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RESEARCH PAPER

A Hierarchical Multi-Period Approach to Integrated Facility Location and Network Design Problem

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Abstract

This study addresses an integrated problem of hierarchical facility location and network design, which involves multiple decisions about the opening of facilities and network links at various levels. We introduce a novel multi-period model that integrates these problems, taking into account budgetary constraints and addressing the specific challenge of optimizing hierarchical upgrades for urban centers and transportation network links within each time period. The aim is to determine the optimal upgrade levels for urban centers and transportation network links in each time period, subject to a predefined budget. The proposed model is formulated as a mixed-integer linear programming problem. To solve the developed model, we employ a heuristic algorithm that combines simulated annealing with different neighborhood structures and fix-and-optimize strategies. The efficiency of the proposed algorithm is demonstrated through various instances, showing superior performance compared to the CPLEX solver, especially for larger problem instances. Furthermore, we illustrate the practical utility of this model in real-world decision-making processes, underscoring its efficacy. By addressing these factors, the proposed model provides valuable insights for organizational managers and planners.

Keywords:
Facility Location,
Network Design,
Hierarchical, Fix and Optimize Algorithm,
Simulated Annealing

Introduction

Facility Location and Network Design Problem(FLNDP) arise from the integration of considerations for facility location and network design. These problems seek to create efficient systems by strategically placing facilities and designing communication links that connect demand nodes to facility nodes. FLNDP has a wide range of applications, including distribution and transportation systems. The primary goal is to design a network and facility locations that collectively minimize the overall costs associated with transportation, facility location, and link construction, while ensuring effective service delivery.

In the current competitive landscape, the strategic design of hierarchical facility location and transportation networks is crucial for optimizing the performance of public service systems. This is especially true for scenarios where budget constraints and multiple time periods are key

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considerations. Governments prioritize achieving equitable public service distribution and enhancing accessibility as core objectives [1][2]. To achieve these goals, the integration of facility location and network design considerations is paramount. Many systems providing services or products inherently exhibit hierarchical structures, considered essential for improving the quality of public services and reducing costs [3][4].Hierarchical systems interconnect facilities based on the nature of services they offer, and decisions on facility locations consider mutual connections and various hierarchy levels. Adopting a hierarchical approach to facility location has been shown to yield substantial enhancements in service performance and cost reduction [5][6], emphasizing the critical importance of strategically siting facilities offering services and products.

Furthermore, the design of transportation networks plays a pivotal role in traffic and transportation planning. Communication networks are crucial for accessing facilities, determining link quality, travel time, and ease of travel to each facility. Therefore, investigating network design issues in a hierarchical manner is imperative due to their substantial economic impact [7]. The integration of these two critical aspects into real-world problems is particularly crucial in engineering contexts, especially within healthcare systems where service delivery directly influences human lives. Beyond service quality, minimizing access time to these facilities is indispensable [8].Consequently, the value of high-quality links becomes pronounced in reducing travel time to facilities, ensuring the timely utilization of services, especially in critical situations such as deliveries, accidents, heart attacks, brain attacks, and other emergencies. The hierarchical structure is a key factor in FLNDP for many real-world applications, particularly in public service sectors like healthcare.

In real-world scenarios, decisions on facility location and network design are significantly influenced by budget constraints aligned with organizational investment policies. To address this, our study proposes a combined policy approach to model budget constraints, assuming separate organizations responsible for investing in facilities and network links, each managing its budget independently. Remaining investments at the end of a period are carried forward. The research aims to identify urban centers and transportation links eligible for hierarchical upgrades within budget limits, enhancing access to services at minimal cost. This study explores the integration of hierarchical facility location and network design, and budget constraints across multiple periods. Assuming all elements start at the first level, the model aims to strategically upgrade centers to higher hierarchy levels. Operating under fixed costs for openings, including operational and transportation expenses, the primary goal is to formulate a comprehensive problem that addresses these complexities simultaneously.

The proposed model, which addresses hierarchical facility location, network design, and budget constraints over multiple time periods, represents a novel contribution to the field. Its application is particularly relevant to public services, where cost-effective and efficient service delivery is paramount. A specific budget for upgrading centers and links is allocated in each time period, leading to a specialized mathematical model. The problem’s inherent decisions regarding location and network design, considering the hierarchical structure and budget constraints, categorize it as NP-hard. This results in computational complexity that grows exponentially. To solve the proposed model, a Simulated Annealing (SA) algorithm has been employed in two modes: generating an initial solution randomly and generating a justified initial solution using fix-and-optimize algorithm. Finally, the results obtained from solving the samples using the simulated annealing algorithm have been compared with the results of the exact solution.

Building upon the insights provided in the research, we have thoroughly explored and examined the following aspects:

- Formulating a hierarchical facility location and network design problem considering varying time intervals.

- Considering budgetary constraints for facility location and network design separately.
- Implementing a simulated annealing algorithm for solving the proposed model.

The structure of the current study is outlined as follows: the next section presents a comprehensive analysis of existing literature on facility location and network design problems, followed by the exposition of the mathematical model in the subsequent section. In Section 4, the employed solution strategies are outlined, and the obtained results from solving the model with various samples are presented. Section 5 concludes the study with a comprehensive overview of findings and recommendations for potential avenues of further research.

Literature Review

The concept of facility location and network design has been extensively investigated in various contexts, primarily due to its profound influence on operational efficiency optimization. However, the confluence of these two problems has been relatively underexplored in academic literature. The following section aims to review the pertinent literature on facility location and network design problems, with a specific focus on integrating facility location and network design considerations. Furthermore, we aim to elucidate the challenges posed by hierarchical structures, multi-period models, and existing budgetary constraints within the FLNDP framework.

Over the past decades, FLNDP models have been the subject of numerous research studies. Daskin et al. (1993) pioneered the field by introducing the uncapacitated transportation network model for FLNDP[9]. Melkote (1996) contributed significantly to the development of three distinct FLNDP models [10]. Subsequently, Melkote and Daskin (2001) expounded upon these models in uncapacitated and capacitated scenarios, establishing foundational frameworks that have become widely referenced in scientific literature[11][9] .Cocking (2008) addressed the uncapacitated FLNDP, and diverse algorithms were studied to enhance both the lower and upper bounds for optimal solutions [12].

Moreover, Drezner and Wesolowsky (2003) delved into a novel network design problem, presenting a model for optimizing the placement of a single facility within a network with candidate links. Each link incurs a specific construction cost, and transport links can be established as unidirectional or bidirectional. To address this challenge, the authors proposed multiple algorithms, including the gradient descent algorithm, SA, tabu search (TS), and genetic algorithm (GA)[13].

Bigotte et al. (2010) formulated a novel optimization model rooted in the FLNDP for the holistic planning of urban hierarchy and transportation networks. Their model concurrently addressed various hierarchical levels of urban centers and network links. The outcomes of their study facilitated the identification of specific urban center and network link types deserving promotion to higher hierarchical levels, ultimately maximizing accessibility across all facility classes[14].

Contreras and Fernández (2012) established a comprehensive framework for addressing general supply chain network design issues, integrating strategic decisions on facility location and link selection into operational allocation and routing decisions for customer demands. Their work encompassed modeling aspects, alternative formulations, and algorithmic strategies for FLNDP[15]. In a subsequent study, they introduced a modified FLNDP version, aiming to minimize the maximum travel time within the network as a new objective. This model offered a generalized approach to the classic p-center problem, demonstrating results on instances with up to 100 nodes and 500 candidate link [16].

In a different approach, Ghaderi and Jabalameli (2013) proposed the Dynamic Uncapacitated Facility Location and Network Design Problem (DUFLNDP), addressing the dynamic (multi-period) facility location problem integrated with network design. This model considered constraints on the investment budget for opening facilities and constructing links during a planning horizon. The authors employed a greedy heuristic and a simulated annealing-based meta-heuristic for problem solution [17]. Furthermore, Rahmaniani and Ghaderi (2013) introduced a mixed-integer model considering different link types with varied capacities, transport, and construction costs to optimize facility location and transportation network design concurrently. The goal was to minimize total transportation and operating costs [18].

Rahmaniani and Shafia (2013) focused on the maximum covering facility location and network design problems under uncertainty. The objective was to locate a predefined number of facilities and optimize the network to maximize the total covered demand points [19]. In a separate study, Shishebori et al. (2013) introduced a mixed-integer nonlinear programming model to formulate the reliable budget-constrained FLNDP[20]. Shishebori and Babadi (2015) formulated a mathematical model for a robust and reliable budget-constrained FLNDP, considering parameter uncertainty and system disruption simultaneously. They utilized a commercial solver to solve the resulting model and illustrated the practical application through a real-world case study in the healthcare system of the Chaharmahal-Bakhtiari province in Iran [21].

In recent years, there has been a growing interest in integrated models addressing FLNDP. These NP-hard problems involve optimizing both facility locations and network design concurrently, playing a crucial role in diverse planning scenarios. Despite their computational challenges, Rahmanian and Ghaderi (2015) [22] introduced three solution methods derived from the variable neighborhood search algorithm to effectively tackle the capacitated FLNDP. These methods, incorporating exact techniques, consistently deliver high-quality solutions across various test instances with up to 100 nodes and 600 links, surpassing commercial solvers in terms of efficiency. Pearce and Forbes (2018) proposed an exact solution approach based on Benders’ decomposition to optimize multi-period facility location and network design considering budget constraints and unlimited capacity[23].

Sadatasl et al. (2016) introduced a complex integer number programming model with fuzzy demands, considering backup facilities and multiple links between nodes [24]. In another paper, the problem of facility location and network design in a fuzzy environment has been explored. The model incorporates various link types with distinct capacities and costs, allowing for the selection of multiple links between nodes. To address customer demand uncertainty, an interactive fuzzy solution approach is applied, and a hybrid meta-heuristic algorithm (FIWO) based on firefly optimization and invasive weeds is developed for problem-solving. The model's performance is empirically tested, and the proposed algorithm is compared with alternative solution methods [25].

Brahami et al. (2020) highlighted the essentiality of devising a sustainable supply chain network within the intensely competitive contemporary landscape. The proposed multi-objective model aimed to concurrently minimize costs and environmental impacts linked to transportation activities, integrating various environmental considerations for the planned network links. To tackle this, they introduced an adapted non-dominated sorting genetic algorithm II, utilizing mixed coding as an effective solution approach [26].

Pourrezaie-Khaligh et al. (2022) conducted a study addressing equity and accessibility in healthcare facility location/network design. The proposed DEL model aims to minimize system costs, enhance availability, and diminish inequality among demand nodes. The real-world case study highlights the model's performance, and a Fix-and-Optimize (FO) approach is introduced for large-scale problems, demonstrating its effectiveness across diverse test scenarios [27].

The problem of hierarchical facility location and transportation design involves deciding on the upgrade of facilities and transportation network links. Despite the significance of hierarchical nature of facilities and network links in some real-world applications, previous research has conducted only a limited number of studies integrating these two issues. Previous research by Narula and Ogbu (1979) focused on models for locating sequential unique and comprehensive hierarchical facilities[28], and Sahin and Sural (2007) explored hierarchical facility location models up to 2004[29]. Antones et al. (2009) developed a multi-period model for locating multi-level facilities in urban hierarchy planning[30], and Ghezavati et al.(2015) presented a hierarchical facility location model for disaster relief supply chain planning under uncertainty[31]. Rastaghi et al.(2018) proposed a multi-objective and multi-service allocation location model with capacity planning for health network design, including a hierarchical health service network model under human resource limitations[32]. In addition, Balakrishnan et al.(1994) investigated multi-level network design issues[33].

All reviewed studies emphasize the significance and complexity of integrating facility location and transportation network design across diverse real-world scenarios, including urban hierarchy planning, disaster relief supply chain planning, and health network design. However, it is notable that only Bigotte et al. (2010) [14] introduced an optimization model capable of concurrently determining upgrades to urban centers and network links, thus impacting hierarchy levels within this integrated framework. Nevertheless, the model presented in this paper extends upon prior work by accounting for operational costs, budget constraints, and time periods. Additionally, this research introduces a heuristic algorithm based on Simulated Annealing with a fix-and-optimize approach to solve the problem. It is essential to acknowledge that real-world facility location and network design challenges frequently involve constraints such as budget limitations, and neglecting these constraints may result in model inaccuracies. Inspired by the existing literature, the model presented in this paper aims to investigate a hierarchical facility location and network design problem that accounts for various time periods and constrained budgets. According to the literature review, this is the first model introduced in the domain of dynamic hierarchical facility location and network design. The following section provides a comprehensive description of the problem under investigation.

Problem Statement and Mathematical Formulation

The current study focuses on the challenge of hierarchical multi-period facility location and network design problem. Initially assuming equivalence across all centers and links at the primary level, the study seeks to optimize the selection of centers and links for elevation to higher levels, aligning with budget constraints to minimize costs. The cost analysis encompasses operating expenses, fixed costs related to facility and link upgrades, as well as transportation costs. The determination of the maximum budget for facility and link upgrades in each period is predicated on an intricate evaluation considering these cost components. Within this framework, origin nodes correspond to urban centers or demand points, while facilities situated at destination nodes are tailored to fulfill the demands originating from other nodes. The problem is articulated as a minimum-cost flow model. To construct this model, a virtual node is introduced into the network, acting as the decisive terminal point regulating the entirety of the network's flow dynamics. The primary objective is to pinpoint the most cost-effective flow between the demand centers and the virtual node. Moreover, the study integrates several critical assumptions crucial to thoroughly addressing the intricacies of this optimization problem.

- Initially, all urban centers and network links are categorized, at a minimum, as level one. For example, in the establishment of health centers, each center begins at level one (health center), and the initial links are, at the least, basic dirt roads.
- Each facility is interconnected across all levels of the urban hierarchy.
- Higher-level centers possess the capacity to offer all services available at lower levels.
- Residents in every urban center require services from all levels and derive benefits from the closest relevant facilities.

- Travel cost or time between two centers is contingent upon the level of the network link connecting them, where higher link levels imply shorter travel times. This parameter remains constant over time.
- The number of urban centers to be upgraded in each hierarchy level is determined by the budget allotted for each period.
- The parameters linked to the continual budget allocation for each period and those related to the expenses of enhancing facilities and links experience variations over time. These alterations are shaped by factors such as fluctuations in inflation rates or specific trends in population growth.
- The virtual node serves as the terminal point for the entirety of the network's flow dynamics.

The notations employed, encompassing sets, parameters, and decision variables, are listed below for reference within the mathematical modeling framework.

Sets

L	Set of service levels denoted by l;
M	Set of link levels indexed by m;
T	Set of time periods indexed by t;
N	Set of urban centers by n;
(i,j)	Link from center i to center j;
(i,s)	Link between center i and the virtual node;
I_1	Set of links (i,j);
I_2	Set of links (j,i);
I_3	Set of links (i,s);
I	Collection encompassing all links;

Decision variables

X_{ijm}^{klt}	Fraction of the demand flow of level l from center k transferred through link (i,j) at level m in period t.
V_{ijm}^{t}	Binary variable with a value of 1 if link (i,j) is upgraded to level m in period t; otherwise, it equals 0.
U_i^{lt}	Binary variable with a value of 1 if center i is upgraded to level l in period t; otherwise, it is 0.
R_{ijm}^{t}	Binary variable with a value of 1 if link (i,j) is at level m in period t; otherwise, it equals 0.
Y_i^{lt}	Binary variable with a value of 1 if center i is at level l in period t; otherwise, it is 0.

Parameters

s_{ijm}^{klt}	Travel cost to demand center k for services at level l through link (i,j) with level m during period t.
f_i^{lt}	Operating cost of center i at level l in period t.
g_i^{lt}	Fixed cost to upgrade center i to level l in period t.
c_{ijm}^{t}	Fixed cost to upgrade link (i,j) to level m in period t.
h_{ijm}^{t}	Operational cost of link (i,j) at level m in period t.
d_k^{lt}	Demand of center k for service level l in period t.
p_{ijm}^{t}	Cost of transferring each unit of flow through link (i,j) at level m in period t.
L_{ij}	Length of link (i,j).
b_1^t	Available budget in period t to upgrade the facilities.
b_2^t	Available budget in period t to upgrade network links.

Mathematical Modeling

Based on the provided notations, the mixed-integer programming model for the studied problem is formulated as follows:

$$Min \quad \sum_{(i,j)\in I}\sum_{m}\sum_{k}\sum_{l}\sum_{t} s_{ijm}^{klt}\ X_{ijm}^{klt} + \sum_{i}\sum_{l}\sum_{t} f_i^{lt}\, Y_i^{lt} + \sum_{(i,j)\in I}\sum_{m}\sum_{t} h_{ijm}^{t}\, R_{ijm}^{t} \tag{1}$$

$$s.t{:}\quad X_{ijm}^{klt} \le R_{ijm}^{t} \qquad \forall\, k \in N\,, (i,j) \in I_1\ l \in L\,, m \in M\ t \in T \tag{2}$$

$$X_{jim}^{klt} \le R_{ijm}^{t} \qquad \forall\, k \in N\,, (i,j) \in I_1\ l \in L\,, m \in M, t \in T \tag{3}$$

$$X_{ism}^{klt} \le Y_i^{lt} \qquad \forall\, k, i \in N\,, l \in L, m \in M\,, t \in T \tag{4}$$

$$\sum_{i}\sum_{m} X_{ism}^{klt} = 1 \qquad \forall\, k \in N, l \in L, t \in T \tag{5}$$

$$\sum_{i}\sum_{m} X_{ijm}^{klt} = \sum_{m} X_{jsm}^{klt} + \sum_{i}\sum_{m} X_{jim}^{klt} \qquad \forall\, \mathrm{j}, k \in N: \mathrm{j} \neq k, l \in L, t \in T \tag{6}$$

$$\sum_{m} R_{ijm}^{t} \leq 1 \qquad \forall\, (i,j) \in I_1, t \in T \tag{7}$$

$$\sum_{l} Y_{i}^{lt} \leq 1 \qquad \forall\, i \in N, t \in T \tag{8}$$

$$Y_{i}^{l,t-1} + U_{i}^{lt} = Y_{i}^{lt} \qquad \forall\, i \in N, l \in L, t \in T \tag{9}$$

$$R_{ijm}^{t-1} + V_{ijm}^{t} = R_{ijm}^{t} \qquad \forall\, (i,j) \in I_1, m \in M, t \in T \tag{10}$$

$$\sum_{t'=1}^{t}\sum_{i}\sum_{l} g_{i}^{lt'} U_{i}^{lt'} \leq \sum_{t'=1}^{t} b_{1}^{t'} \qquad \forall\, t \in T \tag{11}$$

$$\sum_{t'=1}^{t}\sum_{i.j}\sum_{m} c_{ijm}^{t'} V_{ijm}^{t'} \leq \sum_{t'=1}^{t} b_{2}^{t'} \qquad \forall\, t \in T \tag{12}$$

$$X_{ijm}^{klt} \geq 0 \qquad \forall (i,j) \in I, l \in L\ m \in M, t \in T \tag{13}$$

$$Y_{i}^{lt} \in \{0,1\} \qquad \forall\, i \in N, l \in L, t \in T \tag{14}$$

$$R_{ijm}^{t} \in \{0,1\} \qquad \forall (i,j) \in I_1, m \in M, t \in T \tag{15}$$

$$U_{i}^{lt} \in \{0,1\} \qquad \forall\, i \in N, l \in L, t \in T \tag{16}$$

$$V_{ijm}^{t} \in \{0,1\} \qquad \forall (i,j) \in I_1, m \in M, t \in T \tag{17}$$

Equation (1) present the objective function of the problem. In this problem, the objective is to minimize both customer travel costs and operational expenses associated with facilities and network links. Equations (2) and (3) denote that rreceiving the services of a demand center through network links is only possible if these links are built. Hence, the service at demand point k for level l is facilitated through link (i,j) only when it is operational at level m. Equation (4) states that the fulfillment of the demand at point k in level l during period t is contingent upon the existence of a facility at point i with level l. Equation (5) delineates the linkage of flow originating from all demand centers, denoted as k, to the virtual node within the network. Essentially, this constraint ensures the fulfillment of various service demands across different points within each time period. Constraint (6) ensures that the demand flow from center k entering node j in each period must be equivalent to the flow exiting from node j in the same period. Moreover, if a facility is established at node j, it guarantees that the demand from center k will be supplied with located facility in $j(i.e. \sum_m X_{jsm}^{klt} = 1)$. Constraint (7) denotes that the enhancement of the link between node i and node j can only be elevated by a maximum of one level during period t. This implies that each link, within each period, is eligible for an upgrade of up to one level. Constraint (8) similarly asserts that facilities, represented by center i, are subject to an upgrade of no more than one level during period t. This implies that each center is eligible for an upgrade of up to one level within a given period. Equation (9) elucidates the relationship between the upgrading variables of the facilities and their operational status. This means that if facility i is active in period $t-1$ $(Y_i^{l,t-1} = 1)$, the value of the design variable will be zero in period $t(U_i^{lt} = 0)$. Equation (10) outlines a similar condition for network links akin to the ones for facilities. Constraints (11) and (12) concern the constraints related to the budget allocated for upgrading facilities and network links, ensuring that the incurred costs remain within the specified available budget. These constraints also allow any remaining budget in each period to carry over to the next period for utilization. Equations (13) through (17) represent constraints that define different variables of the problem.

Solving Algorithm

Initially, the model, with its objective to minimize costs, is solved using the CPLEX solver. This solver proves efficient for small to medium-sized dimensions, given the integration of location and hierarchical network design problems, as well as the incorporation of time periods. It can deliver an optimal solution within a reasonable computational timeframe. However, for larger dimensions within the solvable scope of CPLEX, an increase in the number of network and link levels results in a substantial rise in solution time. Consequently, the deployment of either exact or heuristic algorithms becomes indispensable for solving the proposed model. Various methods are available for addressing mixed-integer programming problems. In this study, we employed the SA algorithm to solve the model. Figure 1 presents diagram of the solution algorithm.

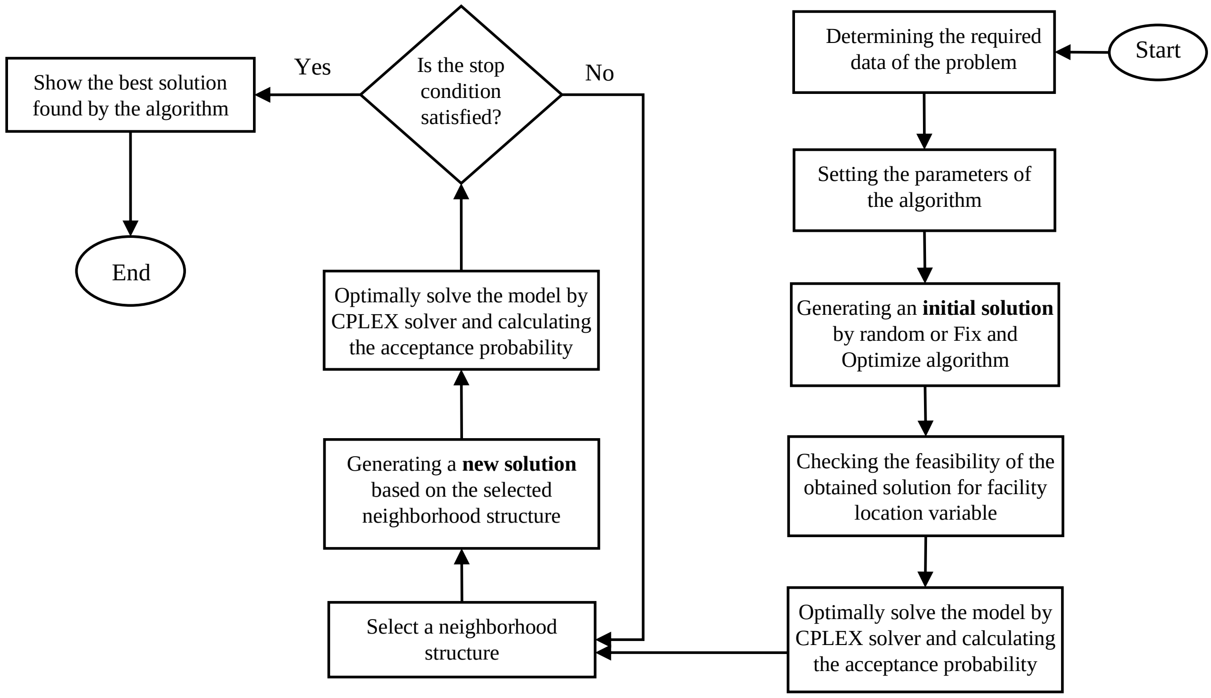

Figure 1. Flowchart of the solution algorithm

Solution Representation

The studied problem, as defined in the mathematical model, encompasses five categories of decision variables, comprising four binary variables and one continuous variable with values between zero and one. Additionally, two of the binary variables serve as auxiliary variables incorporated into the modeling process (variables Y_i^{lt} and R_{ijm}^t). The designed algorithm incorporates a variable associated with facility location in the solution structure, u_{il}^tguiding the exploration of the solution space. The remaining decision variables are determined by solving the mathematical model problem, assuming fixed values for this variable.

Generating an Initial Solution

To initialize the proposed simulated annealing algorithm, an initial solution for the problem is required. We utilize two distinct methods to generate these initial solutions: a simple random approach and an approach based on a fixed and optimized heuristic. The random approach serves as a baseline for comparison, while the fix-and-optimize approach aims to generate a more structurally desirable initial solution that improves the overall performance of the algorithm. Both of these approaches are described as follows.

Random Approach

The process for creating the initial solution involves a random approach, where a solution for the location variable, represented as u_{il}^{t}, is generated. Initially, a random solution is created by assigning values of zero and one to each location variable. Subsequently, the feasibility of the generated solution is assessed, taking into account the defined budget constraint. If the solution proves to be feasible within the specified financial constraints, it is saved as a parameter for the given problem. Following this, the relaxed problem is solved using CPLEX solver and the resulting solution is recorded for subsequent steps of the main algorithm.

Fix and Optimize Algorithm

In the proposed algorithm, generating an initial solution by random may lead to solutions that do not meet the expected desirability. For example, the obtained solution for variable u_{il}^{t} may be feasible in terms of the facility upgrade budget constraint, but may not be structurally desirable for the network. Therefore, finding an appropriate initial feasible solution can improve the performance of the algorithm. For this purpose, in this study, in addition to generating an initial solution randomly, the FO algorithm is also used to find an appropriate initial solution.

The fix and optimize algorithm, designed to determine an initial feasible solution, follows a series of key steps. Initially, the algorithm precisely solves the primary problem, assuming $t = 1$ and employing the CPLEX solver. During this stage, a feasible network is established, incorporating a specific number of edges and active facilities. The algorithm then iteratively increases the time period,t. Subsequently, it progresses through the steps until $t \leq T$, activating and upgrading facilities randomly while adhering to the budget constraint specific to each time period. Finally, the primary problem is resolved, assuming all variables of u_{il}^{t} obtained from previous stages remain constant. This sequential process ensures the generation of an initial feasible solution by iteratively adjusting facility location variables, thereby enhancing the overall effectiveness of the algorithm.

Checking the Feasibility of the Obtained Solution

The solution derived for the hierarchical facility location variable requires budget justification. Subsequently, the feasibility of the obtained solution is assessed. Within this model, during each time period, the budget constraint governing the upgrading of facilities is integral to the obtained solution. To validate the solution's feasibility in each period, the total budget amount is computed based on the summation of the upgrade costs (denoted as g_{il}^{t}) of facilities that have successfully transitioned to the upper level. If the total budget amount derived from the solution is lower than the predetermined budget, the solution is considered feasible. However, if the total budget amount exceeds the budgetary constraint, an iterative refinement process is initiated. During this iterative refinement, a facility is randomly selected, and its value is set to zero. This iterative process continues until a feasible solution is attained, aligning with the stipulated budget constraint.

Neighborhood Structures

One crucial element in neighborhood-based algorithms is the design of their neighborhood structure. In the SA algorithm, the process of generating the neighborhood structure should be formulated to satisfy two essential conditions: the possibility of creating a new neighborhood and the potential to generate all possible solutions. This study defines three neighborhood structures, ensuring compliance with these conditions.

The variable u_{il}^{t} is identified with the (i, l, t) index, where $(3, 2, 2)$: 1 corresponds to $u_{32}^{2} = 1$. The number of elements in this variable, denoted as len, is equal to the product of the sizes of the sets N,L and T. In all the defined neighborhood structures, a consistent procedure has been employed to select an element from the given solution. The key element is chosen randomly from the range $[1, len]$.

To facilitate a better understanding of the neighborhood structures, an example is provided with three nodes, two service levels, and two time periods. It is important to note that, after creating a solution using each neighborhood structure, it must be checked for adherence to the budget constraint and corrected if necessary.

A concise explanation of how each neighborhood structure generates a solution is provided below:

$$|N| = 3\,, |L| = 2, |T| = 2 \;\rightarrow\; u_{il}^{t} = \begin{Bmatrix} (1,1,1){:}\,1 & (2,1,1){:}\,0 & (3,1,1){:}\,0 \\ (1,1,2){:}\,0 & (2,1,2){:}\,1 & (3,1,2){:}\,1 \\ (1,2,1){:}\,0 & (2,2,1){:}\,0 & (3,2,1){:}\,1 \\ (1,2,2){:}\,1 & (2,2,2){:}\,1 & (3,2,2){:}\,0 \end{Bmatrix}$$

First Neighborhood Structure

Two distinct elements from the solution dictionary of u_{il}^{t} are chosen. To generate a new solution using the first neighborhood structure, the value corresponding to the first element is swapped with the value corresponding to the second element. For instance, in the context of the example under study, the process of generating a solution based on the first neighborhood structure unfolds as follows:

In the initial step of the first neighborhood structure, two unique elements are selected from the solution vector of u_{il}^{t}. In this instance, the first element is identified by number 3, and the second element is denoted by number 10. Subsequently, the value associated with the first element is exchanged with the value associated with the second element.

$$\boldsymbol{u_{il}^{t}}{:} \begin{Bmatrix} (\mathbf{1,1,1}){:}\,\mathbf{1} & (\mathbf{2,1,1}){:}\,\mathbf{0} & (\mathbf{3,1,1}){:}\,\mathbf{0} \\ (\mathbf{1,1,2}){:}\,\mathbf{0} & (\mathbf{2,1,2}){:}\,\mathbf{1} & (\mathbf{3,1,2}){:}\,\mathbf{1} \\ (\mathbf{1,2,1}){:}\,\mathbf{0} & (\mathbf{2,2,1}){:}\,\mathbf{0} & (\mathbf{3,2,1}){:}\,\mathbf{1} \\ (\mathbf{1,2,2}){:}\,\mathbf{1} & (\mathbf{2,2,2}){:}\,\mathbf{1} & (\mathbf{3,2,2}){:}\,\mathbf{0} \end{Bmatrix} \dashrightarrow \boldsymbol{u_{il}^{t}}{:} \begin{Bmatrix} (\mathbf{1,1,1}){:}\,\mathbf{1} & (\mathbf{2,1,1}){:}\,\mathbf{0} & (\mathbf{3,1,1}){:}\,\mathbf{0} \\ (\mathbf{1,1,2}){:}\,\mathbf{0} & (\mathbf{2,1,2}){:}\,\mathbf{1} & (\mathbf{3,1,2}){:}\,\mathbf{0} \\ (\mathbf{1,2,1}){:}\,\mathbf{1} & (\mathbf{2,2,1}){:}\,\mathbf{0} & (\mathbf{3,2,1}){:}\,\mathbf{1} \\ (\mathbf{1,2,2}){:}\,\mathbf{1} & (\mathbf{2,2,2}){:}\,\mathbf{1} & (\mathbf{3,2,2}){:}\,\mathbf{0} \end{Bmatrix}$$

Second Neighborhood Structure

Two distinct elements from the solution variable, u_{il}^{t} are chosen. To generate a solution using the second neighborhood structure, if the values of the selected elements are initially zero or one, they are inverted: zeros become ones, and ones become zeros. In simpler terms, if a facility is active, it will be deactivated, and if it is inactive, it will be activated.

For instance, considering the example under study, the process of generating a solution based on the second neighborhood structure is outlined as follows. The selected numbers for elements fall within the range [1, 12] are 1 and 12.

$$u_{il}^{t} = \begin{Bmatrix} (1,1,1){:}\,1 & (2,1,1){:}\,0 & (3,1,1){:}\,0 \\ (1,1,2){:}\,0 & (2,1,2){:}\,1 & (3,1,2){:}\,1 \\ (1,2,1){:}\,0 & (2,2,1){:}\,0 & (3,2,1){:}\,1 \\ (1,2,2){:}\,1 & (2,2,2){:}\,1 & (3,2,2){:}\,0 \end{Bmatrix} \longrightarrow u_{il}^{t} = \begin{Bmatrix} (1,1,1){:}\,0 & (2,1,1){:}\,0 & (3,1,1){:}\,0 \\ (1,1,2){:}\,0 & (2,1,2){:}\,1 & (3,1,2){:}\,1 \\ (1,2,1){:}\,0 & (2,2,1){:}\,0 & (3,2,1){:}\,1 \\ (1,2,2){:}\,1 & (2,2,2){:}\,1 & (3,2,2){:}\,1 \end{Bmatrix}$$

Third Neighborhood Structure

To introduce more perturbation in the solution space, the third neighborhood structure is defined as follows: Three different elements are selected from the u_{il}^{t} variable. Subsequently, the positions of the first and third elements are swapped. Following this, the value of the second element is flipped: if it was initially 0, it becomes 1, and if it was initially 1, it becomes 0. According to the presented example, the result obtained from the third neighborhood structure is equal to:

- The first selected element: 4

- The second selected element: 5
- The third selected element: 11

$$u_{il}^{t}:\begin{Bmatrix} (1,1,1):1 & (2,1,1):0 & (3,1,1):0 \\ (1,1,2):0 & (2,1,2):1 & (3,1,2):1 \\ (1,2,1):0 & (2,2,1):0 & (3,2,1):1 \\ (1,2,2):1 & (2,2,2):1 & (3,2,2):0 \end{Bmatrix} \qquad u_{il}^{t}:\begin{Bmatrix} (1,1,1):1 & (2,1,1):0 & (3,1,1):0 \\ (1,1,2):1 & (2,1,2):0 & (3,1,2):1 \\ (1,2,1):0 & (2,2,1):0 & (3,2,1):1 \\ (1,2,2):0 & (2,2,2):0 & (3,2,2):0 \end{Bmatrix}$$

Based on the third neighborhood structure, elements 5 and 11 are deactivated, and the fourth element is activated.

The Probability and Cooling Functions

To assess the validity of proposed solutions, a function is employed to determine solution acceptance using neighborhood structures in each iteration. In the context of the studied problem, if the objective function yields a value of zero, the solution is deemed invalid and excluded from further calculations. The acceptance criterion for selecting solution $f(j)$ over solution $f(i)$ is given by Equation (18):

$$\boldsymbol{p_r(j)} = \begin{cases} \mathbf{1} & \boldsymbol{f(j) < f(i) \neq 0} \\ \boldsymbol{e^{\left(\frac{f_i - f_j}{c_k}\right)}} & \boldsymbol{else} \end{cases} \tag{18}$$

The parameter c_k is derived from Equation (19):

$$c_{k+1} = \frac{c_k}{1 + \alpha_k c_k} \tag{19}$$

Here, α_k is a factor ranging between zero and one, typically set to 0.995 in most cases. In the SA algorithm literature, c_k is referred to as the cooling function. The algorithm can terminate when reaching a specific temperature, akin to completing a set number of iterations, serving as one of the stopping criteria.

Stop Criterion

The stop criterion in this research is defined as reaching a specific temperature, determined through an empirical trial-and-error approach.

Numerical Experiments

In this study, we examined the performance of the SA and heuristic algorithms by solving the problems of various sizes. The algorithms were implemented in Python environment to solve the proposed model. Sample problems were solved on a computer with a Core i7, 2.5 GHz processor, and 8 GB of internal memory. As there were no similar sample problems available in the literature to use as a benchmark, we addressed this gap by generating random samples for problems of different dimensions. The method of generating problem data in our research closely follows the approaches used in the study by Ghaderi and Jabalameli (2013) [17]. It is important to note that, in the mentioned studies, the purpose of generating data was to simulate real-world situations. Table 1 provides specifications for the problems investigated in this study. In addition, Table 2 presents the values assigned to each parameter.

Table 1. Specifications of Investigated Instances

Number of test problem	The sets for the problem			
	NODES	L	M	T
TP1	10	2	3	3
TP2	10	2	3	5
TP3	10	4	3	3
TP4	10	4	4	3
TP5	10	5	5	5
TP6	20	2	3	3
TP7	20	3	3	3
TP8	20	3	4	4
TP9	50	2	3	3
TP10	50	4	4	3

Table 2. Description of Investigated Instance Parameters

Parameter	Value	Reference
d_l^{kt}	$d_l^{k1} = U[50,100]$ $d_l^{kt} = [1+U(0,0.05)]d_l^{k(t-1)}, t \geq 2$	[17]
p_{ijm}^t	$p_{ij1}^1 = L_{ij}, p_{ij1}^2 = 1.2L_{ij}, p_{ij1}^3 = 1.4L_{ij}$ $p_{ij2}^t = [(0.06, 0.08)]\, p_{ij1}^t$ $p_{ij3}^t = [U(0.05, 0.03)]\, p_{ij2}^t$ $p_{ij4}^t = [U(0, 0.02)]\, p_{ij2}^t$	[17]
c_{ijm}^t	$c_{ij1}^1 = 1.3L_{ij}, \quad c_{ij1}^2 = 1.4L_{ij}, \quad c_{ij1}^3 = 1.5L_{ij}$ $c_{ij2}^t = [1+U(0, 0.05)]c_{ij1}^t$ $c_{ij3}^t = [1.5+U(0, 0.05)]c_{ij2}^t$ $c_{ij4}^t = [2+U(0, 0.05)]c_{ij3}^t$	[17]
h_{ijm}^t	$h_{ij1}^t = [U(0, 0.1)]\, c_{ij1}^t$ $h_{ij2}^t = [U(0.1, 0.3)]\, c_{ij2}^t$ $h_{ij3}^t = [U(0.3, 0.6)]\, c_{ij3}^t$ $h_{ij4}^t = [U(0.6, 0.9)]\, c_{ij4}^t$	[17]
g_{il}^t	$g_{il}^1 = U[100, 500]$ $g_{il}^t = [1+U(0.02, 0.1)]g_{il}^{t-1}, t \geq 2$	[17]
f_{il}^t	$f_{i2}^1 = 0.5g_{i2}^1, f_{i2}^2 = 0.6g_{i2}^2, f_{i2}^3 = 0.7g_{i2}^3$, $f_{i3}^1 = 0.5g_{i3}^1, f_{i3}^2 = 0.6g_{i3}^2, f_{i3}^3 = 0.7g_{i3}^3$ $f_{i4}^1 = 0.5g_{i4}^1, f_{i4}^2 = 0.6g_{i4}^2, f_{i4}^3 = 0.7g_{i4}^3$	[17]
S_{ijlm}^{kt}	$S_{ijlm}^{kt} = P_{ijm}^t d_{lk}^t$	[14]

Small Sample Network

We examine a small-scale example featuring a network with 10 nodes organized into three time periods, along with two-level facilities. The links and facilities are both arranged in a two-level structure. Initially, it is assumed that all centers and links belong to level 1. All relevant data for this instance has been generated according to the specifications outlined in Table 2. The network illustrated in Figure 2 depicts the configuration considered in this example.

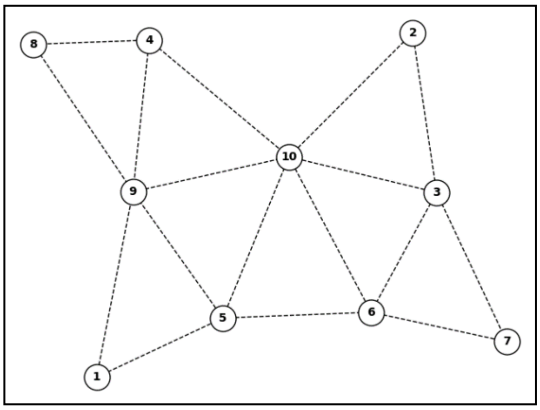

Figure 2. Illustration of the network used in the numerical example

The travel costs fluctuate across various levels, with the highest costs occurring at the lowest level and the lowest costs at the highest level. Similarly, operational and improvement costs for facilities and links adhere to a pattern where higher levels entail increased operational and improvement expenses. Table 3 provides details on the available budget for facilities and links, along with the budget consumed in each period for these respective levels.

Table 3. Available and Used Budget for Links and Facilities per Period

Period	Available Budget		Consumed Budget	
	Facilities	Links	Facilities	Links
1	220	350	215.69	344.43
2	228	370	152.87	350.58
3	230	400	215.92	361.34

After solving the model as illustrated in Figure 2, the following improvements have been made to the links:

- Links (1-9) and (2-3) have been upgraded to level three.
- Links (4-8), (8-9), (1-5), (5-6), (6-7), (3-7), and (2-10) have been upgraded to level 2.

In Figure 3a, the network configuration represents the setup established in the initial period to access level two services, centered at node 5. During this phase, node 5 was elevated to level two. Notably, links (1-9) and (2-3) were upgraded to level three, while other links were elevated to level two.

Figure 3b illustrates the network configuration in the first period to receive level 3 services, with center 9 being upgraded to this level. Similar to the previous instance, links (1-9) and (2-3) were improved to level 3, while other links were upgraded to level 2.

Transitioning to the second period, as depicted in Figure 3c, all previously upgraded links retained their respective levels. While it's plausible for links to ascend to higher levels in subsequent periods, in this case, all existing links maintained their initial levels, while new links were upgraded. Links (3-10), (4-9), and (9-10) were elevated to level three during this period.

Figure 3d, corresponds to the second period, portraying the interconnections between centers and the level of services provided. Notably, center 6 now routes to center 5 to access level 2 services, while all other centers opt for level 2 services at center 1. Consequently, the network configuration underwent alterations in the second period, as depicted in Figure 3e.

Algorithm Performance

The results of solving the problems using the proposed algorithms are presented in Table 4. The table provides a thorough examination of algorithmic performance across various test problems. CPLEX delivers both lower and upper bounds effectively, particularly for smaller instances. In TP7, CPLEX successfully converges to a solution with a relatively low relative error within a time frame of 18,000 seconds. However, the obtained gap for TP6 is relatively large. As the dimensions of the test problem increase, CPLEX becomes unable to find feasible solutions within the allocated maximum CPU time, 76,270 seconds, rendering the problem unsolvable (TP8 to TP10). Nonetheless, we compare the results obtained by the heuristics, considering the lower bound computed by CPLEX as a reference.

In contrast, the SA algorithm, employing both random initial solutions and a fixed-and-optimize approach, consistently produces solutions with lower errors. Notably, the SA with FO initial solution demonstrates commendable performance, achieving solutions with minimal relative errors across test instances TP1-TP7. It also outperforms random SA, reaching solutions with significantly improved quality. Moreover, the FO algorithm efficiently discovers good initial solutions within a short computation time. Consequently, SA consistently outperforms CPLEX in terms of computational efficiency, especially evident in test problems TP8 to TP10. Overall, these results underscore the potential of SA algorithms, particularly those utilizing FO initial solutions, as efficient alternatives for solving intricate optimization problems, surpassing CPLEX in specific scenarios.

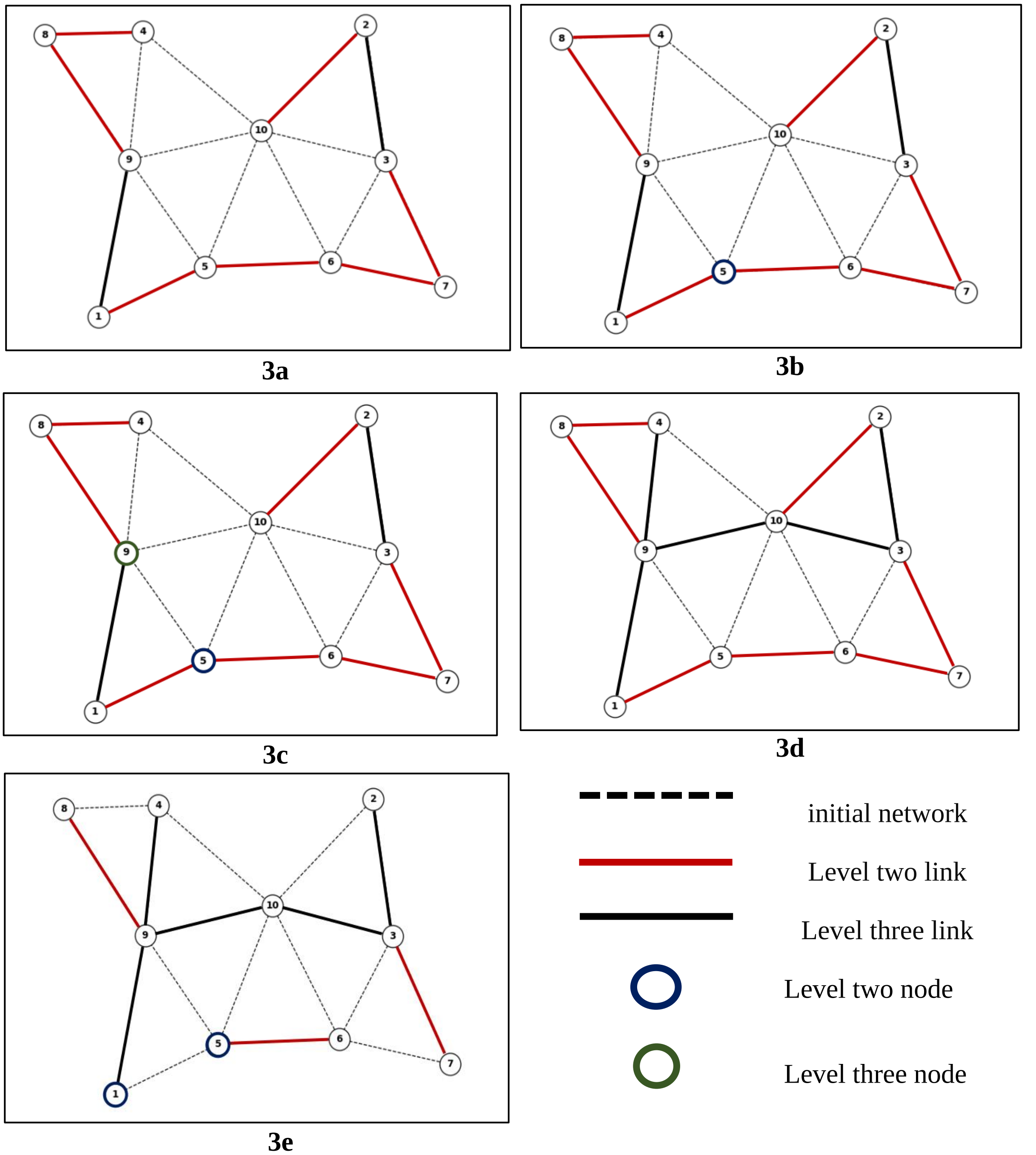

Figure 3. The outcome derived from solving the model depicted across two distinct time period

Table 5 presents the performance of various algorithms in reaching the optimal solution within a specified time for the test problems. As anticipated, SA requires less CPU time to solve the problems compared to CPLEX. Furthermore, SA with FO consistently exhibits faster convergence, achieving solutions in significantly shorter times across the test problems compared to random SA. Overall, these findings underscore the effectiveness of the SA algorithm in efficiently reaching optimal or near-optimal solutions within a shorter timeframe compared to CPLEX across a diverse range of test problem.

Table 6 presents the algorithm's performance in discovering improved solutions for each neighborhood structure during every iteration. In the random SA algorithm, the total number of improved solutions in neighborhood structures 1 to 3 were 65, 72, and 47, respectively.

Correspondingly, these values in the SA with FO algorithm are 48, 71, and 57. Notably, each neighborhood structure has proven effective in enhancing solution search, with the second structure exhibiting superior performance compared to the other two structures.

Table 4. The Algorithms Result from Solving Sample Problems

Test Problem	CPLEX			SA with Random Initial Solution		SA with FO based Initial Solution		FO
	Lower bound	Upper bound	Relative error	The best solution	Relative error	The best solution	Relative error	
TP1	3393798.9	3393798.9	0.0	3393798.9	0.0	3393798.9	0.0	6542783.9
TP2	113150.3	113150.3	0.0	114120.1	0.84	113235.8	0.07	576892.7
TP3	3854027.7	3854027.7	0.0	3855870.9	0.047	3854102.7	0.0019	5452371.8
TP4	4044247.4	4044247.4	0.0	4045879.2	0.04	4044325.0	0.0019	7784521.3
TP5	4612076.0	4612076.0	0.0	4613129.8	0.022	4612105.0	0.0006	9126452.4
TP6	3565.1	4548.9	15.78	4893.2	27	3624.8	1.6	6437852.6
TP7	105775.7	106110.6	1.36	107526.8	1.6	105902.8	0.19	562187.3
TP8	6332.0	NA	NA	18369.1	65	15604.8	59	19572.6
TP9	12255.3	NA	NA	22472.9	45	18864.9	35	22381.7
TP10	1262.3	NA	NA	14527.1	91	9659.8	86	15292.1

Table 5 . Time Comparison for Solving Sample Problems

Test Problem	CPLEX	The time of random SA to reach the best solution	The time of SA with FO to reach the best solution	The time of FO to reach the solution
TP1	168	105	60	7
TP2	98	89	32	6
TP3	430	205	146	10
TP4	760	996	389	9
TP5	14790	1374	985	10
TP6	18000	2293	1784	14
TP7	18000	3725	2925	18
TP8	76270	3657	2521	25
TP9	76270	3635	2125	32
TP10	76270	5632	2458	45
Average	28105.6	2171.1	1342.5	17.6

Sensitivity Analysis

To assess the model's sensitivity to key parameters, variations were introduced in the budget and levels parameters for facility and network line upgrades. Sample problem TP6, representing an average scenario, was chosen for this investigation. Initially, a fixed value was assigned to the facilities and links budget, and subsequently, this fixed value was systematically increased or decreased by a certain percentage to assess the model's sensitivity to these parameters. As illustrated in Table 7, a decrease in the budget parameter correlates with an increase in solution time and objective function value. This observed phenomenon arises from the reduction of the network upgrade budget, encompassing both its lines and facilities. This reduction prompts the utilization of lines with higher transfer costs and an increased distance for service provision. A 40% reduction in the budget results in the depletion of the solution space, rendering the problem infeasible. Conversely, augmenting the budget leads to cost reduction, with this effect being prominent up to a 10% increase in the budget across all time periods. Another noteworthy observation is the prolongation of problem-solving time due to a diminished network upgrade budget. A decrease in budget leads to longer computation times, reflecting the computational complexity associated with smaller resource allocations. Specifically, a 30% budget reduction extends the solution time from 14,760 seconds to 121,587 seconds. In contrast, an increase of over 5% in the budget facilitates the identification of optimal solutions within less than 900 seconds. This intricate relationship between budget variations, cost implications, and computational efficiency underscores the nuanced dynamics at play in optimizing the proposed model.

Table 6. Algorithmic Performance in Discovering Improved Solutions for Each Neighborhood Structure During Every Iteration

Test Problems	**First iteration**			**Second iteration**			**Third iteration**			**Fourth iteration**			**Fifth iteration**			**Type of solution approach**
	First neighborhood	Second neighborhood	Third neighborhood	First neighborhood	Second neighborhood	Third neighborhood	First neighborhood	Second neighborhood	Third neighborhood	First neighborhood	Second neighborhood	Third neighborhood	First neighborhood	Second neighborhood	Third neighborhood	
TP1	0	2	0	1	2	0	0	1	0	0	2	0	1	0	0	**Random SA**
TP2	1	1	0	0	1	0	1	0	0	1	1	1	0	1	1	
TP3	1	0	1	0	2	1	1	0	1	0	0	2	0	2	0	
TP4	1	1	0	0	1	1	2	1	2	2	0	0	0	2	1	
TP5	2	1	0	2	2	0	1	0	0	0	1	0	2	0	0	
TP6	1	3	1	1	2	1	0	2	0	0	2	3	2	1	2	
TP7	2	2	1	3	1	2	2	4	0	2	3	1	3	3	0	
TP8	3	0	1	2	0	3	2	0	2	0	0	4	1	1	1	
TP9	1	3	2	5	0	0	3	2	1	0	2	1	2	3	1	
TP10	4	2	0	0	2	3	2	1	2	3	4	3	2	5	1	
Sum	**16**	**15**	**6**	**14**	**13**	**11**	**14**	**11**	**8**	**8**	**15**	**15**	**13**	**18**	**7**	
TP1	0	0	1	1	0	2	1	1	1	0	0	1	0	1	0	**SA with FO**
TP2	1	1	0	1	0	0	1	1	0	0	1	1	1	2	0	
TP3	0	1	0	0	1	2	0	0	2	0	0	0	2	0	2	
TP4	1	0	1	0	0	1	1	2	1	2	2	0	0	0	2	
TP5	0	0	1	1	2	1	1	0	1	1	2	0	2	2	0	
TP6	3	1	3	1	1	2	0	0	2	0	0	4	0	2	2	
TP7	2	2	2	1	3	0	2	2	2	0	2	1	3	1	2	
TP8	1	1	3	0	0	2	4	2	0	2	0	0	4	1	1	
TP9	1	1	2	3	0	3	0	1	2	1	0	3	1	0	3	
TP10	2	2	2	3	1	4	1	3	0	3	2	3	2	1	3	
Sum	**11**	**9**	**15**	**11**	**8**	**17**	**11**	**12**	**11**	**9**	**9**	**13**	**15**	**10**	**15**	

Table 7. Sensitivity analysis of the model to the budget parameter for TP2

Budget Changes Percent	**Link Budget per Period**			**Facility Budget per Period**			**Objective Function Value**	**CPU Time**
	t_1	t_2	t_3	t_1	t_2	t_3		
40	1200	1150	1050	1250	1200	1150	3504.26	370
30	1125	1075	975	1175	1125	1075	3504.26	369
20	1050	1000	900	1100	1050	1000	3504.26	341
15	1012.5	962.5	862.5	1062.5	1012.5	962.5	3504.26	428
10	975	925	825	1025	975	925	3504.26	529
5	937.5	887.5	787.5	987.5	937.5	887.5	3536.28	812
0	900	850	750	950	900	850	3728.75	14760
-5	862.5	812.5	712.5	912.5	862.5	812.5	4022.18	37191
-10	810	765	675	855	810	765	4164.58	53720
-15	765	722.5	637.5	807.5	765	722.5	4259.87	69631
-20	720	680	600	760	720	680	4548.9	80267
-30	630	646	525	665	630	646	5183.86	121587
-40	540	510	450	570	540	510	NA	-

Conducting another sensitivity analysis on the problem involves varying the number of facilities, link levels, and time periods to comprehensively grasp the impact of these variables. Based on the data provided in Table 11 for TP1, it is evident that as the number of facilities, link levels, and time periods increase, both the objective function's value and CPU time experience a simultaneous increase. This observation implies that the problem becomes more intricate and computationally demanding with a larger number of facilities and link levels, as well as over extended time periods.

Specifically, the objective function value, indicative of the total cost, rises in tandem with the scale of the problem—represented by an increase in the number of facilities, link levels, and time periods. Meanwhile, the CPU time, reflecting the computational effort required to solve the problem, exhibits an exponential increase. In this regard, with a 50% to less than 100% increase in the considered levels, the computational time correspondingly escalates to 1700 and 15000 times higher, respectively.

Table 8. Sensitivity Analysis of the Model to Varying Facility Levels and Links

Number of Facility Levels	Number of Link Levels	Number of Time Periods	Objective Function Value	CPU Time
2	2	2	2905264.8	0.22
2	3	3	3393798.9	168
3	3	3	4230554.8	400
4	4	3	4044247.4	3760
5	5	3	4452472.05	12487

Conclusion

Over the past decades, combinatorial optimization problems have garnered considerable attention. However, research studies have relatively limited coverage of hierarchical facility location and network design problems, which involve facilities and links at different levels. This research addresses this gap by developing a model that incorporates these factors, particularly relevant to the optimization of public service systems. These facilities are interconnected through the services they provide, and enhancing access to these facilities involves identifying links that require upgrading to a higher level. This problem is crucial due to its impact on public services and the need to make efficient use of limited resources. Previous studies in this area have not considered time periods and budget constraints, which are common in real-world scenarios. Therefore, incorporating these factors leads to a more realistic and efficient model that better reflects the complexities of decision-making in real-world applications. Since organization's managers face budget constraints when upgrade networks, it is crucial to consider these constraints in their decisions. Furthermore, network design decisions should be reevaluated at different periods. Hence, this research can greatly assist organizational managers and planners in making network design decisions in hierarchical networks by considering these two issues.

The proposed model was solved using CPLEX solver for small and medium sizes, and its sensitivity to budget parameters was evaluated and analyzed. Our findings indicate that as budget values decrease below a certain threshold, both the objective function value and the solution time increase significantly. In this study, several problems of various dimensions were solved using the SA algorithm to examine the algorithm's performance and the model's behavior, and the results were compared. For solving the developed model, three neighborhood structures were proposed. Based on the presented results, it can be concluded that in the random SA problem, the efficiency of the second neighborhood structure surpasses the first and third ones. Also, the third neighborhood structure has the weakest performance in terms of improvement compared to the other two neighborhoods. The SA algorithm proposed in this study consistently outperforms a random SA algorithm in terms of speed and quality in solving various problems. The results of sensitivity analysis indicate that as budget values decrease below a certain threshold, both the objective function value and the solution time increase significantly. Conversely, as budget values increase, the model's sensitivity to these parameters decreases. In the next step, the model's sensitivity analysis was performed with respect to facility levels. The results show that as the number of facility levels and links increases, both the solution time and the objective function value increase.

The study suggests that by considering more realistic factors such as uncertainty in future applications, models can be improved. While our model assumes deterministic parameters, incorporating uncertainty through methods such as fuzzy logic or stochastic models could enhance the model's realism and provide more robust solutions. We also suggest considering a multi-objective model and allocating human resources to each center as one of the decisions to expand hierarchical facility location and network design models. Moreover, to better meet demand, the objective function can be adjusted to minimize unmet demands. In crisis situations, we suggest that different criteria should be taken into account when determining the location of medical centers. An exact solution method such as the branch and bound method is also suitable for solving the proposed model. The passage concludes by proposing the design of a network that takes crises into account in future research.

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