Advances in Industrial Engineering, June 2025, 59(1): 199-214
DOI: 10.22059/aie.2024.384156.1922
RESEARCH PAPER

Managing Electric Vehicle Charging Networks: Cooperative Servicing Utilizing Mobile Charging Stations

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Received: 21 October 2024, Revised: 13 Novamber 2024, Accepted: 16 Novamber 2024

Abstract

At present, electric vehicles (EVs) are increasingly recognized as a viable alternative to conventional internal combustion engine vehicles, primarily due to their superior environmental sustainability, particularly regarding carbon emissions, and their cost-effectiveness attributed to lower energy consumption. Consequently, the market share of electric vehicles has witnessed substantial growth in recent years, which has in turn heightened the demand for charging infrastructure. Conversely, the rising number of electric vehicles necessitating recharging-especially during peak demand periods-poses challenges such as prolonged waiting times at public charging stations and increased strain on the power distribution network. To address these issues and enhance network efficiency, the concept of Mobile Charging Stations (MCS) has emerged, offering flexible charging solutions in terms of both time and location. This paper introduces an innovative approach for the allocation and deployment of MCSs in areas with high demand, aimed at alleviating the burden on public charging stations. A mathematical model grounded in the Location-or-Routing Problem (LoRP) has been formulated, employing various truck-based and van-based mobile charging stations to collaboratively service demand points near public charging facilities. This strategy seeks to attain various achievements, including the reduction of network load and waiting times at charging stations while simultaneously expanding coverage to improve customer satisfaction. Based on conducted experiments, a comprehensive evaluation and analysis of the proposed model demonstrate that the LoRP significantly outperforms traditional models in terms of both coverage and cost efficiency.

Keywords: Charging Networks, Cooperative Servicing, Electric Vehicles, Location-or-Routing Problem, Mobile Charging Stations, Service Planning.

Introduction

One of the most pressing concerns of contemporary societies is the issue of global warming, considering the increase in air pollutants and their impact on the environment. Among energy consumers, the transportation sector, using fossil fuels that directly contribute to nearly one-third of carbon dioxide emissions, is one of the largest and most influential sectors affecting environmental conditions and pollution. Moreover, with the significant upward trend in the use of fossil fuels, especially in industrial and advanced countries, these energy reserves are rapidly

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depleting, which is another major concern for countries around the world. Therefore, the majority of societies are seeking to discover and utilize clean, secure, and sustainable sources of energy. Among the solutions proposed to overcome this problem is the utilization of renewable energy sources such as solar and wind energy, as well as the use of electric vehicles as a more promising solution to reduce the consumption of fossil fuels in the transportation sector and consequently reduce carbon emissions.

In order to encourage the use of electric vehicles, various methods such as offering subsidies and financial assistance for vehicle purchases by governments, facilitating purchasing conditions by automakers, as well as developing infrastructure and electric vehicle charging networks by investors can be considered. With the continuous increase in the number of these vehicles on roads worldwide, to the extent that by the end of 2035, more than 525 million electric vehicles were sold [1], it is clear that the demand for charging these vehicles is also increasing steadily (it is estimated that global energy demand will increase from 20 billion kilowatt-hours in 2020 to 280 billion kilowatt-hours in 2030), and meeting this demand requires planning, providing charging equipment, and specific provisions for charging networks. It is evident that despite the growth in the market share of electric vehicles and the increased demand for recharging, there are obstacles such as battery capacity and subsequently limited travel distances that have created the biggest concern for drivers, namely range anxiety, long charging times leading to extended waiting times in queues for drivers, as well as limited access to charging stations (which is a major obstacle in encouraging the purchase of electric vehicles), compared to fuel refilling stations for fossil fuels, which, due to their scattered and limited numbers, are effective factors in slowing down the acceptance rate of this type of vehicles.

As a solution, having an adequate number of fixed electric vehicle charging stations (FCS) can significantly reduce the mentioned concerns and problems and help expand the market for these vehicles. Still, the costs associated with this approach in terms of equipment, facilities, and land will be very high. Furthermore, the expansion of a large number of these types of stations, especially in densely populated and high-traffic areas, may have negative implications for the electrical network of different regions during peak energy consumption times, which at this stage, establishing power plants to supply energy for these stations may not be economically and environmentally viable or in some cases may be impossible [2]. Fig 1 illustrates the daily average energy demand, both in total and specifically for fast charging stations [3]. The data presented in this chart indicate that during certain periods of the day, the pressure on the power distribution network increases, resulting in prolonged waiting times at FCSs and a high rate of failures in charging equipment. This situation underscores the need for effective management of the charging network, particularly during peak demand periods.

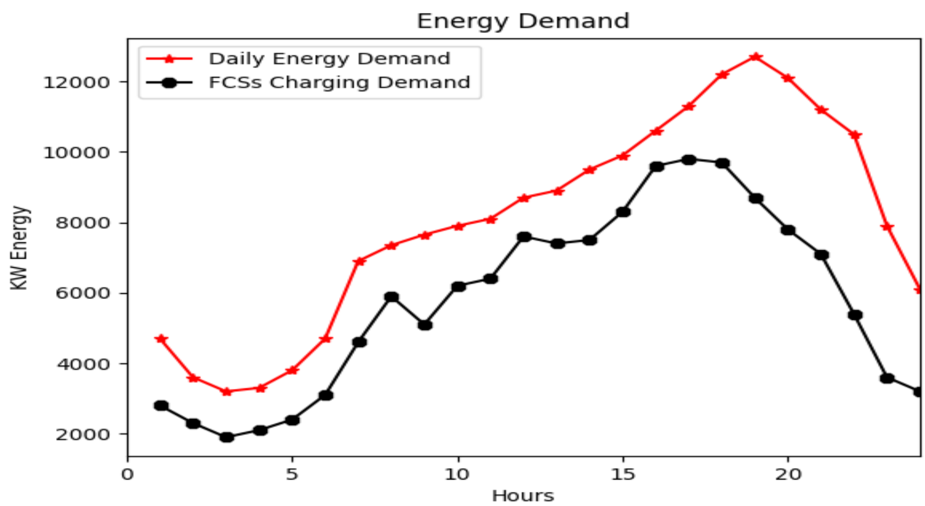


Figure 1. Peak Demand Hours on a Day

On the other hand, some drivers of electric vehicles, due to battery charge shortage, may not be able to reach the nearest public charging station and need recharging on the way, or due to rush and other preferences such as convenience, time efficiency, and the need for rapid charging, they may need to find battery recharging locations in different places and times since establishing fixed stations in all potential locations may be geographically impossible, or may not be economically viable and profitable for investors. Therefore, it is better to explore alternative solutions and other charging methods such as using mobile charging stations or vehicle-to-vehicle charging systems to overcome the shortcomings and limitations of different methods, as well as provide more suitable services to encourage the acceptance of electric vehicles.

Mobile charging stations are vehicles (vans, trucks, etc.) equipped with energy storage systems and fast chargers to transfer energy from their storage source to the batteries of other vehicles. They can charge their storage energy systems during periods of low energy network consumption and dispatch them to different areas to service the demands of drivers at various points and times, as well as relieve pressure on the power network and equipment of fixed stations, and reduce waiting queues for service due to heavy congestion of vehicles seeking charging at FCSs [4]. Based on the discussions outlined in the previous parts, it is worth mentioning that in this research, a mathematical model will be developed to make optimal decisions regarding the management of fleets of MCSs to provide services to high-demand points during peak network load times. These decisions aim to reduce network load pressure and customer waiting times at FCSs, as well as to maximize the coverage of demand.

Consequently, the primary characteristics examined in this study pertain to the heterogeneity of the fleet, in addition to their availability and capacity, Also includes a realistic assessment of coverage parameters and an analysis of cost implications.

In the continuation of this article, the second section will review the literature on studies conducted in the field of mobile charging stations and address research issues in the operations research domain of service systems planning. The proposed problem statement and the developed mathematical model will be elaborated on in the third section. The analysis and presentation of results from numerical examples will be discussed in the fourth section, and finally, in the fifth section, a summary and recommendations for future studies will be presented.

Literature Review

Given the familiarity with the concepts discussed in previous sections of this research and the necessity to present solutions in line with the global expansion of electric vehicle adoption, it is required to delve into a detailed examination of the concepts and applications of this method as well as studies conducted in this field in order to further develop the electric vehicle charging network by relying on the technology of MCSs. Subsequently, the research conducted by scholars in recent years in the field of deployment planning of MCSs will be reviewed. Following that, considering the intersections of this topic with mobile service systems, published articles in the research domain of mobile service provisioning, particularly the issue of location or routing from various aspects, will be studied.

Mobile Charging Stations

In the scope of managerial and technical studies of MCSs, a study by [5] examined the impact of the number of available fleets and the number of charging ports at each station on the waiting time of electric vehicle drivers to receive electric energy. The results demonstrated that in one instance, with an increase in the number of charger ports, the waiting time decreased from 89 minutes to 5 minutes. Additionally, [6] focused on developing an algorithm using process models to shorten the waiting time for visitors, planning suitable deployments for mobile charging stations without energy storage systems and converters and additional charging ports that require connection to the station's power grid. Another algorithm based on process models was developed by [7] to cater to demands exceeding the capacity of fixed stations along highways using mobile charging stations without energy storage systems. Another study investigated the operational principles of mobile charging stations to design an intelligent charging management system and equipment carrier structure for long-distance routes, such as seasonal camping locations [8].

OR in Servicing Systems Planning

The mobile charging method development is a military issue in which the goal is to increase the total distance covered by transferring fuel from one vehicle to another. In this field, a study was conducted by [9] who examined only homogeneous vehicles with consistent fuel capacities and energy consumption rates, leading to a proposed scheme for fuel sequence alignment of vehicles to maximize the overall operational range of the fleet. Subsequently, [10] extended their plan to heterogeneous fleets and presented a linear model that required only O(n) computations for its resolution (linear complexity). Moreover, in the field of mobile refueling, [11] investigated the charging station location problem under uncertainty of traffic flow. For this purpose, a two-stage stochastic programming modeling approach was adopted, where the first-stage decision variable considered the fixed station locations and the second-stage decision variable considered the mobile station locations, and the proposed model results were examined for stations with and without capacity constraints. Given the high applicability and flexibility of mobile services, its concept has been employed in various other domains, with numerous studies conducted in this area. For example, ambulance location-allocation problems with objectives such as maximizing coverage and serviced demand points or minimizing response times and delays have been addressed [12]. Furthermore, the bicycle relocation issue in bike-sharing systems, where bicycles are relocated via other vehicles to better address demand points, can also be considered another example of mobile services [13]. Table 1 is a summary on related papers in the present research area. In Table 1, there is a column titled "Servicing," which indicates that the research predominantly centers on providing service to fixed charging stations (FCSs), electric vehicle owners (EVs), or addressing issues such as emergencies (Others).

In general, issues that are examined through indicators such as demand coverage and response time can be classified into this category. For example, for planning purposes such as providing assistance to drivers of vehicles experiencing technical failures (roadside assistance), servicing passengers by taxi fleets, mobile distribution systems with mobile warehouses, or emergency medical services by ambulances, the utilization of the mobile services approach can be beneficial in appropriately responding to demands.

Table 1. A Summary on Related Papers

Papers	Year	System	Servicing			Problem	Objective(s)	Special Features
			FCSs	EVs	Others			
[14]	2016	Mobile Warehouses			✓	Location Allocation	Min Costs	Scenario-based Stochastic Programming
[15]	2018	Service Charge Requests	✓			Allocation	Max Servicing	Extra Demands
[16]	2020	Ambulance			✓	Location	Min Response Time	Dispatching List, Queuing Approach
[17]	2020	Power Management		✓		Scheduling	Min Candidate Points	Dynamic Programming, Driving Cycle

Papers	Year	System	Servicing			Problem	Objective(s)	Special Features
			FCSs	EVs	Others			
[2]	2020	Service Charge Requests		✓		Routing	Max Servicing	Service Orders
[13]	2021	Bicycle Sharing			✓	Relocation	Min Costs	Dynamic Relocation
[18]	2021	Service Charge Requests	✓			Location	Max Utility	2 Stage Problem, Scenario Sampling
[19]	2021	Mobile Facilities			✓	Scheduling	Min Costs Min Delays	Integrated Production and Routing
[7]	2021	Service Charge Requests		✓		Assignment	Max Profits	Equilibrium Equations
[20]	2021	Mobile Depots			✓	Routing	Min Costs	2-Echelon Distribution
[21]	2021	Service Charge Requests	✓			Scheduling	Min Waiting Times	Queuing Theory, Quadratic Assignment
[22]	2022	Delivery System			✓	Routing	Min Costs Min Pollution	Time Window, Recharging Stations
[23]	2022	Energy Distribution			✓	Location, Allocation	Min Traveling Time	Disaster Recovery Phase
[24]	2024	Health Services			✓	Routing	Min Response Time	Road Extraction
[25]	2024	Service Charge Requests	✓			Scheduling	Min Charger Piles	Scheduling Algorithm Parking Lots
[26]	2024	Service Charge Requests	✓			Location-or-Routing	Max Covering	Coverage Feature
[27]	2024	Drug Delivery			✓	Routing	Min Distance	Two-Step Clustering
Present Paper	2024	Service Charge Requests	✓	✓		Location-or-Routing	Max Covering Min Costs	Vehicle Capacity & Availability, Gradual Coverage Parameter, Heterogeneous Fleet

Problem Definition & Proposed Model

It is evident that at different time intervals during the days of the year, month, week, and especially on a specific day, the average overall demand for receiving electrical energy from electric vehicle drivers varies. Therefore, one of the concerns of energy service providers is proper planning to respond to energy-receiving demands in the best way. Especially considering that peak consumption times, or network peak load times (hours 16-21 based on Fig 1) in residential and densely populated areas coincide with the high traffic flow of vehicles, consequently, the demand for electrical energy from charging stations increases. The high input and demand at FCSs lead to issues such as pressure on the power supply network, damage to station equipment and facilities, as well as creating long waiting queues that result in driver dissatisfaction.

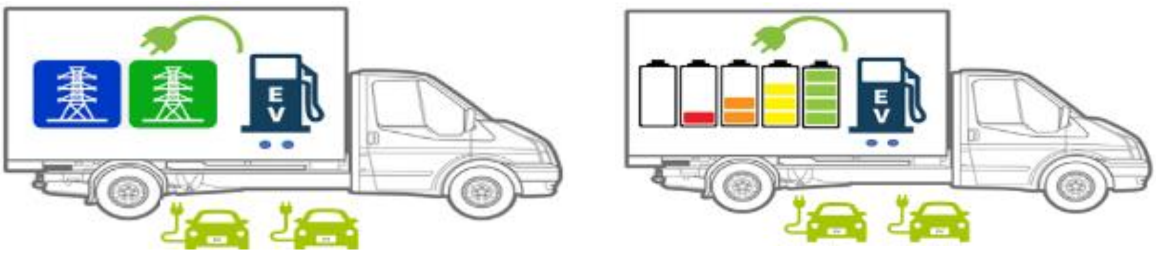
Since one of the proposed solutions is the use of MCSs as a complement to FCSs, another concern for investors is how to use and deploy this fleet. They will now seek to arrange an appropriate plan for the optimal operation of their MCS fleet in addition to supporting fixed stations during peak load times and demands they can cover and respond to the charging requirements of other electric vehicle drivers in different locations. The planning should be carried out in a way that not only increases customer satisfaction (both existing customers at FCSs and potential customers looking to use MCSs) but also focuses on the main goal of investors, which is to reduce operational costs and increase profits from providing services.

To describe the proposed network, we consider a number of FCSs continuously supplying power at fixed locations. In addition, a number of potential sites near these stations have been identified so that, before peak consumption times and network pressure, various MCS fleets are deployed in those locations temporarily. During low consumption periods, their energy storage resources are filled until the specified time, to assist the FCSs and be used as an alternative for them in a way. Now, in this study, in order to efficiently manage the fleet of MCSs whose goal is to allocate these facilities to different potential locations, one of the issues in the field of operations research and planning called the Location-or-Routing Problem (LoRP) is utilized. In the following, this problem will be explained in detail.

Location-or-Routing Problem

The Location-or-Routing Problem -introduced by Arslan 2021- is a type of operations research problem where the facility location and vehicle routing problems are integrated based on the concept of unified customer coverage. In this problem, selected facilities must cover demand points located in their vicinity, while the remaining demands can be serviced by vehicles with capacity (by directly visiting and serving the demands or transferring them to selected facilities) [28]. As the name suggests, each demand can only be addressed by one of the location or routing states because we face the limitation of maximum coverage for facilities in the location state and the limitation of maximum distance or time for routing in the routing state. Applications of this concept include problems such as locating schools along with bus routing or locating vaccination centers along with mobile vaccination routing during the occurrence of epidemics, with the goal of maximizing coverage or service provision. By utilizing Location-or-Routing models and algorithms, stakeholders in the mobile charging station network can efficiently allocate these facilities to potential locations and optimize the routing of MCSs to meet charging demands effectively, ensuring maximum coverage and efficient service delivery to customers.

Based on the discussions presented in this section and Fig 2, the description of the current problem is as follows: Considering the necessity of having support points for FCSs during peak load times on the power network, a type of Truck-based Mobile Charging Stations (TMCS) with high capacity are stationed at these points. These TMCSs respond to some of the demands in that area based on the coverage range characteristics, while other charging requests are served by Van-based Mobile Charging Stations (VMCS) with lower capacity through routing operations.



a. TMCS b. VMCS

Figure 2. Type of used Mobile Charging Stations

It is essential to note that fleet planning and deployment at these points occur during off-peak hours to avoid putting pressure on the energy storage systems. Since the capacity and budget available to energy supply systems to serve all demand points are limited, full coverage in this research topic does not occur (degrading/diminishing coverage), which results in incomplete fulfillment of all requested services due to the distance of facilities from demand points are gradually increasing. To calculate the coverage parameter,

$\Gamma_1(d_{ij}) = 0.5^{\sqrt{(d_{ik})^2+(d_{kj})^2}}$ as a continuous exponential function is employed. This function illustrates that as the distance increases, the costs of servicing demand points also increase significantly. Consequently, this has implications for the capacity and budget constraints of the energy supply systems. The limited capacity and budget inherently mean that not all demand points can be fully covered. Rather, the coverage is degrading and incomplete, reflecting that only a subset of the total demand can be fulfilled due to these constraints. Specifically, when distance increases, not only does the cost increase, but it also affects the feasibility of servicing those more distant demand points. As a result, we observe that the fulfillment of services diminishes as distance increases, leading to a situation where certain demands remain unmet.

For further clarity on this subject, the proposed schematic diagram of the problem is depicted in Fig 3.

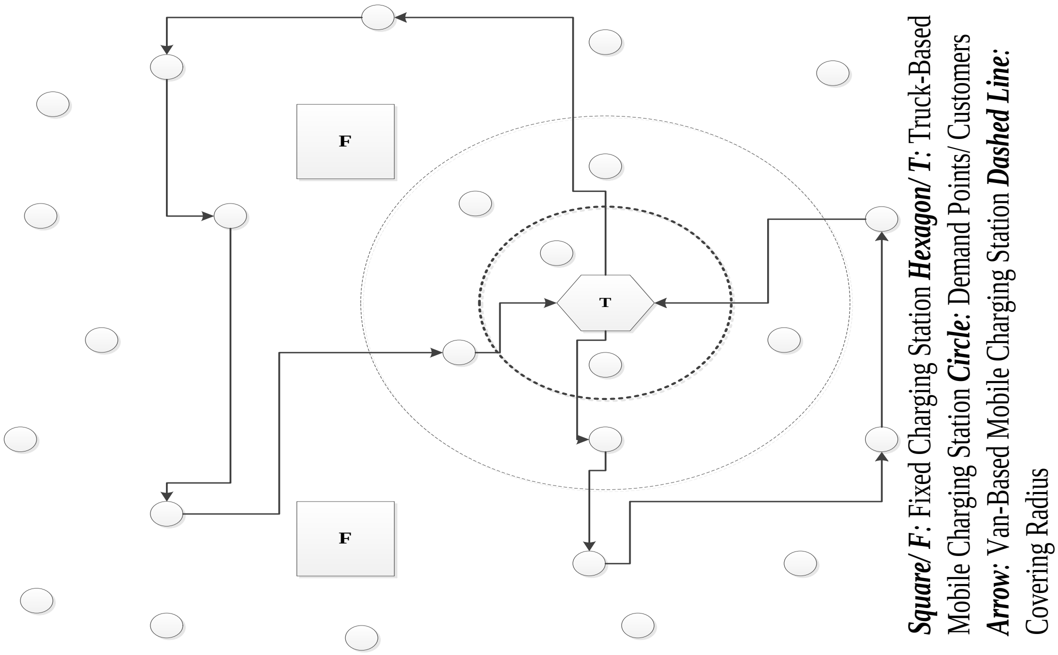


Figure 3. Schematic View of Proposed Network

Proposed Mathematical Model

In this section of the article, a mixed integer linear programming model is presented based on [29]. Initially, the indices, sets, parameters, and decision variables of the problem are defined in Table 2. Subsequently, the objective function and its constraints will be described.

Table 2. Indices, Sets, Parameters & Decision Variables

Indices	**Definition**
i, l	Index of potential points for implementation of TMCSs
r	Index of VMCSs in routing services
j, k	Index of EV demand charging points
m, n	Index of all nodes in the network
Sets	
I	Set of potential nodes for implementation TMCSs
R	Set of VMCSs
J	Set of charging demand points
N	Set of all nodes in the network
A	Set of arcs between nodes in the network

Parameters	
f_i	The implementation cost of *i*th TMCS for covering nodes
g_r	Operating cost of utilizing *r*th VMCS
c_{nm}	Traveling cost between nodes (*n*,*m*)
B	Total available budget
d_{mn}	Distance between nodes(*n*,*m*)
T	Maximum range or time for VMCS
Q	Maximum capacity of VMCS
q_j	Amount of *j*th point charging demand
$\Gamma(d_{ij})$	Gradual covering function
Variables	
v_{ij}	Integer Variable: quantity of the *j*th allocated demand points to the *i*th TMCS
x_i	Binary Variable: implementation of TMCS on the *i*th potential points
w_r	Binary Variable: deploying the *r*th VMCS
y_{nm}^r	Binary Variable: traveling the arcs (*n*,*m*) by the *r*th VMCS
z_{ij}^r	Binary Variable: if the *r*th VMCS starts from *i*th node to serve *j*th demand point
$q_j'^r$	Integer Variable: Amount of covered *j*th demand points by the *r*th VMCS
u_{mn}^r	Integer Variable: Total charging load of *r*th VMCS immediately following its visit to node *m* and its subsequent travel directly to node *n*.

Objective Function

$$Min: \quad M\sum_{i\in I}\sum_{j\in J} v_{ij}\,(1-\Gamma(d_{ij}))+\theta \tag{1}$$

Subject to:

$$\sum_{i\in I} f_i x_i + \sum_{r\in R} g_r w_r + \sum_{n\in N}\sum_{m\in N}\sum_{r\in R} c_{nm} y_{nm}^r \le \theta B \tag{2}$$

$$\sum_{n\in N} y_{nm}^r \le w_r \qquad ;\forall m\in N, r\in R \tag{3}$$

$$\sum_{m\in N} y_{mn}^r = \sum_{m\in N} y_{nm}^r \qquad ;\forall n\in N, r\in R \tag{4}$$

$$\sum_{(m,n)\in N} d_{mn} y_{mn}^r \le T \qquad ;\forall r\in R \tag{5}$$

$$z_{ij}^r \le x_i \qquad ;\forall i\in I, j\in J, r\in R \tag{6}$$

$$\sum_{i\in I} z_{ij}^r = \sum_{n\in N} y_{nj}^r \qquad ;\forall j\in J, r\in R \tag{7}$$

$$y_{ji}^r \le z_{ij}^r \qquad ;\forall i\in I, j\in J, r\in R \tag{8}$$

$$y_{ij}^r \le z_{ij}^r \qquad ;\forall i\in I, j\in J, r\in R \tag{9}$$

$$y_{jk}^r + z_{ij}^r + \sum_{i\in I: i\ne l} z_{lk}^r \le 2 \qquad ;\forall i\in I, k\in J: j\ne k, r\in R \tag{10}$$

$$\sum_{i\in I} v_{ij} \ge q_j - \sum_{i\in I}\sum_{r\in R} q_j'^r z_{ij}^r \qquad ;\forall j\in J \tag{11}$$

$$v_{ij} \le q_j x_i \qquad ;\forall i \in I, j \in J \tag{12}$$

$$q_j'^r \le \sum_{n \in N} q_j y_{nj}^r \qquad ;\forall j \in J, r \in R \tag{13}$$

$$\sum_{i \in I} \sum_{j \in J} y_{ij}^r \le 1 \qquad ;\forall r \in R \tag{14}$$

$$\sum_{n \in N} u_{nj}^r - \sum_{n \in N} u_{jn}^r = \sum_{n \in N} y_{nj}^r q_j'^r ;\forall j \in J, r \in R \tag{15}$$

$$u_{mn}^r \le Q y_{mn}^r \qquad ;\forall (m,n) \in N : m \ne n, r \in R \tag{16}$$

$$\sum_{j \in J} u_{ij}^r = \sum_{j \in J} z_{ij}^r q_j'^r \qquad ;\forall i \in I, r \in R \tag{17}$$

$$\sum_{j \in J} u_{ji}^r = 0 \qquad ;\forall i \in I, r \in R \tag{18}$$

$$u_{jn}^r \le (Q - q_j'^r) y_{jn}^r \qquad ;\forall n \in N, j \in J, r \in R \tag{19}$$

$$u_{nj}^r \ge q_j'^r y_{jn}^r \qquad ;\forall n \in N, j \in J, r \in R \tag{20}$$

$$x_i, w_r, z_{ij}^r, y_{mn}^r \in \{0,1\} \qquad ;\forall i \in I, j \in J, (m,n) \in N, r \in R \tag{21}$$

$$q_j'^r \in Z^+ \qquad ;\forall j \in J, r \in R \tag{22}$$

$$v_{ij} \ge 0 \qquad ;\forall i \in I, j \in J \tag{23}$$

$$u_{mn}^r \ge 0 \qquad ;\forall (m,n) \in N, r \in R \tag{24}$$

$$\theta \in [0,1] \tag{25}$$

The objective function 1 pertains to minimizing uncovered demand based on the feature of diminishing coverage and the percentage of budget spent. Moreover, by utilizing the coefficient *M*, greater priority is given to demand coverage compared to budget consumption. Eq. 2 enforces the adherence to the maximum fixed and variable budget incurred by various MCSs. Eq. 3 pertains to the establishment of routes dedicated to the selection of VMCSs. Eq. 4 expresses the equations of flow conservation, which this principle guarantees that for every node in the network, the quantity of flow entering the node is equal to the quantity exiting it, thereby maintaining a state of balance. Eq. 5 ensures compliance with the maximum allowable time or path length for each VMCS. Eq. 6 indicates that the use of VMCSs is contingent upon the establishment of facilities related to TMCSs. Eq. 7, indicating the connection between two relevant variables, states that if a demand point is visited by a vehicle (VMCS), that demand point is assigned to the origin of the vehicle (TMCS). Eq. 8-10 indicate that the path of each VMCS starts from a specific point and ends at the same point. Eq. 11 refers to calculating the amount of demand allocated from point *j* to the facility at point *i*. It should be noted that out of the allocated demands, only $\Gamma(d_{ij})$*100 of them are actually covered by the TMCSs (as calculated in the objective function). Eq. 12 demonstrates that demand points are allocated to a specific facility (TMCS) only if that facility has been established, and the allocation does not exceed the maximum demand of point *j*. In Eq. 13, the amount of covered demand for each point is determined based on the demand of that point and the VMCSs dispatched to it. Eq. 14 ensures that the service routes for VMCSs start from at most one specific facility. Eq. 15

determines the amount of demand covered for each demand point by the VMCSs. Eq. 16 indicates that the capacity of each VMCS must not be exceeded while moving between points. Eq. 17 and 18 specify the capacity of each VMCS at the beginning and end of its route. Eq. 19 and 20 define the range of the variables related to capacity. The Miller-Tucker-Zemlin (MTZ) constraints are employed to model vehicle capacity and ensure the execution of a single tour. To this end, the variable u_{mn}^{r} is introduced. This non-negative variable represents the total charging load of *r*th VMCS immediately following its visit to node *m* and its subsequent travel directly to node *n*. Thus, constraints 15-20 regulate the capacity limitations of VMCSs and prevent the formation of sub-tours. Eq. 21 to 25 also determine the domain of the variables in the current model.

It should be noted that V_{ij} defined as non-negative variables that denote the amount of demand at location *j* assigned to TMCS *i*. It is important to note that only $\Gamma(d_{ij})$*100 of the assigned users can be effectively served by the TMCS, as dictated by the degrade function. It is important to mention that the model does not necessarily mandate the provision of "service" to every demand point *j*; however, it does ensure that each point is allocated to a TMCS *i*. Specifically, if V_{ij} =1 and $\Gamma(d_{ij})$=0, this indicates that the respective demand point *j* is not covered by the TMCS *i*.

Based on the presented mathematical model, it is observed that Eq. 11, 15, 17, 19, 20 are non-linear. Therefore, in order to linearize the desired model, it is necessary to define variables $\alpha_{ij}^{r} = q_j'^{r} z_{ij}^{r}$, $\beta_{mn}^{r} = q_n'^{r} y_{mn}^{r}$ and reorganize the linear mathematical model as mentioned in Appendix 1.

Computational Results and Sensitivity Analysis

In this section of the paper, the computational results obtained from the proposed model are presented based on various parameters and characteristics, using different numerical examples. In this study, the numerical instances generated randomly by systematically varying the number of potential servicing and demand points to reflect a diverse range of operational scenarios. This involved selecting different configurations that represented realistic geographical distributions and charging demands. Also various fleet sizes and types is incorporated, thereby creating scenarios with heterogeneous fleet characteristics.

To elucidate the concept and applicability of the model, a numerical example has been conducted using the input data presented in Table 3. To enhance the clarity of the results and illustrate the model's functionality, a graph derived from these results is provided in Fig 4. Following the implementation of three TMCSs from a total of five, it is pertinent to note that in the graph, the colored arrows represent the VMCSs, with the numbers on the arrows indicating the load or remaining charge of the VMCSs after servicing the demand points. Additionally, the dashed arrows denote the volume of demands allocated to the TMCSs. It is important to emphasize that only $\Gamma(d_{ij})$*100 of the assigned users can be effectively served by the TMCSs.

Table 3. Input Data & Results for a Numerical Example

I05R03J10T110Q08			
Input Data			Result
$\lvert I \rvert = 5$	f_i = /#i 200/	Q = 8	OFV = **92.9**
$\lvert J \rvert = 10$	g_r = /#r 20/	T = 110	Uncovered Demand = **7.7 %**
$\lvert R \rvert = 3$	q_j = /#j 10/	B = 3000	Consumed Budget = **15.9 %**

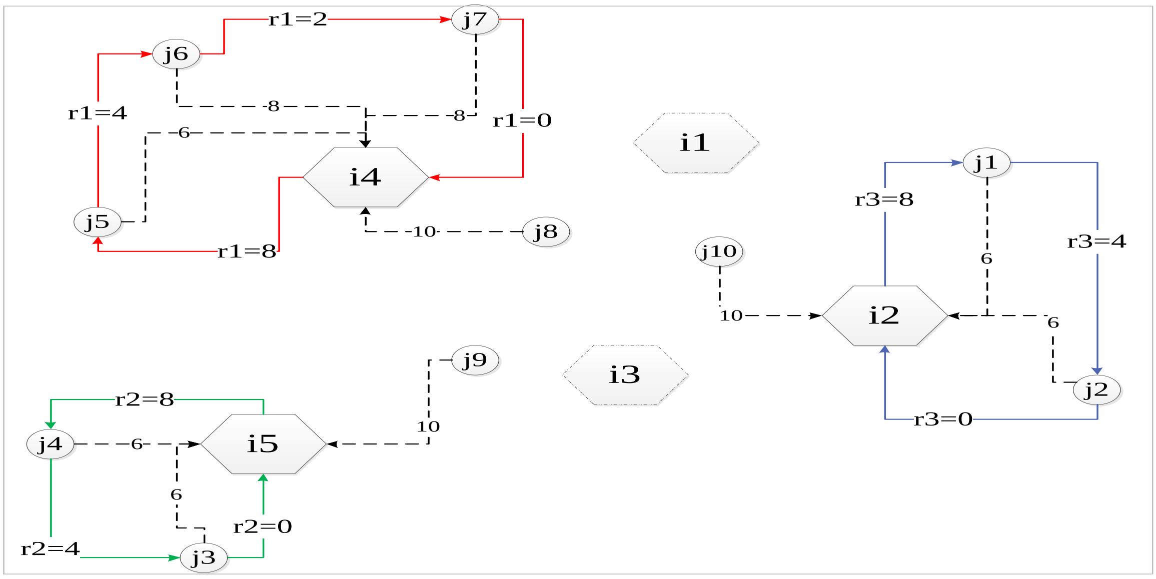


Figure 4. Schematic View of a Resolved Numerical Example

To solve these examples, a CPLEX solver embedded in the form of GAMS commercial software has been utilized. For this purpose, three network examples with varying dimensions in terms of the number of potential charging station locations as TMCSs, the number of VMCSs with different capacities, and the number of charging demand points have been drawn. It is important to note that the commercial exact solver is capable of addressing instances that involve up to 50 combinations of all nodes. The results of solving the various instances are presented in Table 4. Table 4 includes data on the number of facilities utilized (TMCS & VMCS), the percentage of uncovered demand, the ratio of budget consumed, and the solution time for each example. The data in the Table 4 demonstrate that despite the various points in the network and the allocated budget for service operation, the proposed model, after computing all possible scenarios, identifies the most suitable scenario that represents a balance between selecting the number of MCS facilities and the amount of budget expenditure for managing the electric vehicle charging network.

It is worth mentioning that all the results of these example scenarios have also been evaluated from another important perspective called coverage range, as evident in Table 4. It can be observed that with an increase in the coverage radius parameter for the locations of TMCSs, the primary goal of reducing uncovered demands continuously will be achieved. Additionally, as the coverage radius increases, the number of charging facilities utilized decreases, leading to a reduction in the budget consumption as well.

In Fig 5, one can also observe the impact of changes in the coverage radius parameter on the level of budget expenditure. This chart illustrates that with an increase in the coverage radius or service provision of facilities, overall budget consumption related to location cost (which includes the deployment and utilization of TMCSs) and routing cost (associated with deploying VMCSs and traversing routes) experiences a decreasing trend.

At the end, one of the most significant analyses involves comparing the results of the LoRP model with those of established classical operations research models, such as the Maximal Covering Location Problem (MCLP). The analysis of the Table 5, reveals distinct performance differences between the LoRP and MCLP models across short, mid, and long-range coverage scenarios.

In short-range coverage, LoRP implements 53.1% of facilities and achieves a demand coverage of 64.3%, while MCLP, with a higher facility implementation rate of 75.6%, only covers 58.5% of the demand. This demonstrates LoRP's efficiency in converting facility implementation into higher demand satisfaction. The trend continues in the mid-range category, where LoRP implements just 35.2% of facilities yet covers 76.2% of the demand, compared to MCLP's 55.7% implementation with only 64.5% coverage—highlighting LoRP's superior effectiveness in meeting demand with fewer resources. In long-range scenarios, the dichotomy becomes even more pronounced; LoRP achieves an impressive 93.7% demand coverage with a mere 22.4% of facilities implemented, whereas MCLP covers 81.9% of demand with 45.4% of facilities.

This consistent performance across all ranges underscores LoRP's strategic advantage in maximizing demand fulfillment relative to resource deployment, positioning it as a more resource-efficient model for electric vehicle charging networks and similar applications where operational efficiency is paramount. The schematic representation of these results is illustrated in Fig 6.

Table 4. Experimental Results

Covering Range	Instance	Implemented Facilities		Uncovered Demand	Consumed Budget	GAMS Solution Time
		TMCS	VMCS			
Short Range Covering	I05R03J10T090Q08	5	3	10.2%	19.5%	8.1s
	I05R03J10T090Q10	5	3	10.1%	19.3%	8.2s
	I08R06J20T090Q08	8	5	14.6%	22.9%	165.2s
	I08R06J20T090Q10	8	5	14.4%	21.9%	165.0s
	I10R08J30T090Q08	9	8	19.8%	31.9%	784.3s
	I10R08J30T090Q10	9	8	19.5%	30.7%	779.1s
Average	---	**7.33**	**5.33**	**14.83%**	**24.30%**	---
Mid Range Covering	I05R03J10T100Q08	4	3	8.4%	17.8%	8.2s
	I05R03J10T100Q10	4	3	8.1%	17.4%	8.7s
	I08R06J20T100Q08	6	5	11.1%	21.7%	174.2s
	I08R06J20T100Q10	6	5	11.2%	21.0%	177.5s
	I10R08J30T100Q08	8	7	17.3%	29.9%	815.6s
	I10R08J30T100Q10	8	7	16.9%	29.5%	817.9s
Average	---	**6.00**	**5.00**	**12.28%**	**22.75%**	---
Long Range Covering	I05R03J10T110Q08	3	3	7.7%	15.9%	10.3s
	I05R03J10T110Q10	3	2	7.3%	15.6%	10.4s
	I08R06J20T110Q08	5	4	10.1%	18.9%	187.2s
	I08R06J20T110Q10	5	4	10.2%	18.1%	181.1s
	I10R08J30T110Q08	7	7	15.1%	29.4%	891.5s
	I10R08J30T110Q10	7	7	15.0%	28.9%	8872s
Average	---	**5.00**	**4.50**	**10.97%**	**21.03%**	---

***I05R03J10T090Q08: I05:** 5 Potential nodes for implementation of TMCSs, **R03:** 3 VMCs, **J10:** 10 Demand points, **T110:** Maximum time or rout length is 110, **Q08:** Maximum capacity of VMCS is 8.

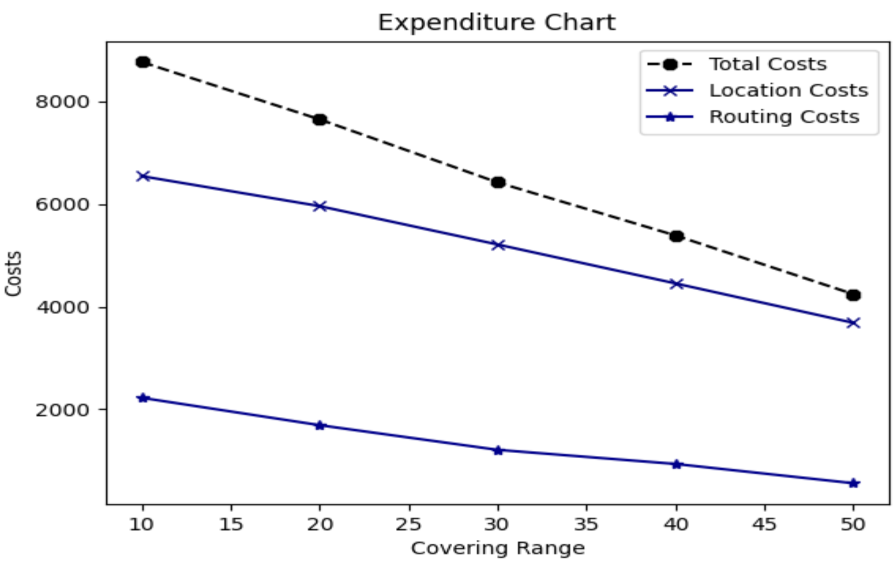


Figure 5. Expenditures Chart

Table 5. LoRP vs MCLP

Case	Implemented Facilities %		Covered Demands %	
	LoRP	**MCLP**	**LoRP**	**MCLP**
Short Range Covering	53.1	75.6	64.3	58.5
Mid Range Covering	35.2	55.7	76.2	64.5
Long Range Covering	22.4	45.4	93.7	81.9
Average	**36.9%**	**58.9%**	**78.1%**	**68.3%**

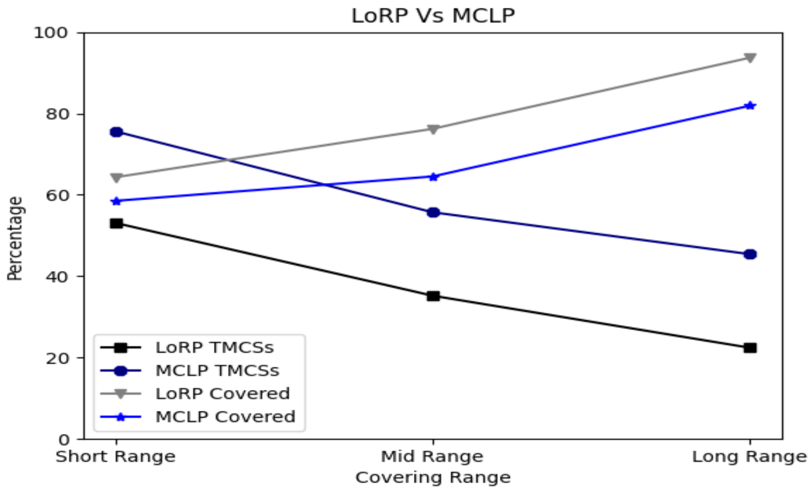


Figure 6. LoRP Vs. MCLP

Managerial Insights

Based on the proposed paper, here are four managerial insights presented as below. These insights emphasize the importance of strategic planning, collaboration, technology, and sustainability in effectively managing electric vehicle charging networks in the presence of mobile charging stations.

Strategic Network Design: Effective management of mobile charging stations requires careful planning of network design. Managers should leverage GIS and predictive analytics to identify high-demand areas, ensuring optimal placement of charging units to enhance user accessibility and satisfaction.

Cooperative Engagement Models: Establishing partnerships with local businesses and municipalities can strengthen cooperative servicing in EV charging networks. This collaborative approach can share operational costs and foster community support, promoting greater EV adoption.

Technology and Communication Infrastructure: A strong digital infrastructure is vital for the success of mobile charging solutions. Managers should invest in IoT technologies that enable real-time communication with users, streamlining payment processes and logistics to enhance overall user experience.

Sustainability and Environmental Impact: Managers must prioritize sustainability by adopting eco-friendly practices, such as utilizing renewable energy sources for charging stations. Communicating these initiatives can improve the organization’s CSR profile, enhancing brand reputation and customer loyalty.

Concluding Remarks & Future Studies

Given the increasing number of EVs on the roads and consequently the growing demand for recharging these vehicles, the need for planning and managing the electric power and vehicle charging network is more critical than ever. In order to assist this network considering the available solutions in this field, the use of mobile charging stations during peak energy consumption times has been proposed as a useful solution. Therefore, with the aim of managing the fleet of MCSs, this article presents a mathematical model of mixed-integer linear programming based on the Location-or-Routing Problem (LoRP). In this problem, two types of mobile charging stations -Truck-based and Van-based Mobile Charging Stations- are utilized for location and routing scenarios integrally. By strategically placing TMCS facilities at potential locations near public charging stations and considering the coverage radius of these facilities, a portion of the demand is served while the remaining demand is accommodated by the VMCSs using routing optimization. After presenting and solving the proposed model, an analysis of the problem and the impacts of various parameters such as coverage radius, vehicle capacity, and availability time were conducted. Due to the nature of the problem being time-dependent and traffic patterns constantly evolving, future research should also consider incorporating time-related aspects and variations into the proposed model. Other areas that require further investigation in future studies include the discussion of uncertainty in customer demand and unforeseen demand fluctuations. Given the expanding network and increasing number of locations and parameters, solving the mathematical model can be time-consuming; therefore, applying high-speed solving methods such as heuristic and metaheuristics specially on the large scale instances could be highly beneficial and efficient.

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Appendix 1.

Linearized mathematical model is as follow:

$$Min: \quad M\sum_{i\in I}\sum_{j\in J} v_{ij}\left(1-\Gamma(d_{ij})\right)+\theta \tag{1}$$

Subject to: (2)-(10), (12)-(14), (16), (18), (21)-(25)

$$\sum_{i\in I} v_{ij} \le q_j - \sum_{i\in I}\sum_{r\in R}\alpha_{ij}^{r} \qquad ;\forall\, j\in J \tag{26}$$

$$\sum_{n \in N} u_{nj}^{r} - \sum_{n \in N} u_{jn}^{r} = \sum_{n \in N} \beta_{nj}^{r} \qquad ;\forall j \in J, r \in R \tag{27}$$

$$\sum_{j \in J} u_{ij}^{r} = \sum_{j \in J} \alpha_{ij}^{r} \qquad ;\forall i \in I, r \in R \tag{28}$$

$$u_{jn}^{r} \leq Q y_{jn}^{r} - \beta_{nj}^{r} \qquad ;\forall n \in N, j \in J, r \in R \tag{29}$$

$$u_{nj}^{r} \geq \beta_{nj}^{r} \qquad ;\forall n \in N, j \in J, r \in R \tag{30}$$

$$\alpha_{ij}^{r} \leq q_j z_{ij}^{r} \qquad ;\forall i \in I, j \in J, r \in R \tag{31}$$

$$\alpha_{ij}^{r} \leq q_j'^{r} \qquad ;\forall i \in I, j \in J, r \in R \tag{32}$$

$$\alpha_{ij}^{r} \geq q_j'^{r} + q_j (z_{ij}^{r} - 1) \qquad ;\forall i \in I, j \in J, r \in R \tag{33}$$

$$\beta_{mn}^{r} \leq q_n y_{mn}^{r} \qquad ;\forall (m,n) \in N, r \in R \tag{34}$$

$$\beta_{mn}^{r} \leq q_n'^{r} \qquad ;\forall (m,n) \in N, r \in R \tag{35}$$

$$\beta_{mn}^{r} \geq q_n'^{r} + q_n (y_{mn}^{r} - 1) \qquad ;\forall (m,n) \in N, r \in R \tag{36}$$

$$\alpha_{ij}^{r} \geq 0 \qquad ;\forall i \in I, j \in J, r \in R \tag{37}$$

$$\beta_{mn}^{r} \geq 0 \qquad ;\forall (m,n) \in N, r \in R \tag{38}$$

while Eq. 31-38, are linearization inaqualities.

Graphical Abstract