1$.