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RESEARCH PAPER

A Graph Theoretic-Based Approach for Optimizing Location-Routing and Crew Assignment in Police Patrols

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Abstract

Ensuring urban safety and preventing crime are paramount responsibilities of municipal managers. Police patrolling plays a crucial role in securing urban areas, yet limited resources and budget constraints necessitate an efficient patrolling system. This study introduces a multi-objective mathematical programming model aimed at maximizing the total effectiveness of a police patrolling system while minimizing associated costs. The approach utilizes a two-stage bi-objective Mixed Integer Linear Programming based on the K-windy postman problem. In the first stage, the model determines optimal locations for constructing police stations, while in the second stage, it allocates vehicle modes, plans patrolling routes, and assigns crew members to each vehicle. For small-size problems, the model is solved using the ε-constraint method, whereas a cluster-based algorithm is proposed for tackling medium- and large-size problems. To illustrate the model's applicability, a real-world case study involving various city zones in Tehran, Iran is examined. The proposed model offers a practical tool for optimizing police patrolling systems in similar urban settings.

Keywords: Chinese Postman Problem (CPP), Multi-Objective Mathematical Modeling, Arc Clustering, Crew Scheduling, Police Patrolling.

Introduction

Physical as well as emotional security plays a substantial role in human life. When all aspects of security are fulfilled, citizens become more dynamic and active in their social activities. Security assurance, in all aspects such as economic, social, and cultural, is one of the main duties of any government. Crime occurrence, which is one of the most important issues strongly affecting the safety of people in a region, can be reduced by police patrolling (Sherman & Weisburd, 1995). Implementing a patrolling system incurs expenditures for governments due to fixed and variable costs. As a result, improving the performance of city patrolling systems is considered a significant challenge.

The efficient allocation of police resources is crucial for maintaining public safety, and this necessitates the application of advanced techniques from operations research and logistics. The police patrolling routing problem is a specialized variant of the vehicle routing problem (VRP), which is a well-studied problem in operations research. The VRP involves determining the optimal set of routes for a fleet of vehicles to traverse in order to deliver goods or services to various locations. Similarly, in the police patrolling context, the objective is to design patrol

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routes that maximize coverage and visibility while minimizing response time to incidents and operational costs. Incorporating principles from vehicle routing and crew scheduling, the police patrolling routing problem also addresses the constraints related to patrol officers' working hours, work shift patterns, and the geographical layout of the patrolled area. Effective patrolling not only deters crime through visible police presence but also ensures rapid response to incidents, thereby enhancing public trust and safety. Advanced algorithms and optimization techniques, such as integer programming, heuristic methods, and metaheuristics, are often employed to solve these complex problems.

Recent advancements in technology, such as Geographic Information Systems (GIS), real-time data analytics, and predictive policing models, have further refined the strategies for police patrol routing. These technologies enable law enforcement agencies to analyze crime patterns, predict potential hotspots, and allocate patrol units more effectively. By integrating these technologies with robust operational research methodologies, it is possible to develop dynamic patrol routing systems that adapt to real-time information, thereby improving the efficiency and effectiveness of police operations. Moreover, the economic implications of police patrolling systems cannot be overlooked. Governments must balance the need for security with budgetary constraints, making the optimization of patrolling routes a critical area of study. Reducing fuel consumption, minimizing overtime costs, and ensuring the equitable distribution of patrol resources are essential considerations in the design of an optimal patrolling strategy.

This paper aims to explore the police patrolling routing problem by examining various models and approaches that have been proposed in the literature. It will delve into the integration of vehicle routing problem methodologies with real-world constraints specific to police patrolling. The rest of this paper is organized as follows: In Section 2, conducted on similar problems are reviewed/ In Section 3, the problem and the mathematical model of the suggested problem are described. Section 4 presents the problem solution approaches. Lastly, Section 5 is dedicated to concluding remarks and directions for future research.

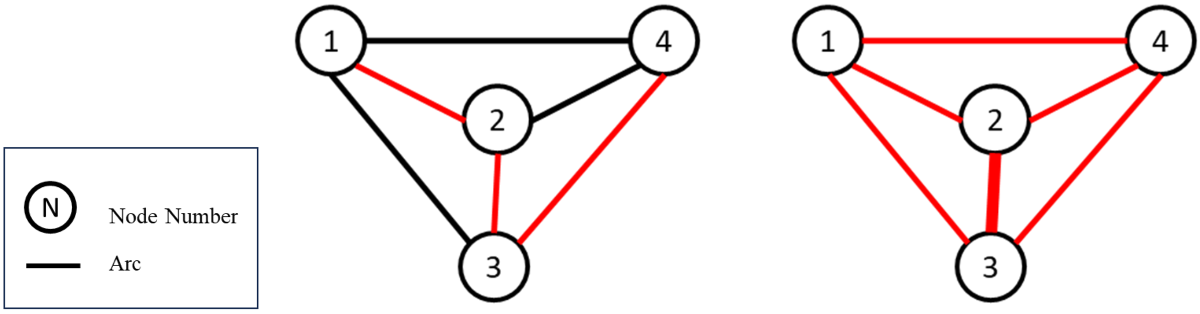
Literature Review

Police patrolling is a critical activity in urban safety and crime prevention, necessitating efficient routing and scheduling strategies to maximize effectiveness while minimizing costs. Recent studies have addressed various aspects of optimizing police patrol routes and schedules using advanced mathematical models and algorithms. In this section, we review the literature related to the problem domain discussed in this research.

Arc Routing Problems

Routing problems are divided into two broad categories: arc routing problems (ARP) and node routing problems (NRP). In node routing problems, the objective is to find a route that covers all or a subset of nodes, while in arc routing problems, the objective is to find a route that covers either all or a subset of a graph. Fig 1 demonstrates how node routing and arc routing work. When the objective is to find the route that covers all nodes, a feasible answer is 1→2→3→4. However, when the objective is to find a route that covers all arcs, a feasible answer is 1→3→2→1→4→2→3→4. In the police patrolling problem, the coverage of all streets is the objective; thus, the problem should be considered as an Arc Routing Problem.

Arc routing problems encompass various applications such as street sweeping (Bodin & Kursh, 1978), newspaper delivery (Applegate et al., 2002)), traffic monitoring (Li et al., 2018), winter road maintenance such as snow plowing and salt spreading ((Perrier et al., 2007) and (Perrier et al., 2008)), garbage collection (Amponsah & Salhi, 2004), prize collecting (Vincent & Lin, 2015), and police patrolling ((Shafahi & Haghani, 2015), (Chawathe, 2007), (Takamiya & Watanabe, 2011)), all requiring efficient routing strategies to cover predefined arcs or edges in a network.



a)Node Routing b)Arc Routing

Figure 1. Node Routing and Arc Routing

A considerable number of arc routing applications are of the Chinese Postman Problem (CPP) type. The CPP, also known as the route inspection problem, was first proposed by the Chinese mathematician Mei-Ko (1962). It involves finding the shortest route for a mailman to cover all the arcs of a closed circuit. In real cases, especially in large networks, allocating one postman (vehicle) to cover the whole network might not satisfy time constraints. Therefore, it may be logical to use more than one vehicle to minimize the total cost or the time of network routing. Considering multiple postmen in the CPP is called the k-CPP. Thomassen (1997) shows that the k-Chinese Postman Problem is NP-hard. In the k-CPP problem, the objective is generally to cover all the arcs in the network using k postmen in minimum time or at minimum cost. Akyurt et al. (2015) presented a mathematical model for road maintenance operations in winter and used a genetic algorithm to solve their model. However, the Min-Max k-CPP (MM k-CPP), which was first mentioned by Frederickson et al. (1976), is also a common objective in the literature. This objective aims to minimize the cost of the most costly tour. Ahr and Reinelt (2002) introduced a heuristic and lower bound, and later in 2006 they introduced a Tabu Search for solving the MM k-CPP (Ahr & Reinelt, 2006).

In the police patrolling system for a city, covering each street is one of the main objectives. The direction of a police vehicle on a two-way street might not be a real issue since the street is covered regardless of vehicle direction, unlike most arc routing problems such as snow plowing, order delivery, salt spreading, and road maintenance. However, in the case of an uphill or downhill street or varied traffic load, the direction of vehicles is important. Furthermore, in the case of having a one-way street, the time or cost for traversing the opposite direction could be set to a large number. Thus, in real cases, the appropriate routing for police patrolling problem could be formulated as a Windy Postman Problem (WPP). In the Windy Postman Problems, introduced by Minieka (1979), the graph is undirected and the cost or time of traversing each arc in each direction might be different. The IP formulation of WPP is as follows where C:= 〚(C_ij,C_ji)_({i,j}∈E) is the cost function of a WPP and x:= 〚(x_ij,x_ji)〛_({i,j}∈E) is the incidence vector of a WP tour of graph G (Win, 1987):

$$
\begin{aligned}
& Min \sum_{\{i.j\}\in E} (C_{ij}x_{ij} + C_{ji}x_{ji}) \\
& x_{ij} + x_{ji} \geq 1 \qquad \forall \{i.j\} \in E \\
& \sum_{\{i.j\}\in \delta(i)} \left(x_{ij} - x_{ji}\right) = 0 \quad \forall i \in V \\
& x_{ij}.x_{ji} \geq 0 \text{ and integer} \qquad \forall \{i.j\} \in E
\end{aligned} \tag{1}
$$

According to Eiselt et al. (1995), WPP contains CPP, DCPP (Directed Chinese Postman

Problem), and MCPP (Mixed Chinese Postman Problem), making it NP-hard. Numerous variants of WPP are found in the literature, used by researchers to address real-world problems. Lum et al. (2017) tackled the Windy Rural Postman Problem with Zigzag Time Windows (WRPPZTW), where some streets can be serviced in a single traversal using zigzag service. They developed a hybrid heuristic that combines insertion and local search techniques with integer programming. Their heuristic was compared to an exact procedure on small instances and tested for scalability on larger grid instances. Keskin et al. (2023) addressed a variant of the Hierarchical Windy Postman Problem (HWPP) with linear precedence and variable service costs. They proposed an integer linear mathematical model and developed a heuristic that adapts a layer algorithm to handle asymmetric costs, demonstrating that it performs faster and more effectively than a commercial solver on test instances. Samanifar et al. (2024) introduced two models of the WPP and rural WPP, incorporating uncertainty theory, which had not previously been applied to these models. They transformed the uncertain problem into a deterministic one and used the Lagrangian heuristic method to solve it. They presented corresponding Lagrangian algorithms for both the WPP and rural WPP, applied them to examples, and solved them using both direct and Lagrangian methods. Khorramizadeh and Javvi (2024) formulated an integer programming model for the Windy Profitable Location Rural Postman Problem (WPLRPP) and proved a theorem regarding the dimension of the polyhedron. They adapted valid inequalities and developed a branch-and-cut algorithm, showing it solves larger instances faster than other algorithms in the literature. In this paper, a bi-objective model is introduced, which considers the minimization of patrolling costs and the maximization of its effectiveness, and a k-WPP based formulation is chosen to closely match real-world cases for the police patrolling problem.

Literature Reviews and Classification Schemes

There is extensive literature on the vehicle routing problem, and numerous review articles have been written on this topic. However, since there are fewer articles on police patrolling routing problems, there are also fewer review articles available in this area. Samanta et al. (2022) conducted a comprehensive review of police patrol methods, focusing on operations research approaches. They presented a novel classification scheme for organizing existing research based on problem type, objective, and modeling approach, revealing practical challenges and opportunities for future research in urban security and smart city planning. Dewinter et al. (2023) reviewed 30 articles on the dynamic vehicle routing problem (DVRP) related to policing to identify suitable methods for the police patrol routing problem (PPRP). They highlighted that hybrid genetic algorithms, routing policies, and local search methods are the most valuable solutions for this problem. Thabet et al. (2023) conducted a systematic literature review to classify common objectives, problems, and solutions in optimizing police patrol routes and predicting crime locations. They analyzed 31 research papers, categorizing the methods into five main types: machine learning, mathematical, simulation, stochastic, and recommendation systems.

Mathematical Formulations and Optimization Models

Each problem can be modeled in various ways depending on different conditions and assumptions, and different solution methods can be used for it. Police patrolling problems are no exception to this rule. So far, a wide range of assumptions, models, and solution methods have been chosen by researchers for this category of problems. Reis et al. (2006) presented an evolutionary multi-agent-based simulation tool named GAPatrol to design effective patrol route strategies. Chen (2012) presented a mathematical formulation for the patrol route planning problem and proposed an algorithm developed from the cross entropy method to solve small-size and an approximate CE algorithm for large-size problems, balancing convergence time with optimality. Takamiya and Watanabe (2011) proposed a route planning method for the police patrol routing problem to minimize the response time estimated using the Network Voronoi Diagram (NVD) and used Local Search to optimize the routes. Willemse and Joubert (2012) presented a heuristic algorithm based on Tabu Search for the problem of road patrolling. Keskin et al. (2012) developed a mixed integer linear programming model for determining patrol routes for state troopers and solved the model using local and Tabu Search-based heuristics.

Chircop et al. (2013) proposed a column generation approach for routing and scheduling patrol boats to provide complete patrol coverage. Li and Keskin (2014) presented a mixed integer linear programming model for the dynamic patrol routing problem for state troopers with the objective of maximizing the critical location coverage benefit and minimizing total costs, and proposed a heuristic algorithm to solve the problem. Shafahi and Haghani (2015) presented a mixed integer programming formulation for security patrolling and discussed four different cases of the model. Dewil et al. (2015) modeled the maximum covering and patrol routing problem (MCPRP) as a minimum cost network flow problem (MCNFP) and showed that several practical additions to the MCPRP, like overlapping work shifts and different origin and destination locations of patrol cars, can be modeled using multi-commodity MCNFP.

Muaafa and Ramirez-Marquez (2017) proposed a multi-objective heuristic optimization approach for patrolling strategy improvement, aiming to minimize vulnerability and cost. Chen et al. (2023) developed a novel real-time patrol route planning algorithm using deep reinforcement learning, specifically the Integrated Double Q-Network (IDQN) method. They empirically tested the algorithm, demonstrating its practical viability and utility in enhancing police patrolling efficiency and urban security management. Joe et al. (2023) addressed the challenge of dynamically dispatching and rescheduling police patrols in response to incidents by developing a Deep Reinforcement Learning-based solution. Their approach uses neural networks with Temporal-Difference learning and a rescheduling heuristic to optimize both incident response times and patrol presence. They also introduced a reward function that balances these dual objectives without needing predefined weights. Jiang et al. (2022) proposed a model to optimize the allocation and patrol paths of city inspectors to minimize average response time and the number of inspectors. They developed a priority patrol and multi objective genetic algorithm (DP-MOGA) and tested it with data from Zhengzhou, China, demonstrating that their algorithm efficiently generates patrol routes and outperforms existing methods in stability and efficiency.

Tohyama and Tomisawa (2022) introduced the police officer patrol problem (POPP), an edge routing decision problem related to the vertex cover problem, where a single police officer must visually confirm all streets from intersections. They proved that solving the POPP on mixed graphs is NP-complete. Katole et al. (2023) developed a distributed online algorithm to balance patrolling between priority and non-priority locations, ensuring that non-priority sites are visited within finite time frames. The algorithm creates offline patrol routes called "Rabbit Walks," which consist of three segments for exploring non-priority locations.

Chen et al. (2022) investigated a patrol routing problem to combat maritime crime by proposing a novel approach to identify suspicious ships and enhance patrol efficiency. They developed three mathematical programming models tailored to different scenarios based on the availability of aerial photographs and conducted extensive numerical experiments to validate their models' effectiveness and efficiency. Rumiantsev et al. (2023) proposed a method to construct optimal patrol routes in terrain using a modified Hamiltonian circuit search algorithm. This method efficiently generates closed paths on terrain maps with over 100 vertices, significantly reducing execution time compared to standard brute-force algorithms. Their algorithm demonstrates 17 times lower time complexity growth, enabling real-time performance for larger graphs. Sá et al. (2022) addressed the challenge of optimizing police patrol routes in crime-prone urban areas by creating the PolRoute-DS dataset. Wong et al.

(2023) addressed the challenge of police patrol scheduling by proposing a Multi-Agent Reinforcement Learning (MARL) solution to manage the Dynamic Bi-objective Police Patrol Dispatching and Rescheduling Problem (DPRP). They utilized an Asynchronous Proximal Policy Optimization-based (APPO) actor-critic method to dynamically reschedule patrols, improving both computational efficiency and solution quality compared to existing heuristic-based RL approaches. Kajita et al. (2024) implemented an optimization algorithm for police patrol routes based on a crime prediction map to deter cable theft in Belo Horizonte, Brazil. Their field experiment over two months showed a 79% reduction in crimes in the targeted area and demonstrated that optimized patrols had a significant crime deterrent effect, even extending to areas not directly patrolled. They used GPS data to assess and validate the impact of these optimized patrols.

Our Contribution

The main contributions of this research may be outlined as follows.

- Model assumptions and mathematical formulation
  Numerous studies have been conducted on police patrolling problems, and various methods have been proposed to solve them. Additionally, different assumptions have been considered for defining the problem. According to our review, the majority of research in the field of police patrol routing focuses primarily on routing and scheduling, while fewer studies address location-routing problems and crew assignment. To the best of our knowledge, no paper examines location-routing and crew assignment for police patrolling simultaneously. Addressing these aspects concurrently could significantly impact police patrolling issues. Determining the optimal locations for police stations throughout the city, identifying the routes that patrol vehicles should follow, and assigning crews based on their skill levels, specialties, and wages are particularly crucial, especially when various scenarios are possible. For instance, specific situations such as bombings or hostage situations may require different expertise. The practical benefit of our model lies in its ability to optimize police station locations, patrol routes, and crew assignments in a unified framework. This is crucial in urban environments where crime patterns vary, and different scenarios such as bomb threats or hostage situations may arise. In such cases, assigning police officers with the right skill sets and expertise to the right locations at the right time can dramatically improve response effectiveness, potentially saving lives. This model also helps reduce operational costs by optimizing routes and scheduling based on wages and skill levels, ensuring that limited resources are used most effectively. The real-world applicability is clear, as law enforcement agencies face constant pressure to improve response times and resource allocation.

- Arc clustering approach
  A vast variety of methods have been introduced for solving police patrolling problems, each demonstrating varying degrees of effectiveness. Given that this issue is a location-routing problem and simultaneously involves the allocation of police stations along with routing, we hypothesized that clustering could yield good performance. Considering this problem as an edge-routing problem, we developed a novel edge clustering algorithm tailored for this type of problem, which exhibits very good performance. In practice, the clustering approach helps law enforcement agencies design patrol routes that are more logical and efficient, reducing overlap and ensuring comprehensive coverage of high-priority areas. This not only improves patrol efficiency but also minimizes unnecessary travel, saving time and fuel costs. Moreover, by clustering streets based on their geographical proximity and risk level, police departments can dynamically adjust patrols to focus on areas with higher crime rates or

urgent situations, making patrolling more adaptive and responsive to real-time conditions.

Problem Description

In the presented mathematical model, a real application of the k-windy postman problem is considered. One of the challenges in security patrolling is determining the optimal number of police stations and their locations to minimize the total cost while maximizing effectiveness. Therefore, a bi-objective mathematical model is presented to optimize the trade-off between costs and effectiveness. Effectiveness is defined as a quality measure that needs to be quantitatively assessed through expert opinion and encompasses several dimensions: response times (how quickly patrol units can respond to incidents), coverage quality (how well the patrol routes cover high-risk or high-crime areas), resource optimization (how efficiently resources like vehicles, personnel, and time are utilized), and incident resolution (how well the assigned crews' expertise matches the type of incident they are responding to, such as bomb threats or hostage situations).

Another challenge is the variety of scenarios in the police patrolling problem. in these types of problems, various scenarios can arise, and the probability of each scenario can be significant. For instance, there may be times when the probability of bombings or terrorist attacks is higher, while at other times, the likelihood of theft is greater. When the probability of a bombing is higher, personnel with bomb disposal expertise can have a greater impact on operations. Conversely, in a theft scenario, those with expertise in theft investigation or high-speed pursuit can be more effective. Additionally, vehicle selection plays a crucial role; bomb-equipped vehicles might be more suitable in bombing scenarios, whereas motorcycles could be more effective in pursuit scenarios. Given the importance of scenario consideration in this problem, we have modeled it as a multi-scenario problem.

The proposed model also considers the possibility of multimodal vehicle selection. It is assumed that the number of each vehicle mode is accessible in the market. Determining how many of each vehicle should be bought from the market might cause a significant cost reduction and/or increase in effectiveness. For example, in security patrolling, different vehicles (e.g., car, motorcycle) could be used, and each of them might differ in their fixed cost, variable cost, effectiveness rate, and capacity. The effectiveness rate of each mode determines the preventive effect on crime reduction in society. The presented model assumes equivalent length for each work shift; therefore, all vehicles are required to return to their depot stations for reporting before the end of the work shift. Furthermore, the minimum number of times each arc should be traversed in each work shift and under each scenario can be set as parameters. This assumption is vital for crime hotspot areas.

The patrolling problem involves various issues, such as crew assignment and scheduling. Several challenges in patrolling crew scheduling lead to more complex situations:

1. More than one person is usually allocated to each vehicle.
2. The wages of personnel are usually high due to the skills they possess, the difficulty of their tasks, and the associated risks.
3. There are different experts (e.g., driver, bomb disposal expert, narcotics officer).
4. The grades of personnel (e.g., officer, major) vary.

We tackled the aforementioned challenges by defining related sets for each crew member. Each crew member is defined by a grade and a set of skills that determine their effectiveness. Additionally, the wage of each crew member and the maximum number of work shifts a crew member is allowed to work in a day can be set as parameters. Also, a crew member cannot work two consecutive work shifts. The minimum number of expertise and grades under each scenario can be defined in the model. The sets and indices, parameters, and decision variables used in the presented model are summarized in Table 1.

Table 1. Sets and indices, parameters, and decision variables

Sets and indices	
V	Set including all nodes in the network
E	Set including all edges in the network
i, j, l	Index of nodes $i, j, l \in V$
k	Index of vehicles $k \in \{1,2, \dots, K\}$
t	Index of work shifts $t \in \{1,2, \dots, T\}$
s	Index of scenarios $s \in \{1,2, \dots, S\}$
p	Index of crew members $p \in \{1,2, \dots, P\}$
e	Index of expertise $e \in \{1,2, \dots, E\}$
g	Index of grades $g \in \{1,2, \dots, G\}$
Parameters	
FS_i	fixed cost of setting a station in node *i*
FV_k	fixed cost of vehicle *k*
N_{ij}^{ts}	minimum number of times that the edge *ij* should be traversed in work shift *t* under scenario *s*
T_{max}	the maximum allowed time for vehicle to finish their tour
T_k	the time that it takes for each vehicle *k* in order to traverse a unit of distance
D_{ij}	the distance between node *i* and node *j*
M	A large number
VV_k	Variable cost for each distance unit for vehicle *k*
VP_k	Variable cost for pollution emission for each distance unit for vehicle *k*
FuC_k	Fuel consumption of for each distance unit for vehicle *k*
$FuCap_k$	Fuel capacity of vehicle *k*
WE_k^s	The effectiveness weight of vehicle *k* under scenario *s*
WP_{ge}^s	The effectiveness weight of each person with grade *g* and expert mode *e* under scenario *s*
WR_{ij}^t	Weight of edge $(i, j) \in E$ in work shift *t* for each distance unit
WG_p	Wage of person *p*
YG_{peg}	A binary 3-dimensional matrix which is 1 if person *p* with expertise *e* has grade *g*
YE_{peg}	A binary 3-dimensional matrix which is 1 if person *p* with expertise *e* has grade equal or above *g*
$MaxS_p$	Maximum allowed work shifts for person *p*
$Vcap_k$	Crew capacity of vehicle *k*
$Pmin_k$	Minimum number of people in vehicle *k*
DE_e^{ts}	Minimum number of expertise *e* needed for work shift *t* under scenario *s*
DG_g^{ts}	Minimum number of grades equal to or above g needed for work shift t under scenario s
φ^s	Probability of scenario s
Decision variables	
x_{ijk}^{ts}	the number of times that vehicle *k* traversed on edge *ij* under scenario *s* in work shift *t*
u_k^{ts}	1 if vehicle *k* is used in work shift *t* under scenario *s*, and zero otherwise
h_{ik}^{ts}	1 if vehicle *k* starts from node *i* in work shift *t* under scenario *s*, and zero otherwise
y_i	1 if a station in node *i* is constructed, and zero otherwise
b_{ik}^{ts}	1 if node *i* visited by vehicle *k* in work shift *t* under scenario *s*, and zero otherwise
f_{ijk}^{ts}	The dummy variable in charge of flow of fictional commodity transferred by vehicle *k* from node *i* to node *j* in work shift *t* under scenario *s*
z_{pk}^{ts}	1 if person *p* is allocated to vehicle *k* in work shift *t* under scenario *s*, and zero otherwise

As mentioned before, some parameters and limitations might change in different situations (e.g., terrorist attack threats, high-ranking officials' visits, official ceremonies, etc.) which necessitates defining multiple scenarios in the model and leads to a multi-stage mathematical formulation. Each scenario has specific constraints, such as the minimum number of times each arc should be traversed, and the different expertise and grades that should be assigned. Considering a multi-stage k-WPP, the problem is formulated as a two-stage mixed-integer linear mathematical formulation. Decision variables are divided into two categories in the two-stage programming approach. The first-stage decisions are not reliant on scenario realization and can be made before a scenario is realized. Second-stage decisions, on the other hand, are scenario-dependent variables that rely on scenario realization. In the presented mathematical model, decision variable y_i (equal to 1 if a station in node iii is constructed and zero otherwise) is scenario-independent (a first-stage variable). The value of all other variables can be determined in the second stage depending on which scenario occurs. The entire mathematical formulation for the aforementioned problem can be seen in the following:

$$VEF^s = \sum_k \sum_t u_k^{ts}\ WE_k^s \qquad \forall s \qquad (2)$$

$$PEF^s = \sum_p \sum_g \sum_e YG_{pge} WP_{ge}^s \sum_k \sum_t z_{pk}^{ts} \qquad \forall s \qquad (3)$$

$$SFC = \sum_i y_i\ FS_i \qquad (4)$$

$$VFC^s = \sum_t \sum_k u_k^{ts}\ FV_k \qquad \forall s \qquad (5)$$

$$PWG^s = \sum_p \sum_k \sum_t z_{pk}^{ts}\ WG_p \qquad \forall s \qquad (6)$$

$$PVC^s = \sum_t \sum_i \sum_j \sum_k x_{ijk}^{ts}\ D_{ij} WR_{ij}^t\ (VV_k + VP_k) \qquad \forall s \qquad (7)$$

$$Max\ EFF = \sum_s \varphi^s\ (VEF^s + PEF^s) \qquad (8)$$

$$Min\ CST = SFC + \sum_s \varphi^s\ (VFC^s + PWG^s + PVC^s) \qquad (9)$$

$subject\ to$

$$\sum_i h_{ik}^{ts} = u_k^{ts} \qquad \forall\ k,t,s \qquad (10)$$

$$\sum_i \sum_{j|(i.j)\in E} x_{ijk}^{ts} \le M\ u_k^{ts} \qquad \forall\ k,t,s \qquad (11)$$

$$\sum_t \sum_k h_{ik}^{ts} \le M\ y_i \qquad \forall\ i,s \qquad (12)$$

$$\sum_k (x_{ijk}^{ts} + x_{jik}^{ts}) \ge N_{ij}^{ts} \qquad \forall\ i,j|(i,j) \in E,t,s \qquad (13)$$

$$\sum_{i|(i.l)\in E} x_{ilk}^{ts} = \sum_{j|(l.j)\in E} x_{ljk}^{ts} \qquad \forall\ l \in V,k,t,s \qquad (14)$$

$$\sum_i \sum_{j|(i.j)\in E} x_{ijk}^{ts}\ D_{ij}\ WR_{ij}^{ts}\ T_k \le T_{max} \qquad \forall\ k,t,s \qquad (15)$$

$$\sum_{j|(j.i)\in E} x_{jik}^{ts} + \sum_{j|(i.j)\in E} x_{ijk}^{ts} \le 2Mb_{ik}^{ts} \qquad \forall\ i,k,t,s \qquad (16)$$

$$\sum_{j|(i.j)\in E} f_{ijk}^{ts} - \sum_{j|(j.i)\in E} f_{jik}^{ts} \le -1b_{ik}^{ts} + Mh_{ik}^{ts} \qquad \forall\ i,k,t,s \qquad (17)$$

$$\sum_{j|(i.j)\in E} f_{ijk}^{ts} - \sum_{j|(j.i)\in E} f_{jik}^{ts} \ge -1b_{ik}^{ts} - M\ h_{ik}^{ts} \qquad \forall\ i,k,t,s \qquad (18)$$

$$Mx_{ijk}^{ts} \ge f_{ijk}^{ts} \qquad \forall\ i,j|(i,j) \in E,k,t,s \qquad (19)$$

$$\sum_i \sum_{j|(i.j)\in E} x_{ijk}^{ts}\ D_{ij}\ WR_{ij}^t\ FuC_k \le FuCap_k \qquad \forall\ k,t,s \qquad (20)$$

$$\sum_p z_{pk}^{ts} \le Vcap_k \qquad \forall\ k,t,s \qquad (21)$$

$$\sum_p \sum_g \sum_k z_{pk}^{ts}\ YE_{peg} \ge DE_e^{ts} \qquad \forall\ t,e,s \qquad (22)$$

$$\sum_p \sum_e \sum_k z_{pk}^{ts}\ YG_{peg} \ge DG_g^{ts} \qquad \forall\ t,g,s \qquad (23)$$

$$\sum_t \sum_k z_{pk}^{ts} \le MaxS_p \qquad \forall\ p,s \qquad (24)$$

$$\sum_{k}(z_{pk}^{ts}+z_{pk}^{t-1.s}) \leq 1 \qquad \forall p,s,t=2,\dots,T \qquad (25)$$

$$\sum_{k}(z_{pk}^{1.s}+z_{pk}^{T.s}) \leq 1 \qquad \forall\, p,s \qquad (26)$$

$$\sum_{p} z_{pk}^{ts} \geq u_{k}^{ts}\, Pmin_{k} \qquad \forall\, k,t,s \qquad (27)$$

$$\sum_{p} z_{pk}^{ts} \leq M u_{k}^{ts} \qquad \forall\, k,t,s \qquad (28)$$

$$x_{ijk}^{ts} \geq 0 \; integer \qquad \forall\, i,j,k,t,s \qquad (29)$$

$$f_{ijk}^{ts} \geq 0 \qquad \forall\, i,j,k,t,s \qquad (30)$$

$$y_{i}, u_{k}^{ts}, h_{ik}^{ts}, b_{ik}^{ts}, z_{pk}^{ts} \in \{0,1\} \qquad \forall\, i,k,p,t,s \qquad (31)$$

Equation (2) shows total vehicle effectiveness, and equation (3) shows total person effectiveness, both of which are used in police patrolling. Equation (4) addresses the station fixed cost, equation (5) concerns the vehicle fixed cost, equation (6) pertains to person wages, and equation (7) involves vehicle variable cost and pollution emission cost. Equations (2)-(3) and (5)-(7) are defined under scenario s. The objective function (8) maximizes the total expected value of total effectiveness, while the objective function (9) minimizes the expected value of total cost. It is evident that objective functions (8) and (9) conflict with each other, as the total cost increases if the total effectiveness of the system increases. Constraint set (10) states that if a vehicle is used, it should start from a node, and if the vehicle is not used, it cannot start from a node. Constraint set (11) ensures that a vehicle can only traverse an edge if it is being used. Constraint set (12) ensures that a vehicle can only start from a node if a station is built at that node. Constraint set (13) enforces that the number of times an edge is traversed is equal to or more than the required number. Constraint set (14) enforces the equality of output and input flow for each node. Constraint set (15) ensures that each vehicle completes its tour before the given time.

Constraint sets (16)-(19) are for sub-tour elimination and discontinuity prevention. Several sub-tour elimination constraints have been proposed in the literature and are discussed by Limon (2015). The formulation of case 1 proposed by Shafahi and Haghani (2015) is used for modeling in the presented paper. However, in case 1, the origins and destinations are the same and are given by users. In the presented model, although the origins and destinations are the same for each vehicle, the origins for each vehicle should be determined by the model. The sub-tour constraints presented by them in case 1 are as follows:

$$\sum_{j} x_{jik} + \sum_{j} x_{ijk} \leq 2M b_{ik} \qquad \forall\, i.k \qquad (32)$$

$$\sum_{j} y_{ijk} - \sum_{j} y_{jik} = -1 b_{ik} \qquad \forall\, i \in (V - O_k).k \qquad (33)$$

$$y_{ijk} \leq M x_{ijk} \qquad \forall\, i.j.k \qquad (34)$$

In these constraint sets, x_{ijk} is a positive integer variable representing the number of times edge ij is traversed by vehicle k, y_{ijk} is a continuous positive variable representing the flow of a fictional commodity being transferred from node i to node j using vehicle k, b_{ik} is a binary variable that equals one if node i is visited by vehicle k and zero otherwise, O_k is the origin for vehicle k, and M is a large number. However, in the presented paper, since the origin for each vehicle is unknown, the constraint is converted into two inequality constraints which remain neutral for a vehicle in the case where node i is a starting node for it.

Constraint set (20) ensures that the route allocated to a vehicle has sufficient fuel capacity. Constraint set (21) ensures that the number of personnel allocated to a vehicle does not exceed its capacity. Constraint set (22) guarantees the minimum number of experts that should be allocated to a work shift. Constraint set (23) ensures scheduling crew members with the required or higher grade. Constraint set (24) ensures that the maximum number of work shifts for a person does not exceed their capacity. Constraints (25)-(26) ensure that a person does not work in two consecutive work shifts. Constraints (27)-(28) ensures that the crew could be assigned to a vehicle if it is being used. Constraints (29)-(31) are restrictions on the variables.

Solution Approach

As the mentioned problem is proven to be NP-hard, and due to the complexity of the model, solving even the average-size problems in a reasonable amount of time is not possible. However, in the case of police patrolling problem, the small-size problems occur practically. Using security patrolling in a college campus, small villages, industrial park, etc. can be formulated by the presented model, and the small-size problem can be solved in a reasonable amount of time. In this research, both exact solution and a heuristic approach is presented in order to solve both small-size as well as medium and large size problems.

The ε-Constraint Method

Using the ε-constraint method, the bi-objective model presented in the previous section can be transformed into a single-objective formulation. The ε-constraint method, first introduced by Haimes (1971), is among the most popular methods for solving multi-objective programming models. This method is particularly suitable when one objective, such as the effectiveness of the patrolling system in our research, is of higher importance than the cost. The ε-constraint method offers several advantages, as outlined by Mavrotas (2009):

- For linear problems, the ε-constraint method alters the original feasible region and can yield non-extreme solutions. In contrast, the weighting method applied to the original feasible region tends to produce corner solutions, potentially requiring many redundant runs. The ε-constraint method efficiently utilizes each run to produce effective solutions.
- In multi-objective integer and mixed-integer linear programming problems, the ε-constraint method can generate supported efficient solutions, whereas the weighting method may not.
- The weighting method heavily depends on finding appropriate scaling factors, which significantly impact the results and lack a specified method for determination. In contrast, the ε-constraint method does not require scaling factors.
- Unlike the weighting method, the ε-constraint method allows for easy specification of the number of generated Pareto optimal solutions by adjusting the number of grid points within each objective function range.

In the ε-constraint method, all objectives except one are transformed into constraints, with upper bounds set for each. This approach simplifies the complexity of solving a multi-objective model by focusing on minimizing or maximizing one objective at a time, while treating the other objectives as inequality constraints.

$$Min_{x\in\chi}\{Z(x) = Z_1(x).Z_2(x).\dots.Z_N(x)\} \quad (35)$$

Where $Z(x)$ represents the vector of all objective functions, x denotes the space of decision variables, and χ is the set of feasible solutions. Using the ε-constraint method, the multi-objective problem formulated in Eq. (35) can be transformed into a single-objective problem in Eq. (36) along with a set of constraints in Eq. (37). In this transformed problem, only one objective is minimized, while the remaining objectives are treated as inequality constraints with upper bounds.

$$Min_{x \in \chi} Z_n(x) \tag{36}$$
$$subject\ to$$
$$Z_i(x) \leq \varepsilon_i \qquad \forall i \in \{1.2. \dots . N\} - \{n\} \tag{37}$$

In our bi-objective model, we apply the ε-constraint method, prioritizing the effectiveness objective function as the primary objective due to its higher importance over the cost function. The cost objective function is transformed into a constraint with an upper bound, represented as Eq. (38) and constraints (10)-(31).

$$Max\ EFF = \sum_{s} \varphi^s \left(VEF^s + PEF^s\right) \tag{8}$$
$$subject\ to$$
$$SFC + \sum_{s} \varphi^s \left(VFC^s + PWG^s + PVC^s\right) \leq \varepsilon \tag{38}$$
$$Constraints\ (10) - (31)$$

Dataset Preparation

Given that the problem is NP-hard, obtaining results for large-scale instances is not feasible within polynomial time. However, for small-scale instances, optimal solutions can be achieved in a reasonable amount of time. To tackle this, we utilized four real-world datasets and employed the General Algebraic Modeling System (GAMS) software, version 24.1.

The small-scale problems were solved using the ε-constraint method, applied to instances from various areas depicted in Fig 2. For example, in Fig 2(c), the node set $v \in \{1.2.5.8.11.12.15.17\}$ are potential candidates for station locations (with fixed costs set high for other edges). The edge sets $\{2.3\}. \{3.4\}. \{4.5\}. \{5.19\}. \{17.18\}$ *and* $\{18.2\}$ must be traversed at least 3 times during a work shift, while other edges should be traversed at least once during a work shift.

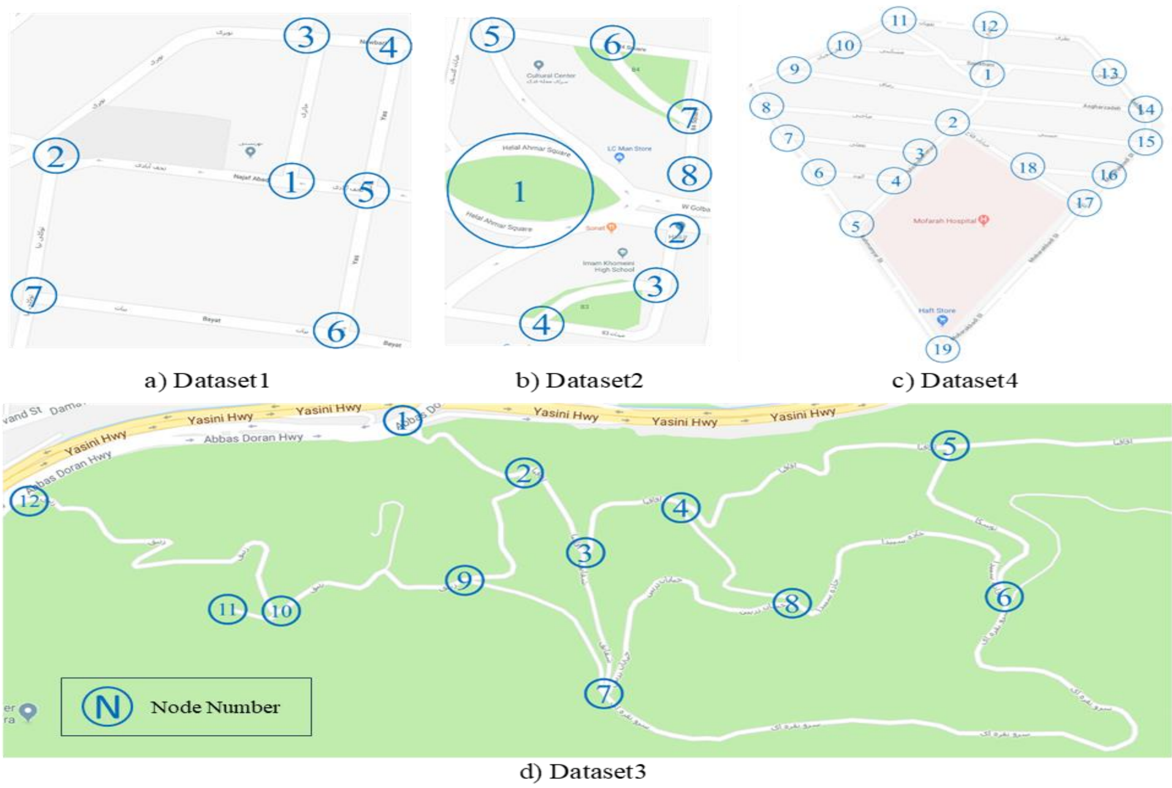


Figure 2. Dataset representation

Tradeoff Between Effectiveness and Total Cost

Examining the impact of varying cost tolerance (ε) on patrolling effectiveness is crucial. Analyzing the tradeoff between cost and effectiveness empowers decision makers to make informed decisions across different scenarios. Details regarding the datasets and numerical results are provided in Table 2, while Fig 3 illustrates the Pareto front curve derived from these results. ε represents the maximum allowable total cost, allowing decision makers to tailor patrolling strategies according to the available budget.

Table 2. Obtained numerical results

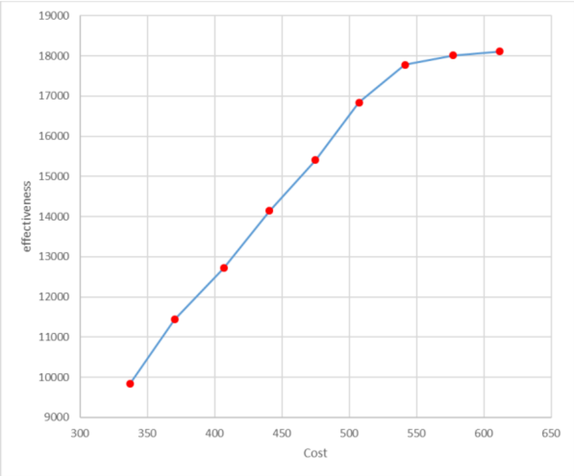
Dataset	Indices *i*k*t*s*p*e*g*	ε	*LB*	*UB*	Runtime (seconds)
Dataset1	7*4*3*2*6*3*3	611.700	18116.000	18116.000	15.523
		577.180	18013.000	18013.000	166.681
		541.500	17784.000	17784.000	141.796
		507.360	16844.000	16844.000	111.484
		474.960	15404.000	15404.000	143.599
		440.720	14141.000	14141.000	176.194
		406.880	12721.000	12721.000	403.51
		370.380	11438.000	11438.000	118.373
		337.080	9838.000	9838.000	33.987
Dataset2	8*8*3*4*7*2*2	2623.720	88332.000	88332.000	24.463
		2373.280	82294.000	82294.000	128.638
		2121.880	75696.000	75696.000	121.856
		1873.300	66627.000	66627.000	218.61
		1623.400	57009.000	57009.000	144.293
		1372.500	47402.000	47402.000	119.018
		1123.700	37424.000	37424.000	157.894
		872.080	27148.000	27148.000	121.381
		624.400	14748.000	14748.000	54.254
Dataset3	12*6*3*1*8*2*3	8620.132	315476.977	315476.977	448.808
		7593.462	288194.977	288194.977	863.124
		6566.792	260007.283	260007.283	814.547
		5540.121	231787.806	231787.806	721.946
		4513.451	199533.357	199533.357	1485.231
		3486.781	153903.522	153903.522	803.917
		2460.111	108640.463	108640.463	759.88
		1433.440	63010.520	63010.520	1030.91
		406.77	10908.057	10908.057	126.28
Dataset4	19*6*3*1*8*2*3	14707.460	540450.181	540450.181	534.486
		13032.444	495501.805	495501.805	7345.237
		11357.429	449282.564	449282.564	8732.62
		9682.413	403178.393	403178.393	7612.002
		8007.397	354947.877	354947.877	6519.43
		6332.381	280506.249	280506.249	6998.023
		4657.366	206428.724	206428.724	4528.972
		2982.350	127351.894	127351.894	4329.922
		1307.334	36007.043	36007.043	429.475

Cluster-First, Route-Second

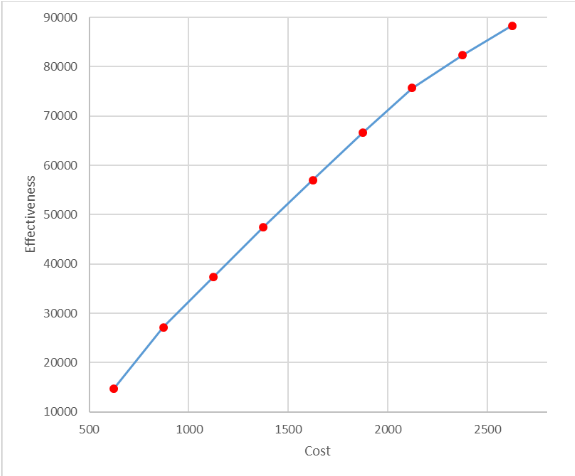
As previously noted, the problem is established as NP-hard, making it impractical to find optimal solutions for large-scale problems within a reasonable timeframe. Therefore, heuristic and/or meta-heuristic approaches are employed to find feasible solutions. Arc routing problems fall under the category of vehicle routing problems (VRP). Bowerman et al. (1994) classified heuristic approaches for VRP into five categories:

- Cluster-first, route-second
- Route-first, cluster-second
- Improvement/exchange
- Simpler mathematical representation

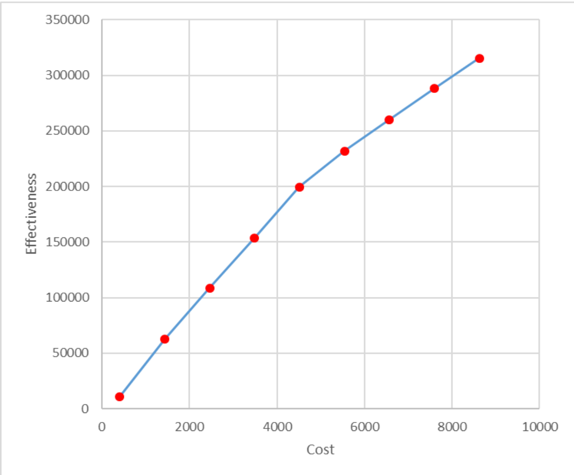
- Savings/insertion



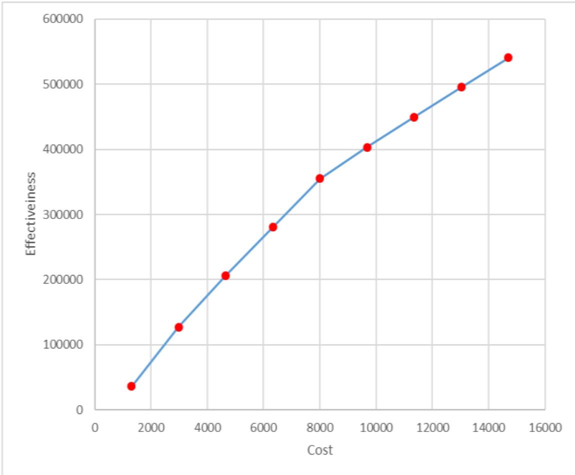
a) Dataset1 Pareto front



b) Dataset2 Pareto front



c) Dataset3 Pareto front



d) Dataset4 Pareto front

Figure 3. Datasets Pareto front

Each of these categories is suited to problems with distinct characteristics. In this study, the optimization of a police patrolling problem is pursued, where it is typically expected that each vehicle traverses adjacent streets in the optimal solution, returning to its starting point. Therefore, employing a clustering method for geographic areas is logical. Among the clustering procedures mentioned, the cluster-first, route-second method appears most effective (Dondo & Cerdá, 2007). This algorithm initially groups nodes into clusters and assigns each vehicle to a specific cluster. Subsequently, it determines the optimal vehicle route for each cluster independently. However, clustering nodes may lead to some edges being removed. For instance, let graph $G = (V, E)$ clustered into $G_1 = (V_1 . E_1)$ and $G_2 = (V_2 . E_2)$. In node clustering, the edge $\{e_{16} . e_{58}\}$ must either be removed or assigned to a specific cluster (Fig 4), highlighting the significance of cluster assignment decisions.

Cluster-based algorithms in vehicle routing problems operate by selecting adjacent nodes and/or arcs. In the police patrolling problem, where all arcs must be traversed by vehicles, it is practical to choose arcs that are on average closer to a subgraph. Therefore, our proposed algorithm clusters the set of all edges and creates new subgraphs while ensuring that no edges are removed. The procedure of our proposed arc clustering algorithm is outlined as follows (Fig 5):

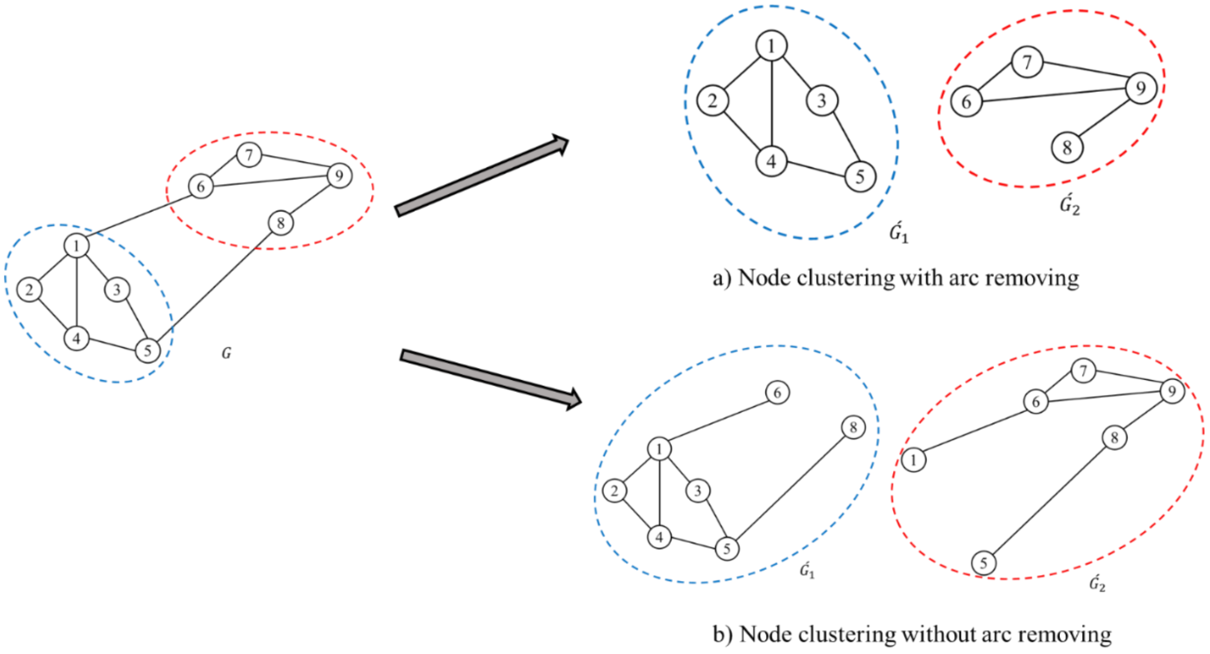


Figure 4. Node clustering

Step 0: let graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. Each vertex v_i is represented as coordinates $(x_i . y_i)$ in a 2D dimension.

Step 1: for each edge e_{ij} connecting vertices $(v_i . v_j)$, calculate the midpoint using $v_{ij}' = (\frac{x_i+x_j}{2} . \frac{y_i+y_j}{2})$.

Step 2: Cluster the midpoints obtained in Step 1 using the k-means clustering method. Each cluster yields a set of arcs, each connecting two nodes.

Step 3: For each cluster, create a subgraph using the set of arcs and nodes obtained in Step 2.

Step 4: let $\acute{G}^c = (\acute{V}^c . \acute{E}^c)$ denote a cluster obtained in Step 3. For each cluster let $\ddot{G}^c = G - \acute{E}^c$.

Step 5: remove all vertices from $\ddot{G}^c$ where $degree(\ddot{v}_i^{\,c} = 0)$.

Step 6: Add an edge between each pair of nodes where $\{\ddot{v}_i^{\,c} . \ddot{v}_j^{\,c}\} \subseteq \ddot{G}^c \cap \ddot{G}^d$. $\forall d \neq c$ with $\ddot{w}_{ij}^{\,c} = Shortest\ path\ from\ v_i\ to\ v_j$ if the shortest path does not exist in $\ddot{G}^c$.

Step 7: Solve the problem separately for each graph $\ddot{G}$ separately using the exact method ensuring that the right-hand side of Constraint (13) for edges added in Step 6 is set to zero, and adjust the right-hand side of Constraint (15) to $\frac{T_{max}}{Number\ of\ Clusters}$.

The proposed algorithm is used in order to solve dataset3 and dataset4. Both datasets are clustered into two segments. The result of clustering dataset3 and dataset4 is shown in Figure 6.

The ε-constraint method for each clustered dataset is used as a solving approach which is explained in Section 3.1. The numerical results are shown in Table 3 and the Pareto fronts for the clusters are shown in figure 7.

Let graph G, which is clustered into k subgraphs, be used. To solve each subgraph, the exact solution approach using the ε-constraint method is applied. Consequently, each cluster produces a set of Pareto front points. To estimate the curve-fit function of each set, the curves are fitted as second-order polynomials using the "Polyfit" function of MATLAB software. It is evident that the total cost equals the sum of each cluster's cost, and similarly, the total effectiveness equals the sum of each cluster's effectiveness. Therefore, for each dataset, the sum of the curve-fit functions of each cluster is compared to that dataset's curve-fit function, as shown in Fig 8. The average runtimes using both the exact method and the proposed algorithm are presented in Table 4.

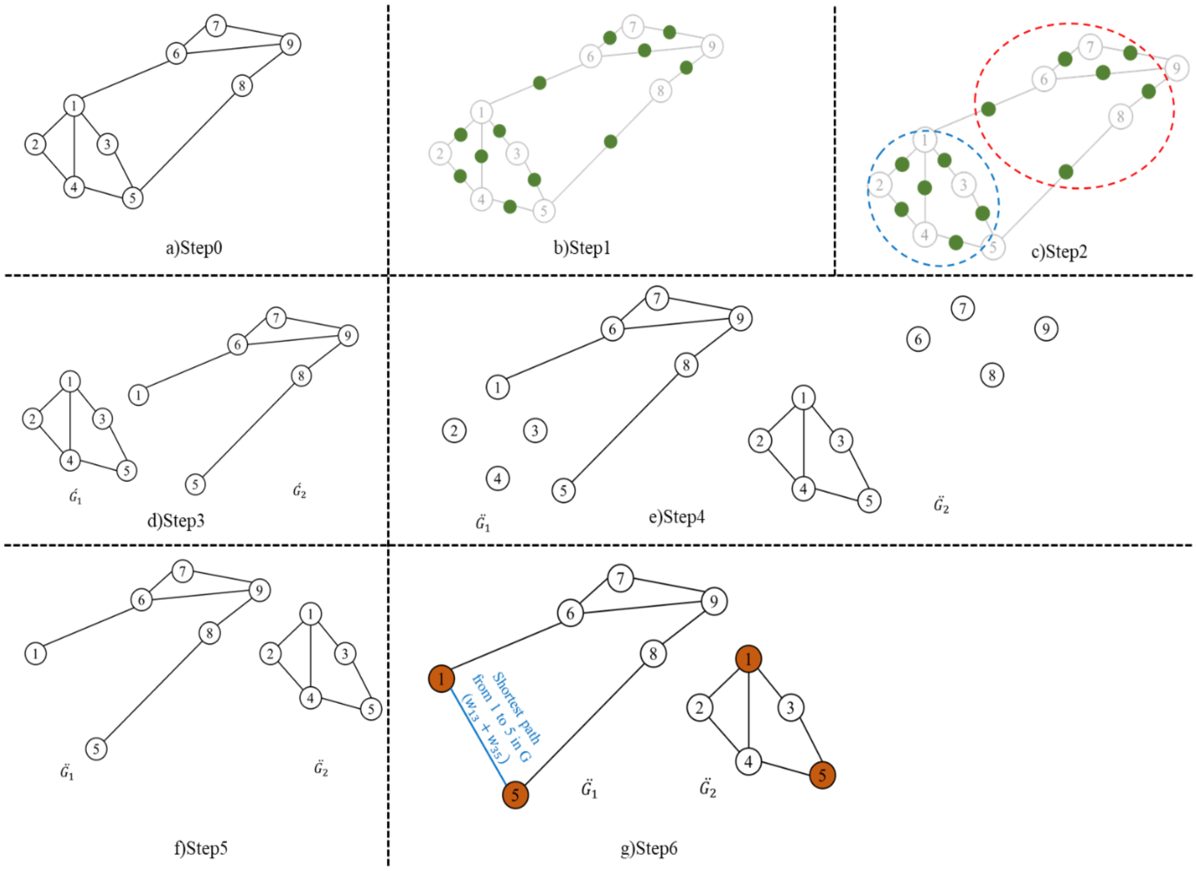


Figure 5. Proposed clustering algorithm

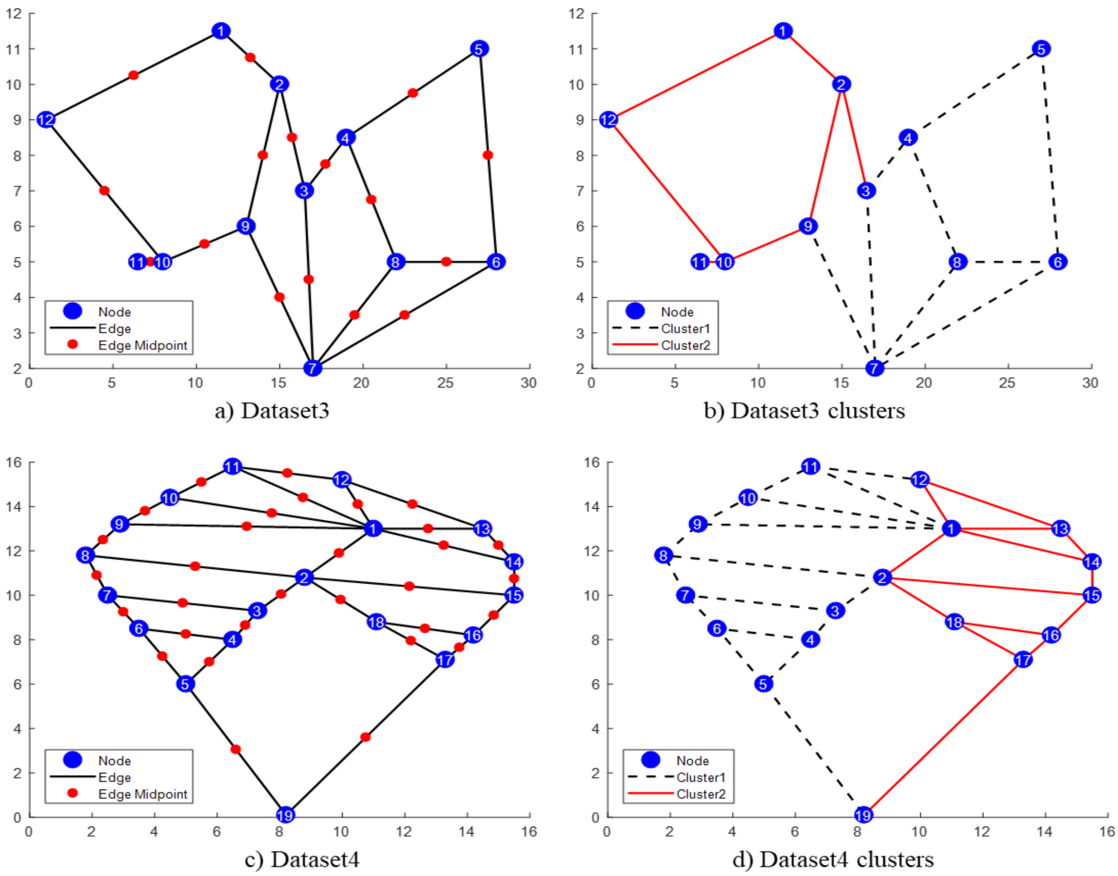
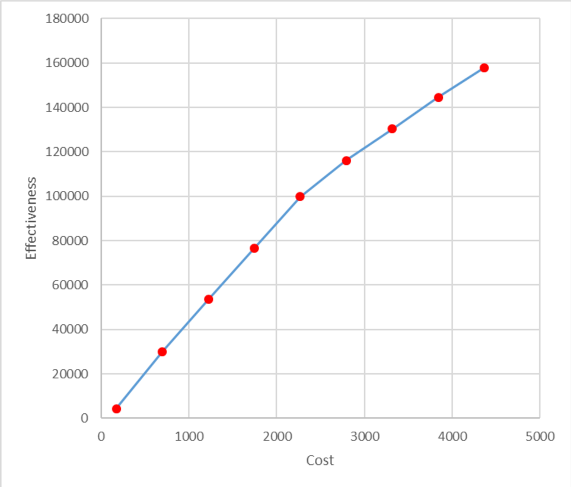
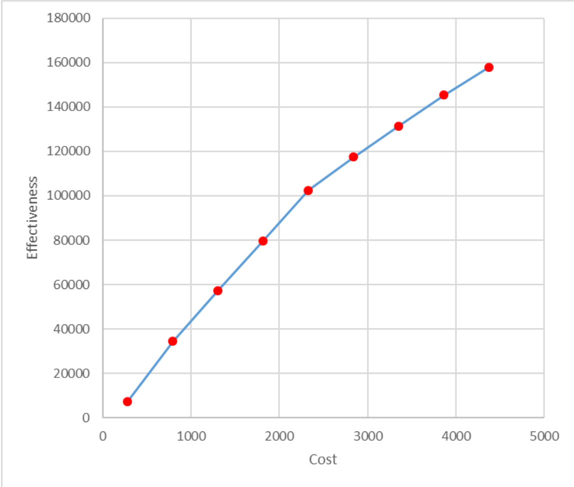


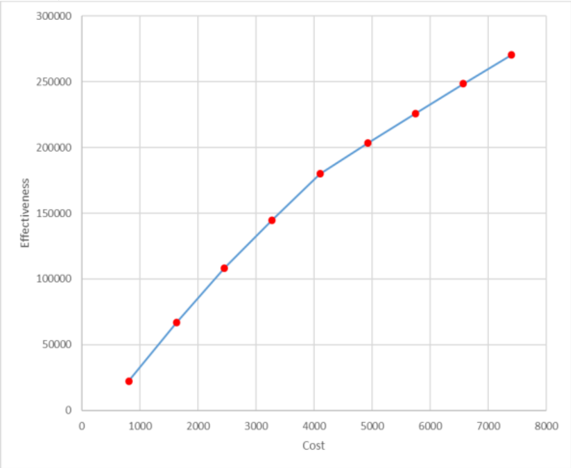
Figure 6. Dataset clusters



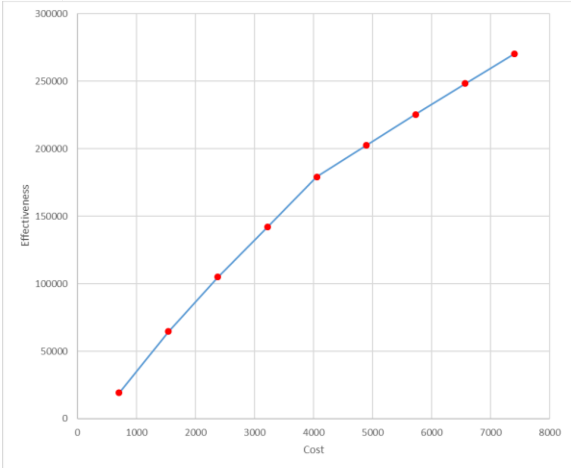
a) Dataset3-cluster1 Pareto front



b) Dataset3-cluster2 Pareto front



c) Dataset4-cluster1 Pareto front



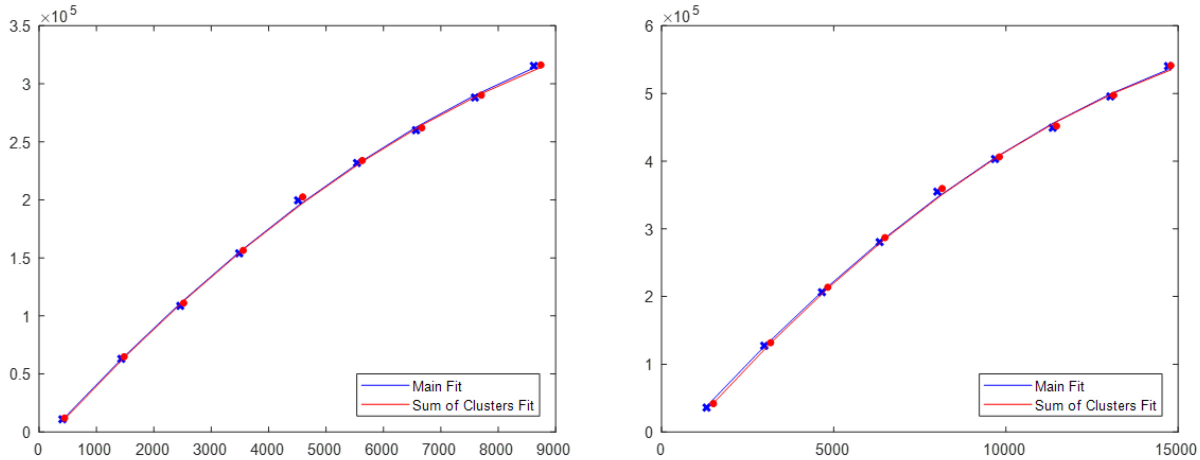
d) Dataset4-cluster2 Pareto front

Figure 7. Pareto Fronts for each clustered dataset

Table 3. Numerical results for clustered datasets

Dataset	**Indices** *i*k*t*s*p*e*g*	ε	***LB***	***UB***	***Runtime (seconds)***
		4367.197	157976.026	157976.026	56.262
		3842.472	144619.81	144619.81	143.562
		3317.747	130383.283	130383.283	214.895
		2793.021	116108.130	116108.130	143.002
Dataset3-cluster1	7*6*3*1*8*2*3	2268.296	99748.556	99748.556	154.764
		1743.571	76427.123	76427.123	83.642
		1218.846	53472.722	53472.722	105.653
		694.120	30145.244	30145.244	4.691
		169.395	4314.302	4314.302	3.958
		4379.811	157959.185	157959.185	60.034
		3867.364	145308.186	145308.186	179.231
		3354.917	131417.829	131417.829	202.317
		2842.470	117477.833	117477.833	195.76
Dataset3-cluster2	7*6*3*1*8*2*3	2330.023	102492.070	102492.070	133.252
		1817.575	79716.325	79716.325	121.402
		1305.128	57307.718	57307.718	83.674
		792.681	34442.252	34442.252	14.113
		280.234	7393.157	7393.157	3.843
		7392.814	270405.435	270405.435	363.452
Dataset4-cluster1	13*6*3*1*8*2*3	6570.117	248487.241	248487.241	1160.001
		5747.419	225929.954	225929.954	1142.317

		4924.722	203294.263	203294.263	967.202
		4102.025	180062.117	180062.117	1343.269
		3279.327	144683.371	144683.371	1283.45
		2456.630	108304.607	108304.607	1045.998
		1633.932	66686.981	66686.981	914.113
		811.235	22226.539	22226.539	130.276
		7403.719	270417.957	270417.957	45.852
		6566.281	248359.921	248359.921	514.56
		5728.844	225452.693	225452.693	463.874
		4891.406	202479.610	202479.610	425.863
Dataset4-cluster2	10*6*3*1*8*2*3	4053.968	179110.834	179110.834	532.753
		3216.530	141892.409	141892.409	698.86
		2379.093	105036.419	105036.419	597.972
		1541.655	64713.891	64713.891	476.853
		704.217	19170.483	19170.483	26.144



a) Dataset3 b) Dataset4

Figure 8. Datasets and clusters curve fit comparison

Table 4. Average runtime for exact method and proposed algorithm

	Exact method	Cluster1	Cluster2	Clustering Function	Proposed Algorithm
Dataset3	783.8492	101.1588	110.4029	0.516	212.0777
Dataset4	5225.574	535.2229	420.3034	0.682	956.2083

Conclusion

This paper presents a two-stage bi-objective Mixed Integer Linear Programming approach for location-routing and crew scheduling based on the K-windy postman problem in police patrolling. In the first stage, the model decides on the nodes for constructing police stations, while in the second stage, it determines the vehicle modes, patrolling routes, and crew assignments. The objective functions aim to maximize patrolling effectiveness and minimize total cost. The proposed model is validated using realistic datasets, with small-size problems solved using the ε-constraint method implemented in GAMS software. Given the NP-hard nature of the problem, an arc clustering algorithm is proposed to tackle medium and large-size problems efficiently. To validate this heuristic algorithm, results from two datasets are compared against an exact approach. The findings demonstrate that the cluster-based algorithm achieves satisfactory solutions in shorter runtimes. Due to the NP-hard complexity, evaluating the performance of the proposed algorithm against the exact solution for large-size problems remains impractical. This underscores the necessity for developing metaheuristic or alternative heuristic algorithms to benchmark against the cluster-based heuristic model proposed in this study.

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